Onsager's Pancake Approximation for Fluid Flow in a Gas Centrifuge

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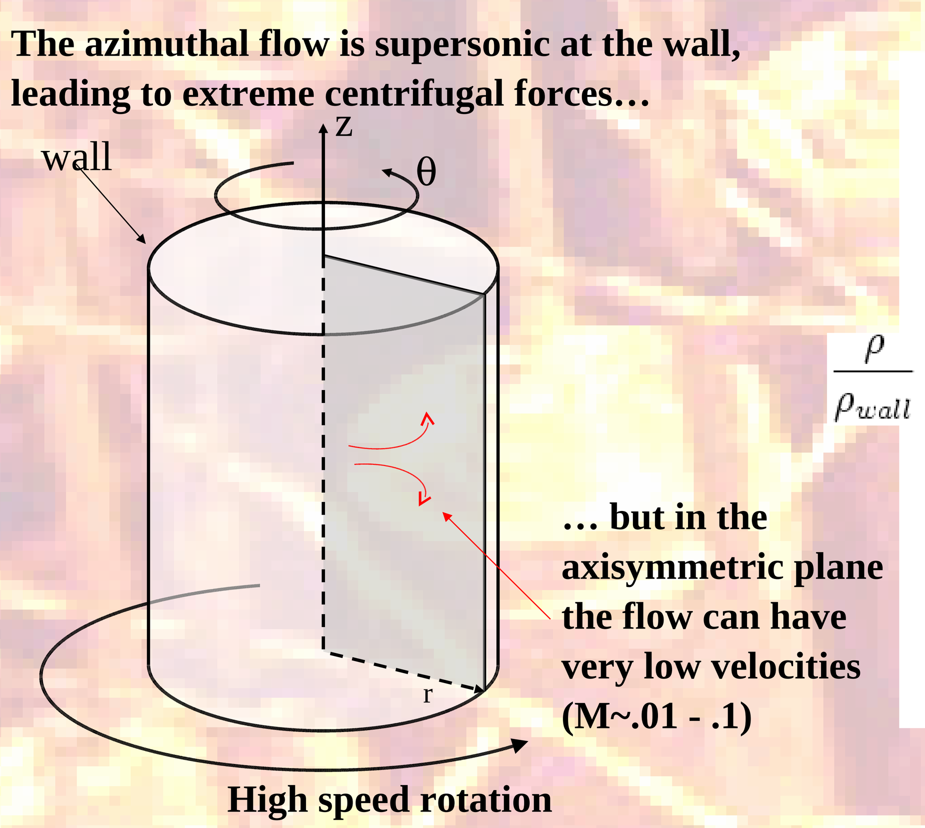
Abstract

Onsager's Pancake Approximation is used to construct a simulation to model fluid flow in a gas centrifuge. The governing 6th order partial differential equation will be broken down into a coupled system of three equations and then solved using Overture software. The results obtained from this simulation will be compared with those from other centrifuge models to quantify the increases in predictive fidelity resulting from greater model complexity. They will also be used as an initial condition for a 3-D Navier-Stokes simulation.

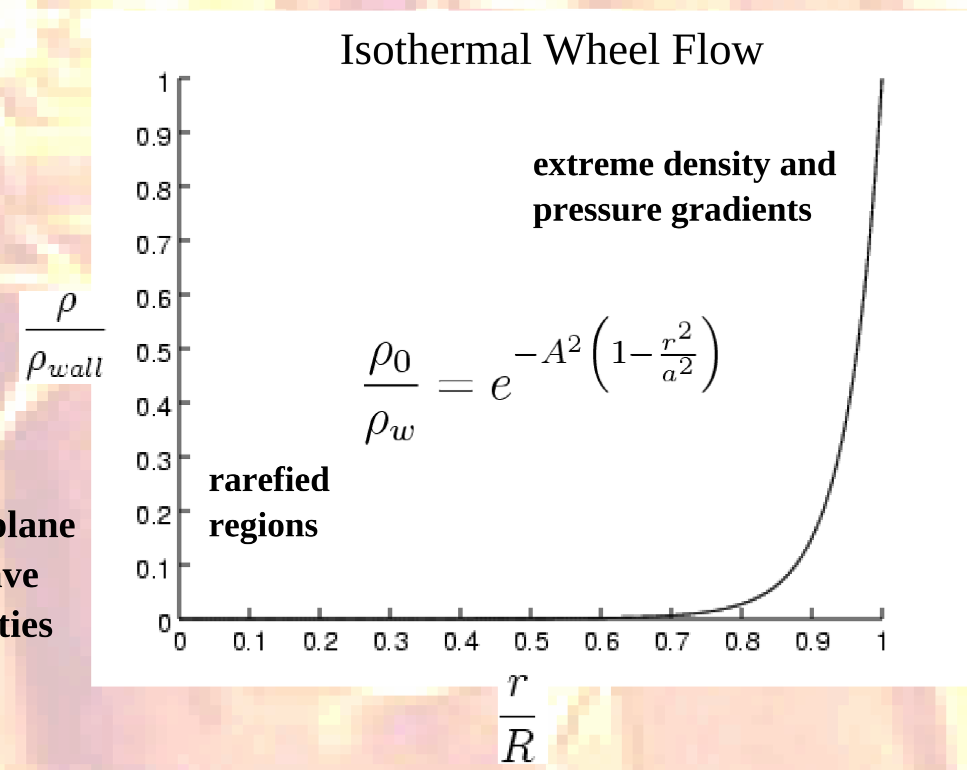
Introduction

Gas centrifuges are used to separate gas isotopes. They exhibit very complex fluid behavior.

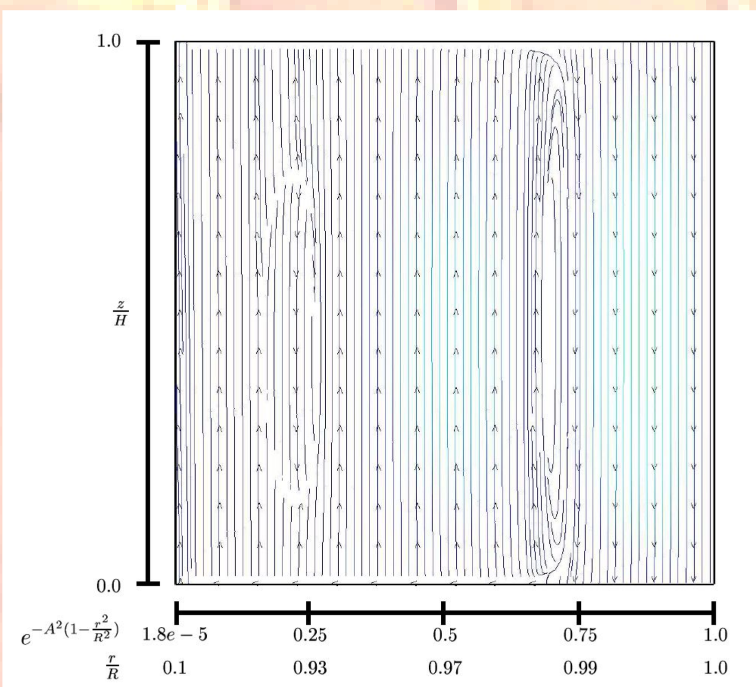
- **Flow consists of a rarefied region, a transition region, and a region with an extreme density gradient**
- **Flow is moving at hypersonic speeds with shock waves present**
- **Flow is subsonic in the axisymmetric plane**



Operating Centrifuge with Axisymmetric Plane Shown



Radial Density Profile



Flow Streamlines Near The Rotor Wall

Governing Equation

After linearizing the Navier-Stokes equations and reducing terms, Onsager's Master Equation is obtained

$$\frac{\partial^2}{\partial \xi^2} e^{\xi} \frac{\partial^2}{\partial \xi^2} e^{\xi} \frac{\partial^2 \chi}{\partial \xi^2} + \frac{1 + A^2 Pr \frac{\gamma-1}{2\gamma}}{16 A^{12} \epsilon^2 (\frac{Z}{a})^2} \frac{\partial^2 \chi}{\partial \eta^2} = 0$$

$$\psi = \frac{1}{r}\frac{\partial \chi}{\partial r}$$

$$r\rho_0 v = -\frac{\partial \psi}{\partial z}$$

$$\rho_0 u = -\frac{1}{r}\frac{\partial \psi}{\partial r}$$

This equation governs the fluid flow inside the gas centrifuge in the axisymmetric plane subject to the relevant assumptions.

Implementation

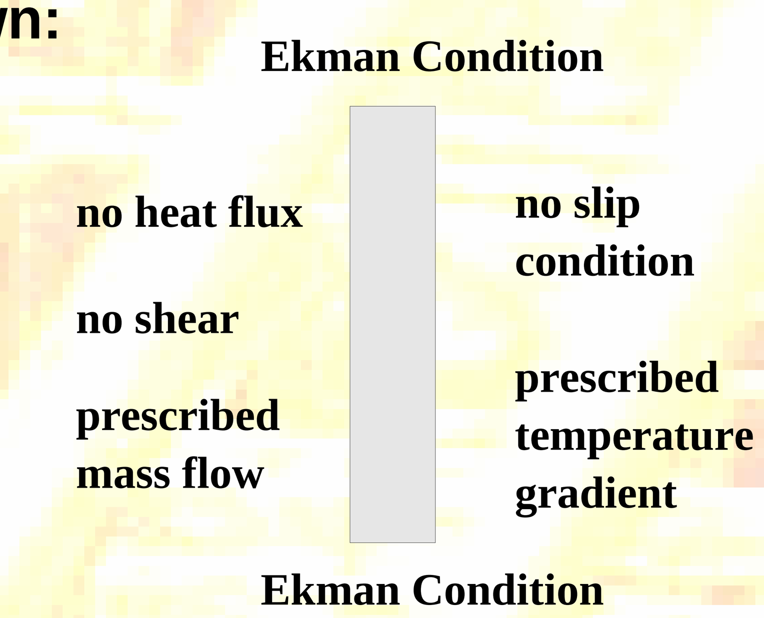
The complex governing equation will be broken down into 3 coupled partial differential equations with boundary conditions as shown:

$$\frac{\partial^2 \psi}{\partial \xi^2} + B \frac{\partial^2 \chi}{\partial \eta^2} = 0$$

$$\frac{\partial^2 \varphi}{\partial \xi^2} - e^{-\xi}\psi = 0$$

$$\frac{\partial^2 \chi}{\partial \xi^2} - e^{-\xi}\varphi = 0$$

These equations will be solved with the appropriate boundary conditions to obtain the velocity field in the radial plane.



Boundary Conditions

Formulation Advantages

- **Easier to implement in Overture**
- **More robust formulation when used with Overture**
- **More compact approximation**
- **Allows more general cases to be considered than an eigenfunction expansion**

Verification

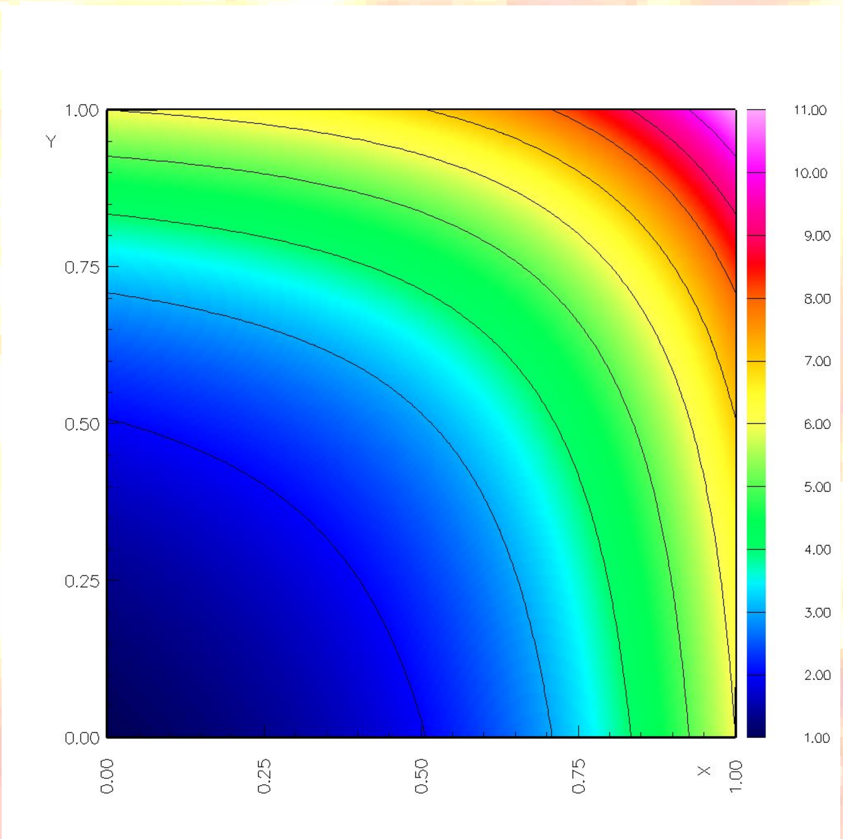
- **Manufactured Solutions**
- **Eigenfunction expansion solution**
- **Navier-Stokes code solution**

h	max error in u	max error in v
0.05	8.68E-3	7.69E-2
0.025	2.47E-3	2.72E-2
0.0125	6.21E-4	6.61E-3
0.00625	1.54E-4	1.54E-3

Order of convergence for u: 1.94

Order of convergence for v: 1.90

Order of Convergence Test for 2 Coupled PDEs



Manufactured Solution: Polynomial Function

Project Status

- **Software is currently being developed to solve Onsager's Master Equation with realistic boundary conditions**

The background image shows a centrifuge cascade from URENCO

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