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Dynamics of three-dimensional turbulent wall plumes and implications for estimates of submarine glacier melting

Ekaterina Ezhova*

Institute for Atmospheric and Earth System Research/Physics, Faculty of Science, University of Helsinki, Finland.

Claudia Cenedese

Physical Oceanography Department, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts, USA

Luca Brandt

Linné FLOW Centre and SeRC (Swedish e-Science Research Centre), KTH Mechanics, Stockholm, Sweden

**Corresponding author address:* Ekaterina Ezhova

E-mail: ekaterina.ezhova@helsinki.fi

ABSTRACT

Subglacial discharges have been observed to generate buoyant plumes along the ice face of Greenland tidewater glaciers. These plumes have been traditionally modelled using classical plume theory and their characteristics parameters, i.e. velocity, are employed in the widely used three-equation melt parametrization. However, the applicability of plume theory for three-dimensional turbulent wall plumes is questionable due to the complex near-wall plume dynamics. In this study, corrections to the classical plume theory are introduced to account for the presence of a wall. In particular, the drag and entrainment coefficients are quantified for a three-dimensional turbulent wall plume using data from direct numerical simulations. The drag coefficient is found to be an order of magnitude larger than that for a boundary layer flow over a flat plate at a similar Reynolds number. This suggests a significant increase in the melting estimates by the current parametrization. However, the volume flux in a wall plume is found to be half that of a conical plume which has double the buoyancy flux. This suggests that the total entrainment (per unit area) of ambient water is the same and that the plume scalar characteristics, i.e. temperature and salinity, can be predicted reasonably well using classical plume theory.

1. Introduction

Subglacial discharge is among the major factors controlling submarine melting of Greenland's tidewater glaciers (Straneo and Cenedese 2015). Turbulent plumes generated by fresh water at the freezing temperature discharged at the glacier base enhance melting of the ice face. In Greenland the ice tongue has broken off in most tidewater glaciers and the ice face is quasi vertical, thus, subglacial discharge plumes are usually modelled as a turbulent buoyant plume propagating along a vertical ice face (Straneo and Cenedese 2015).

Current ice-ocean models quantify melting employing the three-equation formulation by Holland and Jenkins (1999), where the effect of plume turbulence is parametrized through the friction velocity u_*, a fundamental parameter defining wall-bounded turbulence, and the melting rate is assumed to be proportional to u_*. To estimate the friction velocity from the mean velocity profile a drag coefficient is typically used

$$C_d = \frac{\tau}{\rho U_{ref}^2} = \frac{u_*^2}{U_{ref}^2}, \tag{1}$$

where $\tau = \rho < u_i' u_j' >$ is the mean turbulent stress parallel to the ice face, U_{ref} the reference velocity, and ρ the water density. The drag coefficient is usually taken to be of the order $C_d \sim 0.001$, a value close to that of a turbulent boundary layer flow over a flat plate at high Reynolds numbers (e.g. Monin et al. 1971). However, a three-dimensional wall plume can be expected to exhibit strong lateral spreading similarly to what reported for the more extensively investigated wall jets (Launder and Rodi 1983). This effect is attributed by these authors to the secondary flows in the jet as well as to the gradients of turbulent stresses, the latter being more important (Craft and Launder 2001). Assuming the turbulent stresses to be similar to those in the two-dimensional boundary layer flow over a flat plate is thus not justified. A recent study (Slater et al. 2016) used a larger value of $C_d = 0.01$ following Jenkins et al. (2010) who found that this larger value of C_d

was necessary to predict the observed melt rates of an ice shelf in Antarctica. In addition, current estimates of entrainment in wall plumes are based on experimental data and theoretical models for conical free plumes (Cowton et al. 2015; Slater et al. 2016; Mankoff et al. 2016).

The main focus of this study is to compare the modification of the classical plume theory for a three-dimensional turbulent wall plume against Direct Numerical Simulations (DNS) and to quantify the drag and entrainment coefficients consistent with the theory using data from DNS and existing experiments. An appropriate drag coefficient is obtained by applying the modified plume theory to our simulations and for this we use an analytical solution which, to our knowledge, is novel for 3D flows (2D analogues are reported by Gayen et al. 2016). As a first step, we consider a turbulent plume along a vertical wall without the meltwater feedback, i.e. we assume that the wall is neither a source of mass nor of buoyancy.

2. Wall plume theory

Following Cowton et al. (2015) we consider the wall plume as half of a conical plume and assume that it can be described by the classical system of equations suggested by Morton et al. (1956) (this approach will be justified later by means of DNS). This theory is referred hereafter as a modified MTT theory: the conservation equations for volume $\tilde{Q}$, momentum $\tilde{M}$ and buoyancy $\tilde{F}$ fluxes, are written, following Cowton et al. (2015) and Slater et al. (2016), as

$$\frac{d\tilde{Q}}{d\tilde{z}} = \frac{d}{d\tilde{z}}(\pi\tilde{b}^2\tilde{u}/2) = \pi\alpha\tilde{b}\tilde{u}, \tag{2}$$

$$\frac{d\tilde{M}}{d\tilde{z}} = \frac{d}{d\tilde{z}}(\pi\tilde{b}^2\tilde{u}^2/2) = \pi\tilde{b}^2 g'/2 - 2C_d\tilde{b}\tilde{u}^2, \tag{3}$$

$$\frac{d\tilde{F}}{d\tilde{z}} = \frac{d}{d\tilde{z}}(\pi\tilde{b}^2\tilde{u}g'/2) = 0. \tag{4}$$

In the above, $\tilde{b}$ is the dimensional plume radius, $\tilde{u}$ the dimensional plume velocity (assuming a top-hat velocity profile), $g' = g\Delta\tilde{\rho}/\tilde{\rho}_0$ the reduced gravity, C_d the drag coefficient and α the entrainment coefficient. The latter is defined as $\tilde{u}_e = \alpha\tilde{u}$, where $\tilde{u}_e$ is the entrainment velocity. Note that, due to the presence of a wall, α is not necessarily equal to that for a conical plume. Moreover, to account for a possible asymmetry in the plume shape, we introduce an 'equivalent' radius $\tilde{b}$ (to be defined in terms of momentum and volume fluxes).

The system (2)-(4) is non-dimensionalized introducing the following variables $Q = \tilde{Q}/\left(\tilde{b}_0^2\tilde{u}_0\right)$, $M = \tilde{M}/\left(\tilde{b}_0^2\tilde{u}_0^2\right)$, $F = \tilde{F}Fr_0^2/\left(\tilde{b}_0\tilde{u}_0^3\right)$ and $z = \tilde{z}/\tilde{b}_0$, where $Fr_0 = \tilde{u}_0/\sqrt{g_0'\tilde{b}_0}$ is the source Froude number and the subscript '0' indicates values at the source. Equations (2)-(3) can be rewritten as

$$\frac{dQ}{dz} = \sqrt{2\pi}\alpha M^{1/2}, \tag{5}$$

$$\frac{dM}{dz} = \frac{F_0 Q}{Fr_0^2 M} - \sqrt{8/\pi} C_d M^{3/2} Q^{-1}. \tag{6}$$

When neglecting the effect of a wall on the plume dynamics (hereafter 'free' plume), the drag term in (6) is assumed to be zero and

$$\frac{dM}{dQ} = \frac{F_0 Q}{\sqrt{2\pi}\alpha Fr_0^2 M^{3/2}}, \tag{7}$$

which has the following analytical solution

$$M^{5/2} = M_0^{5/2} + \frac{5F_0(Q^2 - Q_0^2)}{4\sqrt{2\pi}\alpha Fr_0^2}. \tag{8}$$

However, in the presence of a wall the drag term in (6) should be considered, which leads to

$$\frac{dM}{dQ} = \frac{F_0 Q}{\sqrt{2\pi}\alpha_w Fr_0^2 M^{3/2}} - \frac{2}{\pi\alpha_w} C_d \frac{M}{Q}. \tag{9}$$

The solution therefore becomes (see supplementary material for derivation details)

$$M^{5/2} = \frac{5F_0 Q^2}{4\sqrt{2\pi}\alpha_w Fr_0^2 (1 + \dfrac{5C_d}{2\pi\alpha_w})} + \left(M_0^{5/2} - \frac{5F_0 Q_0^2}{4\sqrt{2\pi}\alpha_w Fr_0^2 (1 + \dfrac{5C_D}{2\pi\alpha_w})} \right) \left(\frac{Q_0}{Q}\right)^{\dfrac{5C_d}{\pi\alpha_w}}, \tag{10}$$

where α_w is the entrainment coefficient in the presence of a wall.

The first term on the r.h.s. of (10) grows with Q, while the second decreases; thus, for $Q >> Q_0$, i.e. sufficiently far from the source, the second term on the r.h.s. of (10) can be neglected. Thus, in the far field (i.e. for $M >> M_0$ and $Q >> Q_0$), the ratio $M^{5/2}/Q^2$ is constant for both the free (8) and wall (10) plume, and the drag and turbulent entrainment coefficients define the difference between these two cases. Since C_d is taken to be small in current models (Cowton et al. 2015; Slater et al. 2016), the wall plume is assumed to behave as a half-conical free plume. This, however, should be treated with caution. We show in what follows that the drag coefficient is an order of magnitude larger than it can be expected when compared to the boundary layer flow over a flat plate.

The entrainment coefficient for a free plume can be obtained from the MTT theory, $b = 6/5\alpha z$, if one knows the evolution of the plume radius with the distance from the source. The far-field asymptotic solutions for the wall plume radius and velocity can be obtained substituting the first term on the r.h.s. of (10) in (5) and combining the solution with the definitions of the volume and momentum fluxes:

$$b_w = 6/5\alpha_w z, \tag{11}$$

$$u_w = \left(\frac{\alpha}{\alpha_w}\right)^{2/3} \frac{u}{\left(1 + \dfrac{5C_d}{2\pi\alpha_w}\right)^{1/3}}, \tag{12}$$

where $u = \left(\dfrac{5F_0}{4\pi\alpha Fr_0^2}\right)^{1/3} \left(\dfrac{6\alpha z}{5}\right)^{-1/3}$ is the classical MTT self-similar solution for a conical plume in a homogeneous fluid and the subscript 'w' indicates wall plume properties.

In what follow, we quantify the entrainment and drag coefficients using data from DNS. In particular, we use the radius dependence on the distance from the source to define the entrainment coefficients for free and wall plumes, and then quantify the drag coefficient based on the far-field solutions of (8) and (10).

3. Results

Two simulations of a turbulent vertical lazy plume in a homogeneous fluid were performed: one conical plume and one wall plume. The conical plume is generated by a source volume flux $2\tilde{Q}_0$ exiting from a round source of radius $\tilde{b}_0$. The source Froude number of the plume is $Fr_0 = \frac{\tilde{u}_0}{\sqrt{g_0'\tilde{b}_0}} = 0.66$ and the Reynolds number $Re_0 = \frac{\tilde{u}_0\tilde{b}_0}{\nu} = 1000$, where ν is the kinematic viscosity. The Froude number chosen here corresponds to that of a lazy plume, typical of those generated by a subglacial discharge. Note that a lazy plume gains velocity near the source due to its buoyancy (e.g. Fischer et al. 1979), and for the Froude number used here the equivalent top-hat velocity near the source becomes approximately twice as large as the source velocity, and consequently, the effective Reynolds number near the source also increases nearly twice. The wall plume is generated from a half-round source of radius $\tilde{b}_0$ attached to a wall with a total discharge $\tilde{Q}_0$ (see visualization in Fig. 1a).

The DNS has been performed using the spectral element code Nek5000 (Fischer et al. 2008). We consider an incompressible fluid with buoyancy modelled by the Boussinesq approximation. A cylindrical domain is used to simulate the conical plume, whereas a half-cylinder is used for the wall plume, with an increased resolution close to the wall. The domain radius is $10\tilde{b}_0$, while the vertical length is $29\tilde{b}_0$. The resolution is less than $0.01\tilde{b}_0$ near the wall (or in terms of inner scaling $\Delta x \leq 0.8x_\nu$, where $x_\nu = (Re_0 \cdot u_*)^{-1}$ is the viscous length scale, except in a small domain in the vicinity of the symmetry axis where the resolution is $\Delta x \approx x_\nu$) and close to $0.01\tilde{b}_0$ in the plume, thus, we resolve the viscous sublayer as well as the plume up to the Kolmogorov scale of this flow, estimated as $\tilde{b}_0/Re^{3/4} \sim 0.01\tilde{b}_0$. The total number of nodes is about 29 million for the wall plume and 46 million for the conical plume. We use the open (zero-gradient) boundary conditions for the vertical velocity and density, combined with a sponge layer for the density fluctuations and horizontal velocity at the top outflow boundary, open boundary conditions for all variables and a sponge layer for the density fluctuations on the open domain sides (cf. Ezhova et al. 2017). Finally, we set zero velocity and zero buoyancy flux at the wall.

a. Comparison between wall plume theory and DNS results. Estimates of drag and entrainment coefficients using the wall plume theory.

The DNS results show that a wall plume indeed behaves similarly to a wall jet, being wider in the direction parallel to the wall and narrower perpendicular to the wall, as illustrated by Figs. 1b, c.

The volume and momentum fluxes are computed at horizontal cross-sections at each vertical z-level as $Q = \iint U dxdy$ and $M = \iint U^2 dxdy$. The non-dimensional mean vertical velocity U is an average over 50 non-dimensional time units (the eddy turnover time near the top boundary is approximately 2 time units for the wall plume and 1.7 for the conical plume, where time is non-dimensionalized using $t = \tilde{b}_0/\tilde{u}_0$). The values for a half-conical free plume are obtained by dividing by 2 the values from a conical plume.

The volume flux of the wall plume is almost identical to half of the volume flux pertaining the conical plume, whereas away from the source due to the wall friction the momentum flux of the wall plume is reduced by $\approx 15\%$ when compared to that of the free plume due to the wall friction (Fig. 2). A similar result, i. e. same volume fluxes and significant reduction of momentum flux in the presence of a wall, has been reported for three-dimensional turbulent wall jets by Namgyal and Hall (2016). In agreement with the modified MTT theory solutions for wall plumes (dashed lines in Fig. 2), the volume and momentum fluxes increase with distance from the source as $Q \sim z^{4/3}$ and $M \sim z^{5/3}$ (see supplementary material for a detailed derivation).

The 'equivalent' plume radius is calculated at each vertical z-level as $b = \sqrt{\frac{2Q^2}{\pi M}}$ and using the relationship $b = 6/5\alpha z$ we determine the entrainment coefficient α for the two cases considered (Fig. 3). The entrainment coefficient pertaining the wall plume is slightly larger than that for the conical plume, $\alpha_w = 0.110$ and $\alpha = 0.102$, respectively. Using the entrainment coefficients, we therefore proceed with the estimate of the drag coefficient by means of (8) and (10). As discussed above, we neglect the second term on the r.h.s. of (10) and use the far-field formulations of (8) and (10) to obtain the ratio $M^{5/2}/Q^2$ for both free and wall plumes, which is given by the slope of the two curves in Fig. 3, right panel. The ratio of these two slopes, $(1 + \frac{5C_d}{2\pi\alpha_w})\frac{\alpha_w}{\alpha} = 15.2/9.6$, gives a value of the drag coefficient $C_d \approx 0.065$, which is an order of magnitude larger than that for a boundary layer flow over a flat plate at a similar Reynolds number.

The most striking result of the simulations, which was not expected given the complex dynamics of the wall plume, is the similarity of the volume fluxes for a wall plume and half a conical plume (Fig. 2, left panel). In light of the latest works on jet and plume turbulence (e.g. Burridge et al. 2016), one may speculate that the turbulent structures defining the entrainment in a wall plume remain similar to those in a conical plume, while only the shape of the plume 'boundary' changes. To support this hypothesis, the maximum velocities in the free and wall plumes are similar, and the geometric scales of the fluctuations of the plume 'boundaries' are similar (Fig. 4). However, the wall acts to reduce the average velocity in the wall plume as compared to the conical plume (Fig. 2) and, given the similarity of volume fluxes, the 'equivalent' plume radius at any given height must be larger for a wall plume (Fig. 3, left panel). Finally, given $b = 6/5\alpha z$, the latter produces an increase in entrainment coefficient for a wall plume.

b. Estimates of the drag coefficient for a wall plume using the measured velocity profiles.

To support the finding that the drag coefficient for a wall plume is an order of magnitude larger than than that for a boundary layer flow over a flat plate, we estimated the drag coefficient from the mean velocity profiles at two different z cross-sections, $z = 15$ and $z = 18$.

We fitted the velocity profiles in the vicinity of the wall with a linear function to get the slope defining the turbulent stresses (or friction velocity). The fitting function is $U = x(Re_0 \cdot u_*^2)$, corresponding to the inner scaling in the viscous sublayer. Then it is straightforward to calculate the viscous scale $x_v = (Re_0 \cdot u_*)^{-1}$. Fig. 5 displays the velocity profiles in the inner coordinates $x_+ = x/x_v$ and $U_+ = U/u_*$ at fixed y-coordinate and in the cross-sections $z = 15$ and $z = 18$. Note, that u_* and x_v are different for the profiles at different fixed y-coordinates. We also show the $(U_+ = x_+)$ dependence, characteristic of the viscous sublayer, and the classical log-law dependence $(U_+ = \ln(x_+)/0.41 + 5)$.

As can be seen, all the velocity profiles follow the dependence typical of a viscous sublayer up to $x_+ \approx 5$, in agreemant with other studies on turbulent boundary layers (e.g. Monin et al. 1971). However, further from the wall all the velocity profiles are lower than the classical log-law dependence. Note that even for the simpler case of a plane wall jet there is a discrepancy in log-law constants in different studies (e.g. Banyassady and Piomelli 2015), not all studies report the classical values for the parameters $\kappa = 0.41$ and $B = 5$. We are not aware of any studies comparing the log-law dependence with the velocity profiles in 3D plumes or jets. However, the boundary layer structure of a 3D plume is more complicated when compared to that of a 2D flow. The maximum of the wall-parallel velocity in each cross-section $y = const$ moves further away from the wall as the flow propagates in the z-direction, and also in each cross-section $z = const$ as the plume spreads horizontally, at $|y| > 0$. Similar behaviour is reported by Namgyal and Hall (2016) for a 3D wall jet. This can be considered as a smooth detachment of the flow from the wall and, in analogy with the separating (Falkner-Skan) boundary layer, might be the reason for the lower mean wall-parallel velocity in the log-law zone as compared to the classical boundary layer flow.

Table 1 summarizes the drag coefficients based on the maximum vertical velocity for each profile, $C_{dm} = (u_*/U_{\max})^2 = 0.008 - 0.024$. Further, we estimate the drag coefficients, $C_d = (u_*/u)^2$, for all the profiles based on the cross-sectional average vertical velocity, as used in the modified MTT theory. The cross-sectional average vertical velocity, defined as $u = M/Q$, yields $u_{15} = 1.45$ and $u_{18} = 1.39$ in the cross-sections at $z = 15$ and $z = 18$, respectively. The drag coefficient C_d can be estimated from the friction velocity as $\int u_*^2 dy = 2C_d bu^2$. Introducing the local drag coefficient for each cross-section, $C_{d,loc}(y) = (u_*(y)/u)^2$, one can obtain $\int C_{d,loc}(y)dy = 2C_d b$. We have values of $C_{d,loc}(y)$ in 7 y-cross-sections (the drag coefficient is calculated at $y = 0,1,2,3$ and due to symmetry $C_{d,loc}(-y) = C_{d,loc}(y)$). Therefore, with the distance $\Delta y = 1$ between the different cross-sections, one can get an estimate for the integral: $\int C_{d,loc}(y)dy \approx \sum C_{d,i}\Delta y = (7\Delta y)(\sum C_{d,i}/7) = C_{d,avg} \cdot 2B$, where $2B = 7\Delta y$ and $C_{d,avg}$ is the average value of the local drag coefficient (see Table 1 for $C_{d,i}$). Thus, $C_d = C_{d,avg} \cdot (B/b) \approx 0.04$ both for $z = 15$ and $z = 18$, which is lower, but still of the same order as the results obtained in the subsection 3a using the modified MTT equations. It is important to note that the value of the drag coefficient depends on the choice of the reference velocity U_{ref}, as follows from its definition, i.e. eq (1). Using the maximum and average vertical velocity as the reference velocity in the calculation above lead to differences in the drag coefficient of a factor of 3, a significant difference comparable to that obtained when changing the Reynolds number by 3-4 orders of magnitude. Hence, the choice of the drag coefficient should be consistent with the choice of the reference velocity when employing the MTT equations to obtain the subglacial discharge plume vertical velocity used in the melt parameterization.

c. Estimates of the drag coefficient for a wall jet.

In this subsection we estimate the drag coefficient using the experimental data obtained for a three-dimensional wall jet by Namgyal and Hall (2016). The drag is defined by the turbulent shear stresses, which have been observed to be similar for conical jets and plumes (van Reeuwijk et al. 2016), thus one could expect similar results for wall jets and plumes. These estimates can be used to test the sensitivity of the results to the Reynolds number, which in the experiment is $Re = 250000$, i.e. two orders of magnitude larger than in the DNS discussed in this section.

The solution of equation (9) for a turbulent jet is

$$M = M_0 \left(\frac{Q_0}{Q} \right)^{\frac{2C_d}{\pi \alpha_{wj}}}, \tag{13}$$

where α_{wj} is the wall jet entrainment coefficient. The above expression gives the momentum flux evolution with distance from the source:

$$M = M_0 \left(\sqrt{2\pi}\alpha_{wj}(1 + \frac{C_d}{\pi\alpha_{wj}}) M_0^{1/2} Q_0^{-1} z \right)^{-\frac{2C_d}{\pi\alpha_{wj}} \frac{1}{(1+\frac{C_d}{\pi\alpha_{wj}})}}. \tag{14}$$

Opposite to the wall plume results, the evolution of the wall jet 'equivalent' radius involves a dependence on the drag coefficient: $b_{wj} = \sqrt{\frac{2Q^2}{\pi M}} = 2\alpha_{wj}\left(1 + \frac{C_d}{\pi\alpha_{wj}}\right) z$. The 'equivalent' radius and momentum flux of the wall jet from the experiment of Namgyal and Hall (2016) are shown in Fig. 6. A best fit of the data in the far field allows to determine the entrainment coefficient $\alpha_{wj} \approx 0.052$, reduced when compared to the typical entrainment coefficient for a round jet found in the literature, $0.065 < \alpha_j < 0.082$ (e.g. Fischer et al. 1979), and the drag coefficient $C_d \approx 0.032$, larger than for a flat-plate boundary layer. The difference in C_d for the wall plume ($C_d = 0.065$) and jet ($C_d = 0.032$) can be related to the difference in Reynolds number between the simulations ($Re = 1000 - 2000$) and the experiments ($Re = 250000$) and probably to near-wall buoyancy effects, absent in the case of wall jets.

d. Implications of the results for the estimates of submarine glacier melt rates

The drag coefficient obtained in the present study is 6.5 times larger than the value used by Slater et al. (2016), $C_d = 0.01$, much higher than that used by Cowton et al. (2015), $C_d = 0.0025$, and in general an order of magnitude larger than that for a boundary layer flow over a flat plate. A large drag coefficient is expected given the relatively low Reynolds numbers, however the difference is too large to be explained exclusively by the effect of the Reynolds number. The well-known von Karman law for the boundary layer flow over a flat plate is $\frac{1}{\sqrt{c_f}} = \frac{1}{\kappa\sqrt{2}}(\ln(Re_z\sqrt{c_f}) + B_5)$, where $B_5 = 1.7$, $Re_z = Uz/\nu$ and $c_f = 2C_d$ (Monin et al. 1971). If, for example, we take the cross-section at $z = 15$ in the region with developed turbulence, the mean vertical plume velocity increases by a factor 1.5 from its initial value, and we obtain $Re_z \approx 20000$. For this Reynolds number the drag coefficient obtained from the above von Karman law for a flat plate is approximately $C_d = 0.005$, an order of magnitude lower than what we obtain in the simulations. Hence, the simulations results and the reasoning above suggest that also for larger Re we should expect an increased drag coefficient for a wall plume. This increase in C_d is a critical factor in the current parametrization for submarine melting.

The present study suggests that $C_d = 0.001$ is an inappropriate estimate of the drag coefficient when using the modified MTT model with a top hat velocity profile. The drag decrease with increasing Reynolds number can be expected to be similar to that following from the von Karman law and reliably quantified for the boundary layer flow over a flat plate (e.g. Monin et al. 1971). The von Karman law suggests a 4-5 times decrease of the drag coefficient from the low ($Re \sim 10^4$) to high ($Re \sim 10^9$) Reynolds numbers, thus, the value of $C_d = 0.065$ obtained for $Re_z = 20000$ corresponds to a value $C_d = 0.01 - 0.02$ for the large Reynolds numbers, relevant to geophysical flows. This is in agreement with the value 0.01 used by Slater et al. (2016). It is worth noting, that the lower value of the drag coefficient due to a larger *Re* obtained for a wall jet in subsection 3c is also consistent with that predicted by the von Karman law. In addition, given that some important phenomena, such as sediment load within the subglacial discharge plumes and glacier surface roughness, are not considered in our study, the drag coefficient relevant to geophysical flows is likely larger than 0.01-0.02.

We finally discuss the implications of the larger value of the drag coefficient obtained using the modified MTT theory, which is often implemented to calculate the subglacial discharge plume velocity used in the melt rate parametrizations. From the three-equation melt formulation (Holland and Jenkins 1999) the melt rate is proportional to the friction velocity, or using eq. (1) to the mean vertical plume velocity: $\dot{m} \sim uC_d^{1/2}$, where $C_d = 0.01$ is used by Slater et al. (2016) (0.0025 by Cowton et al., 2015) and u can be obtained from MTT theory neglecting the wall effects. Within the same framework, improved with eq. (12) to account for the presence of the wall, the melting rate can be written as

$$\dot{m}_w \sim u \left(\frac{\alpha}{\alpha_w} \right)^{2/3} \frac{C_d^{1/2}}{(1 + \frac{5C_d}{2\pi\alpha_w})^{1/3}},$$

with the velocity u from MTT theory without a wall. This dependence of the melt rate on the drag coefficient is illustrated in Fig.7, with the melt rate normalised with that obtained with the frequently used drag coefficient $C_d = 0.0025$. Thus, the estimate of melt rate for $C_d = 0.01 - 0.02$ is more than twice that obtained using $C_d = 0.0025$. Moreover, from van Kessel and Kranenburg (1996), it follows that up to a three times increase in the drag coefficient can be expected when sediments are present in the flow. Thus the melt rate can grow further to yield as much as $\sim 4 - 5$ times that for $C_d = 0.0025$ if the sediment load and a roughness of the glacier surface are taken into account. Given the non-negligible change in melt rates, additional investigations are therefore needed to characterize the dependence of C_d on Reynolds numbers and its sensitivity to the sediment load.

4. Conclusions

We have shown that classical plume theory can form the basis of improved models of three-dimensional wall plumes if the wall drag is accounted for and the entrainment coefficient is corrected. The volume flux evolution of a wall plume is well captured already by considering half of that obtained for a conical plume, which implies that the dilution of the wall plume fluid, i.e. the salinity and temperature evolution with depth, should also be predicted reasonably well when neglecting drag effects. The difference is only in the momentum flux which is overestimated by about 10-20% if the wall drag is not accounted for. However, the coefficients parametrizing turbulence effects for entrainment, drag and scalar transfer are important for the predictions of melting rates, as these coefficients appear in the widely used three-equation melt formulation (Holland and Jenkins 1999). We have shown that a consistent estimate of the drag coefficient based on the modified MTT theory plume velocity, and a corrected vertical velocity for wall plumes which takes into account a non negligible drag coefficient (eq. 12) substantially increases the predictions for melting rates near an ice wall. Furthermore, we have shown for the first time that the wall plume spreads horizontally parallel to the wall and loses its axisymmetric shape (Fig. 1b, c). This important aspect will produce an increase in melting when compared to that obtained with a half-conical plume due to the larger area covered on an ice face by the wall plume.

Adding the mass and buoyancy fluxes associated with melting into the wall plume model is not expected to alter our results significantly. Generally, a subglacial discharge is characterized by a volume flux $Q \sim 100$ m^3 s^{-1}, corresponding to a 'convection-driven melting' regime (Jenkins 2011), where the contribution of submarine melting to the plume buoyancy is small. It is only for a small discharge, ≈ 10 m^3 s^{-1} (Mankoff et al. 2016; Ezhova et al. 2017), that the effect of submarine melting on the plume buoyancy flux cannot be neglected. Both drag and entrainment are mainly influenced by the turbulent characteristics of the wall plume which, for substantial subglacial discharges, should remain unchanged.

Our study shows that the increase in C_d for a modified MTT model of a three-dimensional wall plume at large Reynolds numbers can be as high as 10 times as compared with that associated with a 2D turbulent boundary layer flow ($C_d = 0.001$) and thus, can not be ignored while calculating melting rates.

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LIST OF TABLES

Parameter, $z = 15$	$y = 0$	$y = 1$	$y = 2$	$y = 3$
u_*	0.30	0.26	0.19	0.13
x_v	0.0033	0.004	0.005	0.008
$U_{\max}$	3.38	2.72	1.80	0.84
C_{dm}	0.008	0.009	0.011	0.024
$C_{d,i}$	0.043	0.033	0.018	0.008
Parameter, $z = 18$	$y = 0$	$y = 1$	$y = 2$	$y = 3$
u_*	0.28	0.25	0.20	0.15
x_v	0.0036	0.004	0.005	0.0065
$U_{\max}$	3.10	2.59	1.90	1.27
C_{dm}	0.008	0.009	0.011	0.015
$C_{d,i}$	0.041	0.033	0.021	0.012

TABLE 1. Parameters of the logarithmic near-wall flow.

LIST OF FIGURES

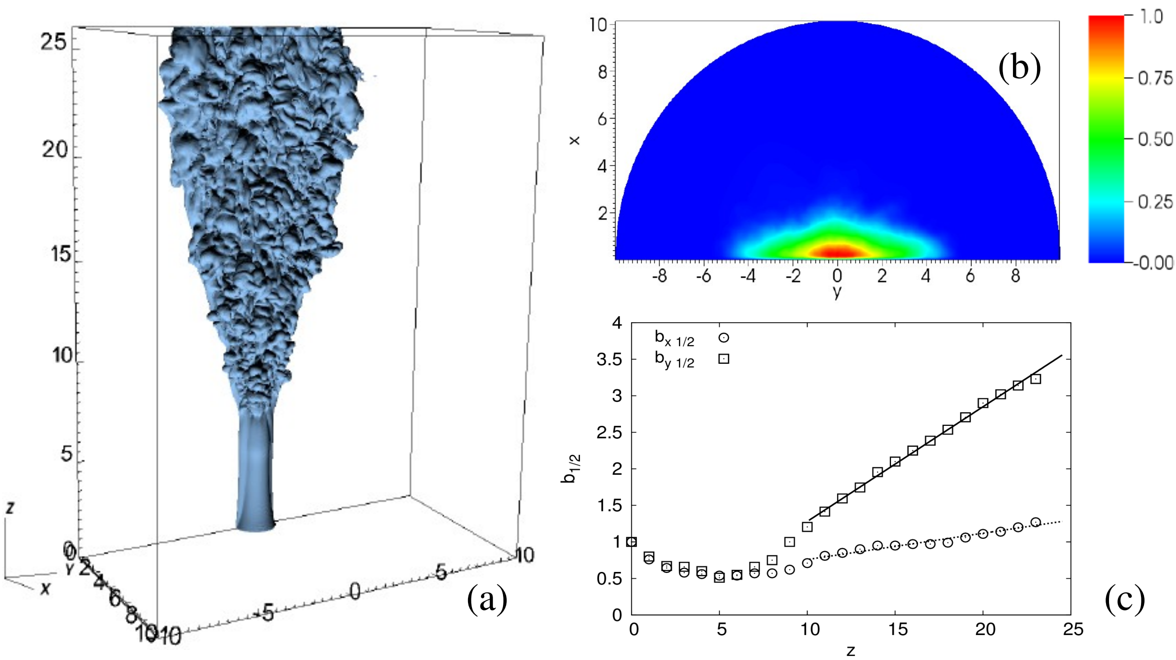

FIG. 1. (a) Wall plume visualized by the density contour $\rho = (\tilde{\rho} - \tilde{\rho}_{pl})/(\tilde{\rho}_{amb} - \tilde{\rho}_{pl}) = 0.99$, where $\tilde{\rho}_{pl}$ is the plume density and $\tilde{\rho}_{amb}$ is the density of the ambient fluid. (b) Mean vertical velocity at the cross-section $z = 20$ normalized with the maximum velocity in this cross-section. (c) Characteristic radii of the wall plume, $b_{1/2}$, in the x and y directions vs vertical coordinate z. $b_{1/2}$ is defined as the radius where the mean maximum velocity is halved. Best fits of the data for $z \geq 10$ have the slopes $s_x = 0.036$ and $s_y = 0.157$.

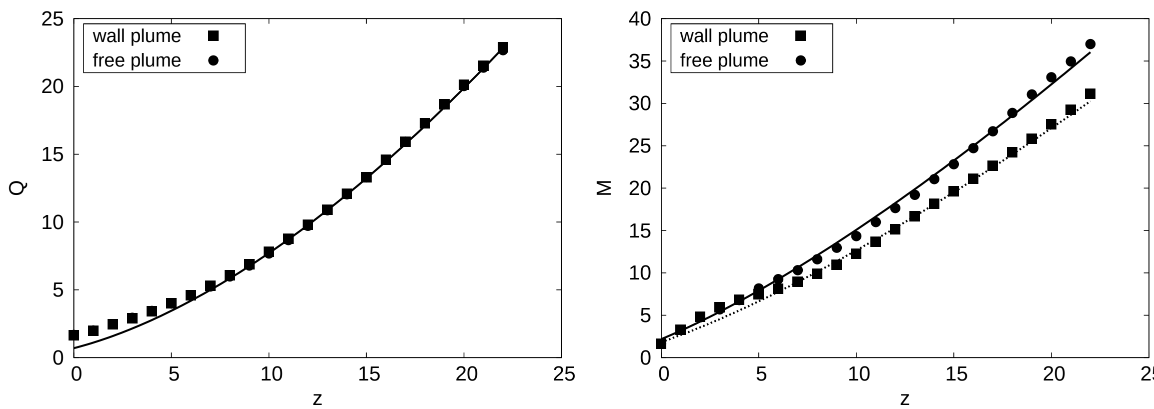

FIG. 2. Volume flux (left) and momentum flux (right) vs the vertical coordinate for the half-conical free and wall plumes. The volume flux of the wall plume is almost identical to half that of the conical plume, hence the two symbols lie on top of each other and the circles on the left panel are below the squares. Solid curves indicate the asymptotic scaling following from the classical MTT theory and valid for the conical plume; dashed curves indicate the asymptotic scaling following from the modified MTT theory and valid for the wall plume (see supplementary material). Both theories give $Q \sim z^{5/3}$ and $M \sim z^{4/3}$. The difference is in the coefficients: $Q_w/Q = \left(\frac{\alpha_w}{\alpha}\right)^{4/3} \frac{1}{\left(1+\frac{5C_d}{2\pi\alpha_w}\right)^{1/3}} = 0.97$ (dashed and solid curves are on top of each other in the left panel); $M_w/M = \left(\frac{\alpha_w}{\alpha}\right)^{2/3} \frac{1}{\left(1+\frac{5C_d}{2\pi\alpha_w}\right)^{2/3}} = 0.82$ with C_d, α and α_w obtained from DNS in the present study.

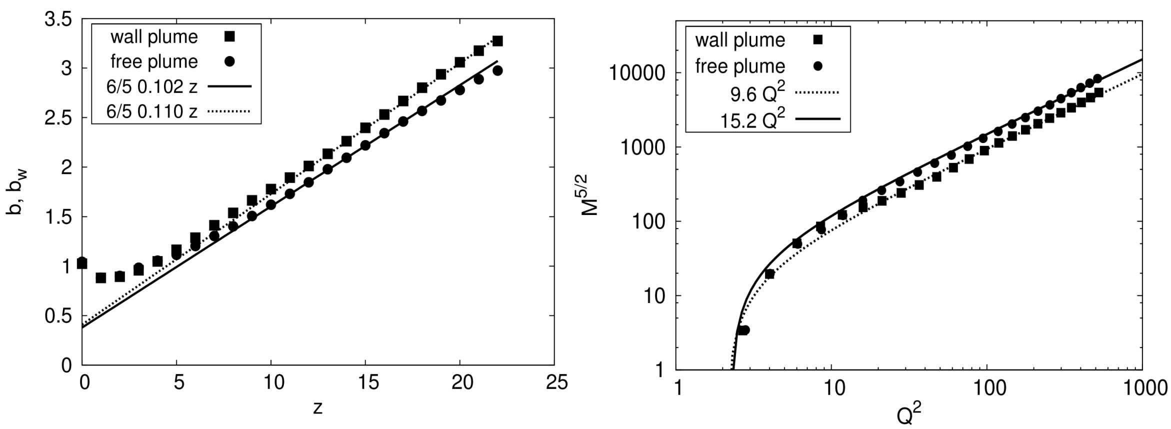

FIG. 3. Left: free and wall plume radii ('equivalent' plume radius for the wall plume). Lines indicate the radius solution $b = \dfrac{6}{5}\alpha z$ for two different values of the entrainment coefficient. Right: $M^{5/2}$ versus Q^2 for the free and wall plumes.

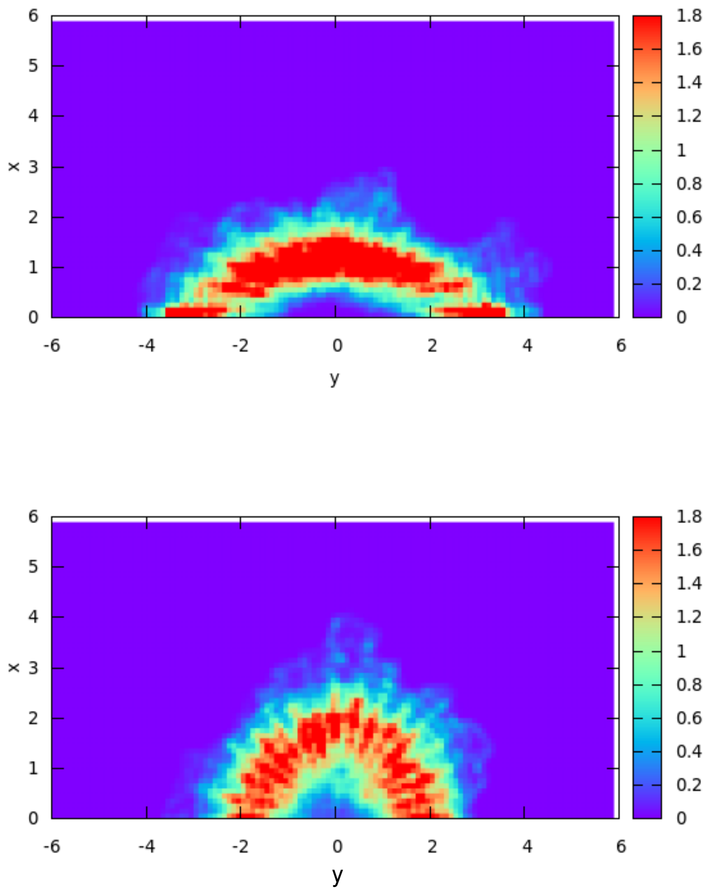

FIG. 4. Statistics of the turbulent plume boundary location at $z = 15$: wall plume (upper panel) and half a conical plume (lower panel). The figures illustrate the frequency of finding the plume boundary at a certain location (in a square 0.1×0.1). Given the turbulent structure of the plume, the boundary is not always a single simple closed curve, as it encompasses turbulent eddies. The plume boundary is defined by the contour of density $\rho = (\tilde{\rho} - \tilde{\rho}_{pl})/(\tilde{\rho}_{amb} - \tilde{\rho}_{pl}) = 0.97$, where $\tilde{\rho}_{pl}$ is the plume density and $\tilde{\rho}_{amb}$ is the density of the ambient fluid.

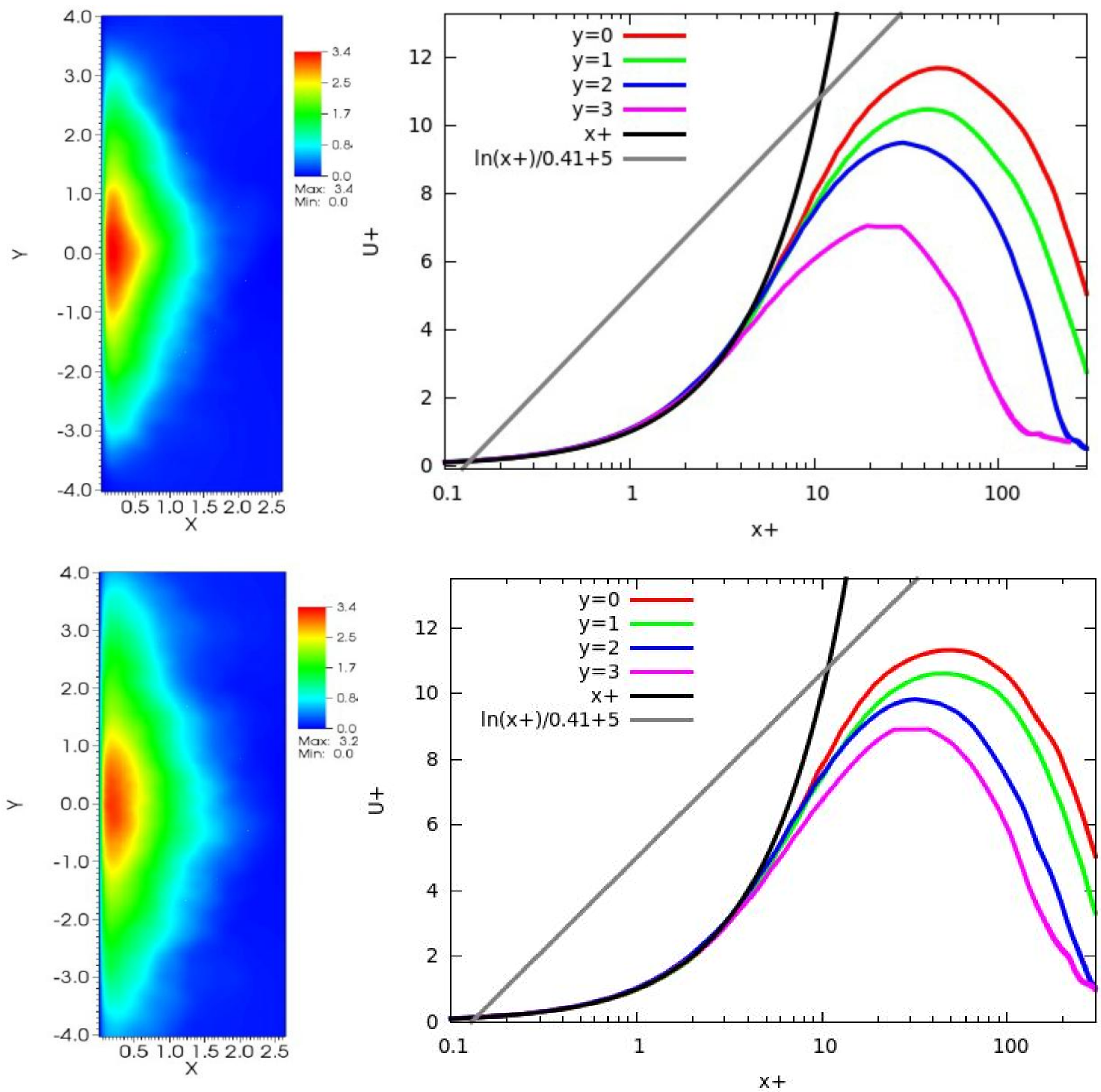

FIG. 5. Horizontal cross-sections (left) and profiles (right) of the mean velocity parallel to the wall $U = \sqrt{(<u>^2 + <v>^2)}$ in the inner coordinates at different y locations. Upper panel: $z = 15$, lower panel: $z = 18$.

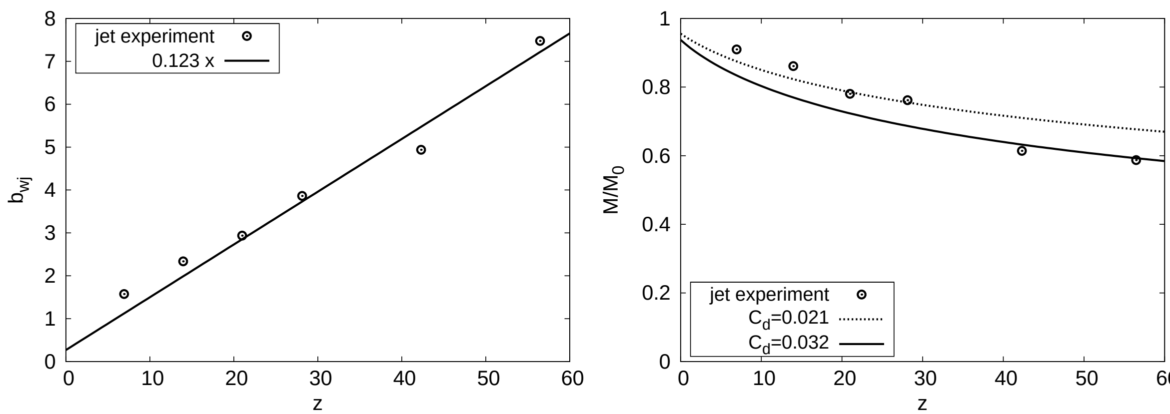

FIG. 6. Left: 'equivalent' wall jet radius vs vertical coordinate. Right: momentum flux vs vertical coordinate. Dashed and solid curves represent approximations to near-field and far-field data respectively. The data are taken from the wall jet experiment by Namgyal and Hall (2016).

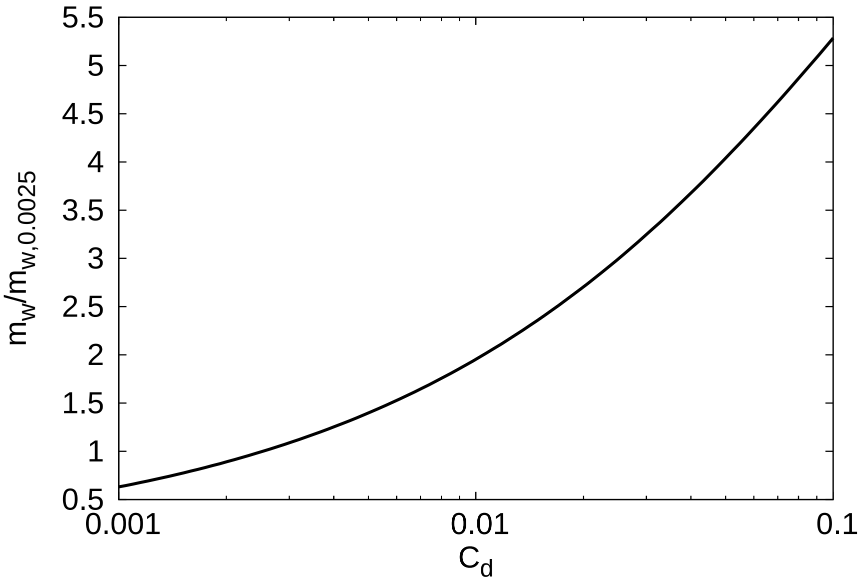

FIG. 7. Melt rate dependence on the drag coefficient. The melt rate is normalized with that obtained using $C_d = 0.0025$.