Possible exam questions, SG2214 Fluid Mechanics

1. (10 p.) Define what is a stream line, path line and streak line. Under what special conditions do they coincide? Give a general description of a possible procedure to visualize these three types of lines in an experiment.

2. (10 p.) Given a field variable $F(t, \mathbf{x})$ in Eulerian coordinates, what is meant with the material time derivative? Give a physical description in words. How is it related to the partial time derivative of $F(t, \mathbf{x})$? Write down the components of the material time derivative of the velocity vector field given in Eulerian coordinates $u(t, \mathbf{x})$. Give a physical example which shows that the material time derivative and the partial time derivative generally are not the same.

3. (10 p.) State the Reynolds transport theorem. Use the Reynolds transport theorem to derive the equations for conservation of mass and momentum in a given fixed, otherwise arbitrary, fluid volume V. Give a physical interpretation(description) in words to each of the terms in the two equations equations derived. Make sure to define all introduced variables and parameters.

4. (10 p.) State the Reynolds transport theorem. Use the Reynolds transport theorem to derive the equation for conservation of energy in a given fixed, otherwise arbitrary, fluid volume V. Give a physical interpretation(description) in words to each of the terms in the equation. Make sure to define all introduced variables and parameters.

5. (10 p.) Given the conservation equations of mass and momentum in a fixed, otherwise arbitrary, fluid volume V,

$$\frac{d}{dt}\int_V \rho dV + \oint_S \rho\, u_k n_k\, dS = 0,$$

and

$$\frac{d}{dt}\int_V \rho u_i dV + \oint_S \rho u_i (u_k n_k) dS = \int_V \rho g_i dV + \oint_S \left(-p\delta_{ik} + \tau_{ik}\right) n_k dS,$$

derive the differential forms of these equations in a Newtonian fluid (also, you need to define what is a Newtonian fluid). Derive also simplified versions of the derived equations valid for an incompressible fluid. Give a physical interpretation(description) in words to each of the terms in the equations derived.

6. (10 p.) Given the conservation equation of energy in a fixed, otherwise arbitrary, fluid volume,

$$\frac{d}{dt}\int_V \rho(e+\frac{1}{2}u_i u_i) dV + \oint_S \rho(e+\frac{1}{2}u_i u_i)(u_k n_k) dS = \int_V \rho g_i u_i dV + \oint_S u_i \left(-p\delta_{ik} + \tau_{ik}\right) n_k dS - \oint_S q_i n_i dS,$$

derive the differential form of this equation in a Newtonian compressible fluid with heat conduction according to Fourier's law(also, you need to define what is a Newtonian fluid and to state Fourier's law). Give a physical interpretation(description) in words to each of the terms in the equation derived.

7. (10 p.) Define the so called dissipation function of the energy equation and show, using the constitutive relation for a Newtonian fluid, that its value cannot be negative, not even for a compressible fluid.

8. (10 p.) State the boundary layer equations for incompressible flow over a solid, weakly curved boundary of a Newtonian fluid. What approximations are done compared to the full Navier-Stokes equations and under what conditions are these approximations valid.

9. (10 p.) What is described by the Blausius equation for boundary layers? Given the general boundary layer equations,

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho U_e\frac{dU_e}{dx}+\mu\frac{\partial^2 u}{\partial y^2},\quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$$

derive the Blausius equation.

10. (10 p.) The so called Blausius equation is given by

$$f'''+\frac{1}{2}ff'=0$$

What is described by this equation? What boundary conditions are used to obtain a solution? Given the solution to the Blausius equation, $f(\eta)$, obtain the "skin friction" and "drag".

11. (10 p.) Define the concepts of displacement thickness and momentum thickness and show how they are related to the conservation equations of mass and momentum of a control volume for the flow over a flat plate.

12. (10 p.) Give the definition for the point of separation of a boundary layer. Make a sketch of the velocity profile at the point of separation, and just before and after the point of separation. Give a physical explanation as to why the boundary layer separates. In this regards, what is the typical difference between a laminar and a turbulent boundary layer? How can separation of the boundary layer affect the drag on a body?

13. (10 p.) Derive Kelvin's circulation theorem and state under what conditions it is valid. Also, explain the consequences of Kelvin's theorem if the fluid is initially irrotational everywhere.

14. (10 p.) Derive the vorticity equation from the Navier-Stokes' equations using tensor notation. With reference to the derived equation, identify sources for generation of vorticity in a fluid volume.

15. (10 p.) Derive the unsteady form of Bernoulli's equation from the Navier-Stokes' equations and state when it is valid.

16. (10 p.) State the Cauchy-Riemann equations for the velocity potential and stream function of a two-dimensional, incompressible flow. Show that if these equations are fulfilled the velocity potential and stream function both satisfy the Laplace equation. Also, show that iso-levels curves of these functions are orthogonal.

17. (10 p.) What is meant with the Kutta condition for irrotational flow? Give a physical description of why a real flow should satisfy the Kutta condition.

18. (10 p.) Specify five typical properties of turbulent flow. State what is the Reynolds decomposition. Define the Reynolds stress tensor and the average turbulent kinetic energy.

19. (10 p.) Derive the Reynolds averaged Navier-Stokes' equations for turbulent flow.

20. (10 p.) Define the averaged turbulent Reynolds shear stresses. Give a clear physical description of their origin. Define the averaged turbulent heat flux density. Give a physical description of its origin.