Motivation
Effect of small momentum excess

It is sometimes stated that a small amount of injected momentum during acceleration/deceleration changes wake behavior qualitatively towards longer lifetime and large coherent structures. Is this correct?

Wake studies usually consider steady-state towed or self-propelled cases despite this being an unrealistic assumption due to unsteady upstream conditions, slight imbalances in thrust and drag, and the presence of maneuvers.

Prandtl number effect
Is the assumption of Pr=1 adequate in numerical models although presence of maneuvers.

Prandtl number effect
Excess momentum: towed wake
- \( \text{Pr}=0.2 \)
- \( \text{Pr}=1 \)
- \( \text{Pr}=7 \)

Prandtl number effect
Computational cost per simulation
- \( \text{Pr}=0.2: 448 \text{ processors, } 14,000 \text{ computing hours} \)
- \( \text{Pr}=1,0.2, 2: 40 \text{ processors, } 800 \text{ computing hours} \)
- \( \text{Excess momentum: } 24 \text{ processors, } 200 \text{ computing hours} \)

Prandtl number effect
Computational cost scales as \( \text{Pr}^3 \) using DNS
- \( \eta_m = \eta/\sqrt{\text{Pr}} \)

Formulation
Physical Domain

Governning equations
Momemtum
\[ \frac{\partial u}{\partial t} + \frac{\partial (u u)}{\partial x} + \frac{\partial (u p)}{\partial x} = -\frac{\partial \mu}{\partial x} + \frac{1}{Re} \frac{\partial^2 u}{\partial x^2} \]

Density
\[ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0 \]

Prandtl number effect
Computational domain size
- \( \text{Pr}=7: 3584 \times 1024 \times 512, 1.88 \text{ billion grid points} \)
- \( \text{Pr}=1,0.2, 2: 1280 \times 512 \times 256, 168 \text{ million grid points} \)
- \( \text{Excess momentum: } 1024 \times 384 \times 192, 75 \text{ million grid points} \)

Computational cost per simulation
- \( \text{Pr}=7: 448 \text{ processors, } 14,000 \text{ computing hours} \)
- \( \text{Pr}=1,0.2, 2: 40 \text{ processors, } 800 \text{ computing hours} \)
- \( \text{Excess momentum: } 24 \text{ processors, } 200 \text{ computing hours} \)

Excess momentum: jet
5\% ΔM jet similar to SP case, 20\% ΔM jet behaves qualitatively like SP but with smaller wake dimensions

Excess momentum: towed wake
5\% ΔM tow similar to SP case

Turbulent quantities unaffected by small excess momentum

Do large eddies form in the late wake?
Self-propelled
Large scale eddies are not observed in the late wake with a small amount of excess momentum

Conclusions
Effect of small momentum excess

- A small amount of excess momentum added to a self-propelled profile does not dominate wake evolution
- Turbulent quantities unaffected by small excess momentum
- Wake dimensions and defect velocity weakly affected by small excess momentum

Prandtl number effect
Principal conclusion: For \( \text{Pr} < 1 \) or \( \text{Pr} > 1 \), the wake behaves qualitatively the same as \( \text{Pr}=1 \)

Future work
- Determine parameters governing transition between momentum dominated and self-propelled wake
- Perform spatially evolving simulations with a constant forcing
- Higher Reynolds number simulations

Acknowledgments
Matthew de Stadler received support from the Department of Defense (DoD) through the National Defense Science & Engineering Graduate Fellowship (NDSEG) Program and from a Jacobs School Fellowship (UCSD). Partial support was provided by ONR grant N000140710133.

Simulation of the Wake of an Accelerating Body in a Stratified Fluid

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Advisor: Prof. Sutanu Sarkar

Figure from K. Brucker, Ph.D. Thesis, UCSD, 2009

Computational Domain

Excess momentum simulations were conducted with \( \text{Re}=5,000, \text{Pr}=1, \text{and Fr}=2 \)

Towed
Late wake vorticity: \( \omega \) (\( x_3=0 \)) at \( t=800 \)

Effects of small excess momentum
Principal conclusion: Neither large coherent structures nor a longer lifetime were observed

Physics of Fluids, 2010 (under review).