

Simulation of a self-propelled wake with small
excess momentum in a stratified fluid

Matthew de Stadler and Sutanu Sarkar
University of California San Diego

Features of Turbulent Wakes in Stratified Fluids

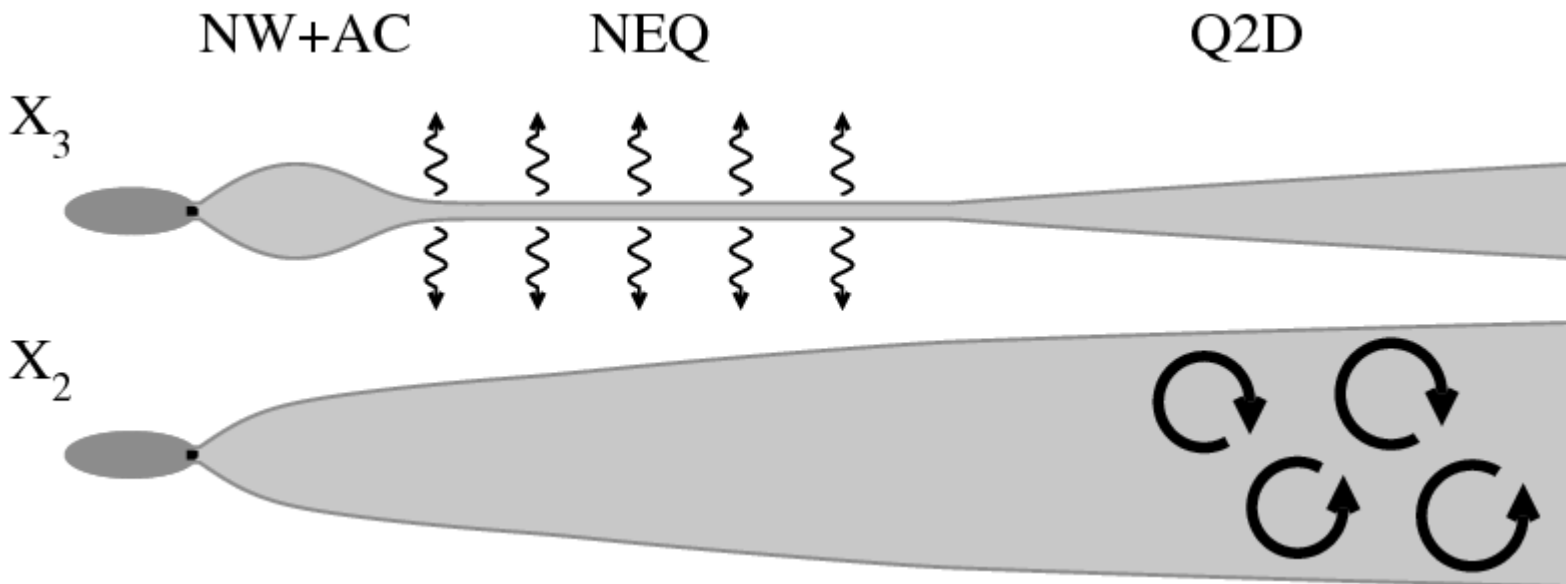
Stratification breaks radial symmetry: vertical motion inhibited

Transfer between kinetic and potential energy

Internal waves radiated

Late time quasi-2D flow

Very different than unstratified wake

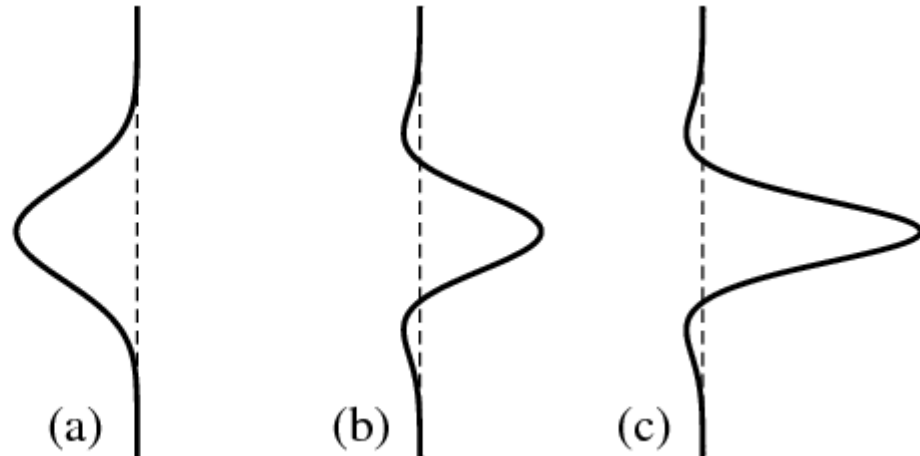
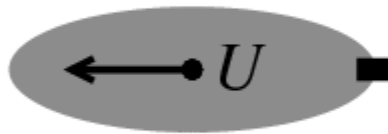


Classes of wakes

Two fundamental classes of wake

Towed: Drag only

Propelled: Thrust and Drag



(a) Towed wake,

only drag

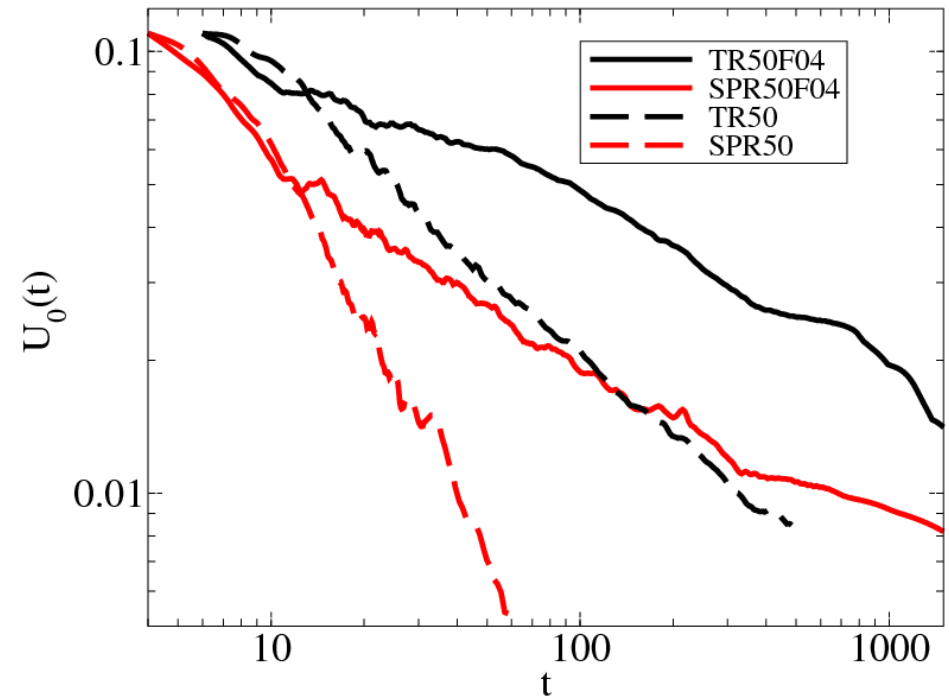
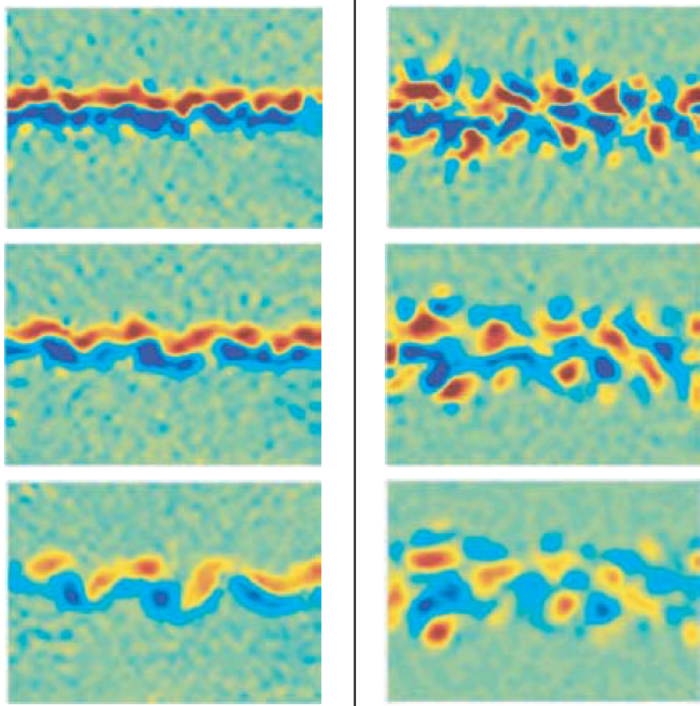
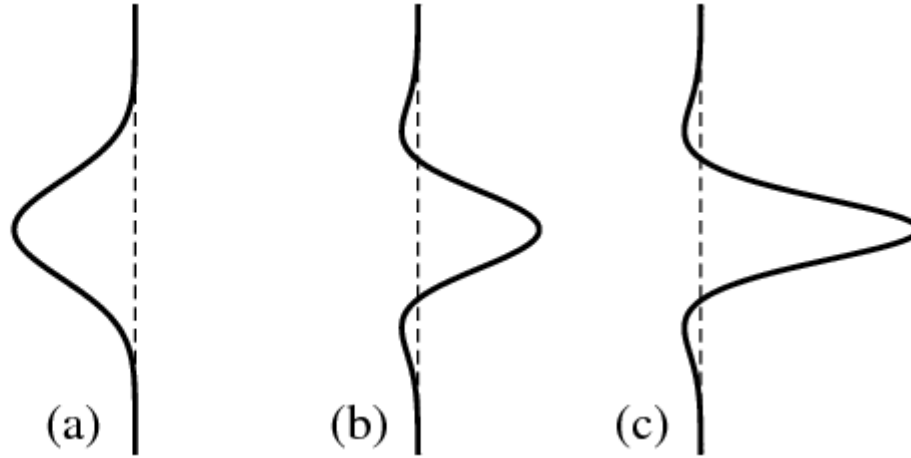
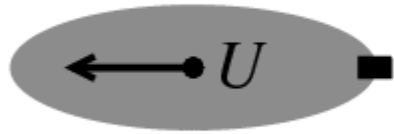
(b) Self-propelled wake,

thrust=drag, net momentum=0

(c) Excess momentum wake,

thrust > drag

Difference between self-propelled and towed wake



Vorticity data from Meunier and Spedding (2006)
(Left) Momentum wake (right) SP wake

Data from Brucker and Sarkar (2010)

Why this problem?

A self-propelled (SP) wake with zero net momentum is unrealistic

Unsteady upstream conditions

Slight imbalances in thrust

Maneuvers

The self-propelled wake with zero net momentum is qualitatively different from a towed (drag wake)

Assuming self-similarity, Tennekes and Lumley (1972) determined that an unstratified wake with any excess momentum at late time is dominated by the excess momentum component

Meunier and Spedding (2006) found qualitatively different results with $> 2\%$ excess momentum in the wake vorticity field

Brucker and Sarkar (2010) found that a self-propelled wake decayed much faster than a towed wake. SP wake has smaller vortical structures.

Voropayev et al. (1999) and Voropayev and Fernando (2010) show the formation of large coherent structures in the late wake after an accelerating maneuver

Study focus

How is the wake affected by excess momentum?

Amount of excess momentum

Shape of excess momentum

Do larger structures form in the late wake with small amounts of excess momentum?

Interested in the effect of excess momentum on

Defect velocity

Wake dimensions

Vorticity dynamics

Internal wave flux

Mean and turbulent kinetic energy and associated budgets

Does a small amount of excess momentum qualitatively change the wake evolution?

Problem approach

Use DNS to solve the Boussinesq Navier-Stokes equations in a temporally evolving frame

First DNS of a self-propelled wake with excess momentum

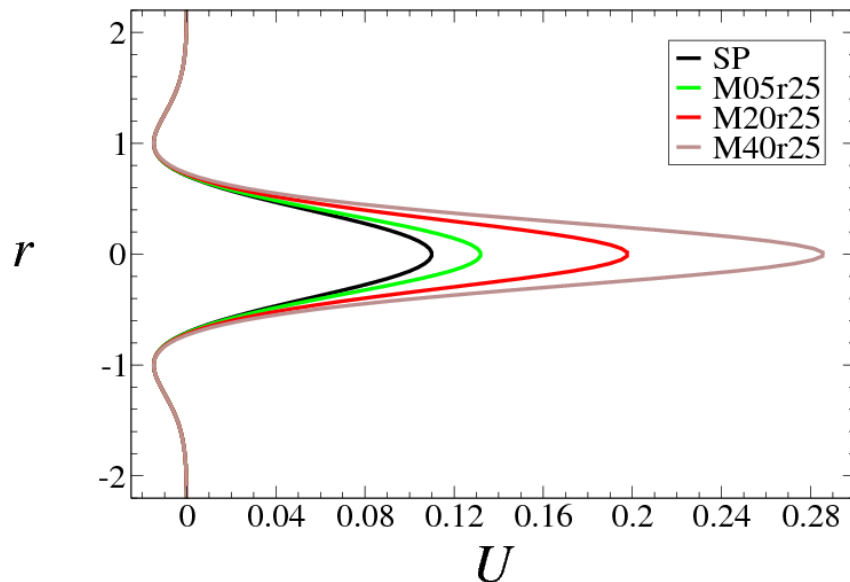
Start with a self-propelled profile

Impulsively add momentum with a Gaussian profile

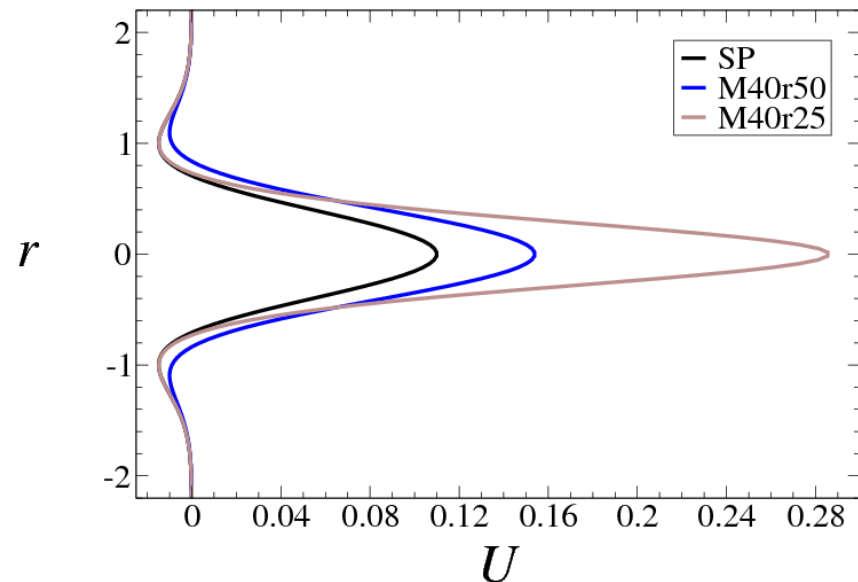
Linear density stratification

Simulation parameters:

$Re = 10,000$, $Fr=3$, $Pr= 1$

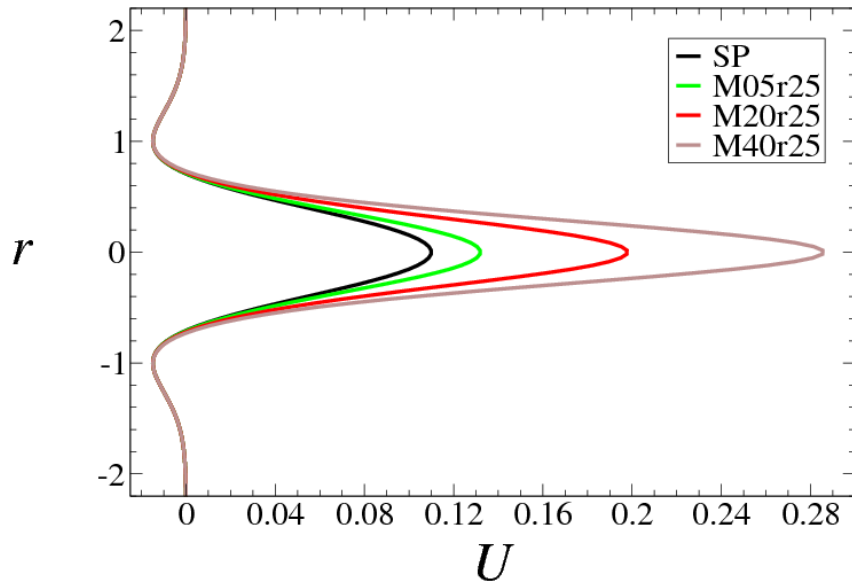


Vary amount of excess momentum



Vary shape of excess momentum

Vary amount of excess momentum



Increasing % M

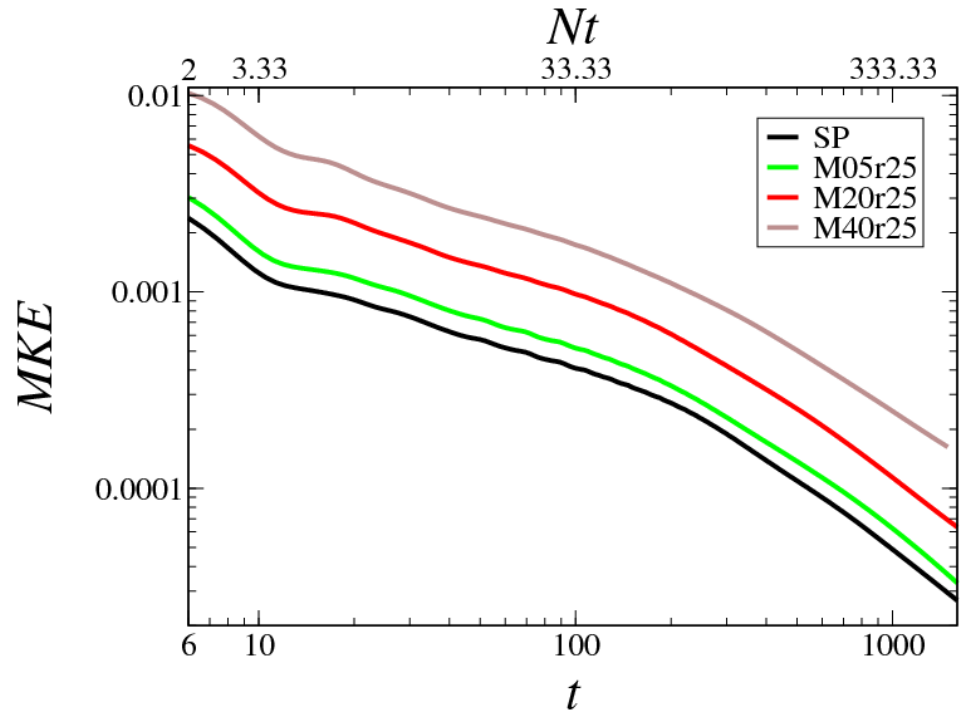
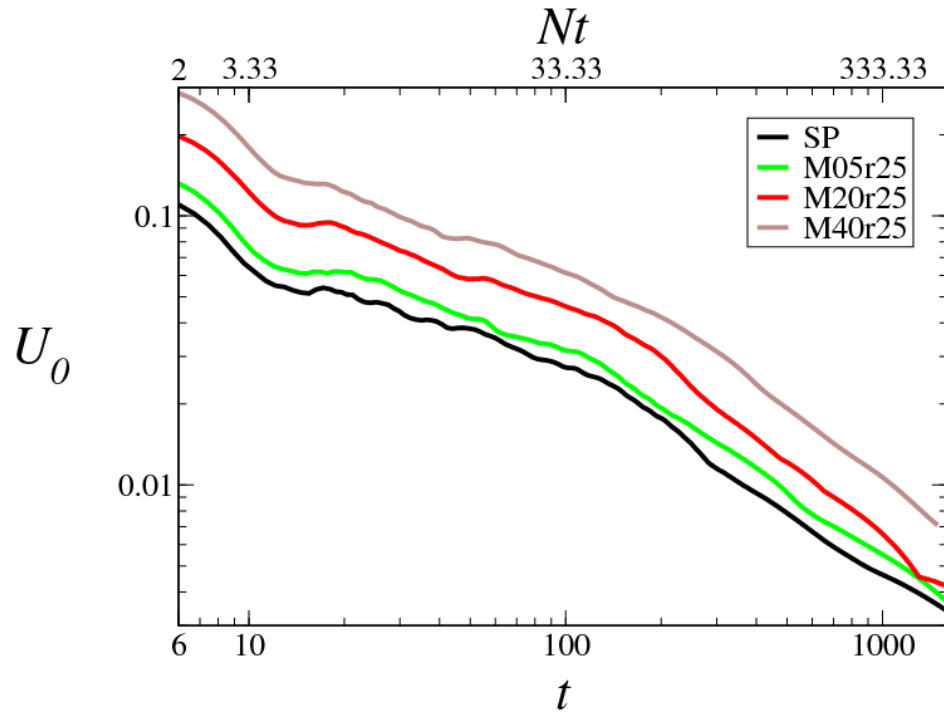
Increases defect velocity

Increased MKE

Increases velocity gradient

Vary amount of excess momentum

Defect Velocity and MKE

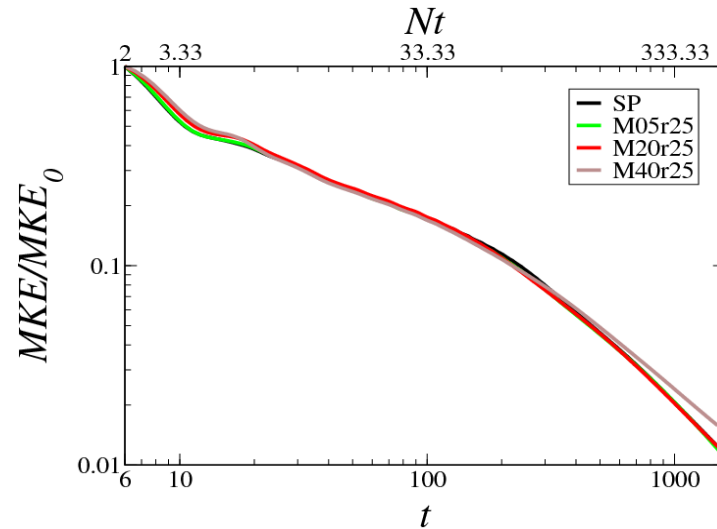
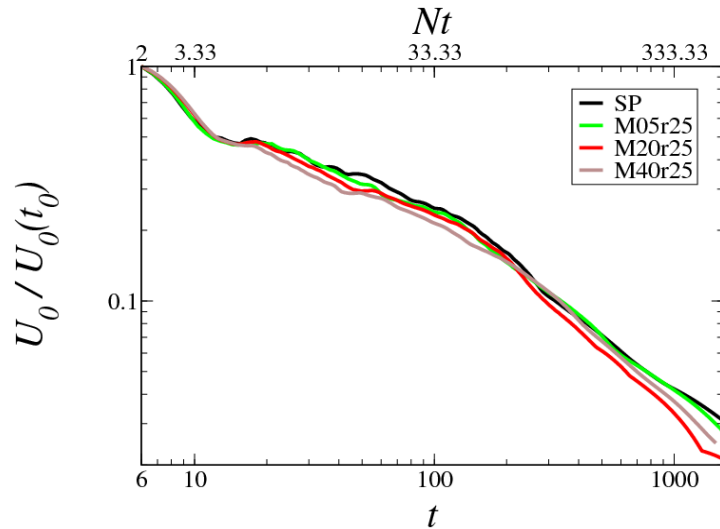
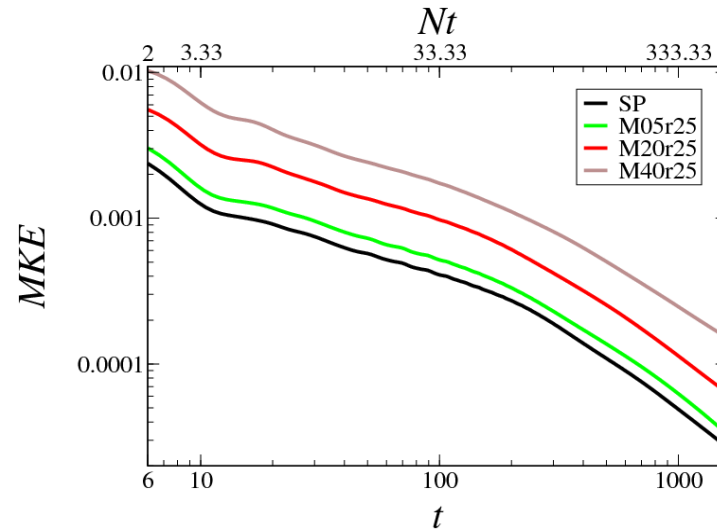
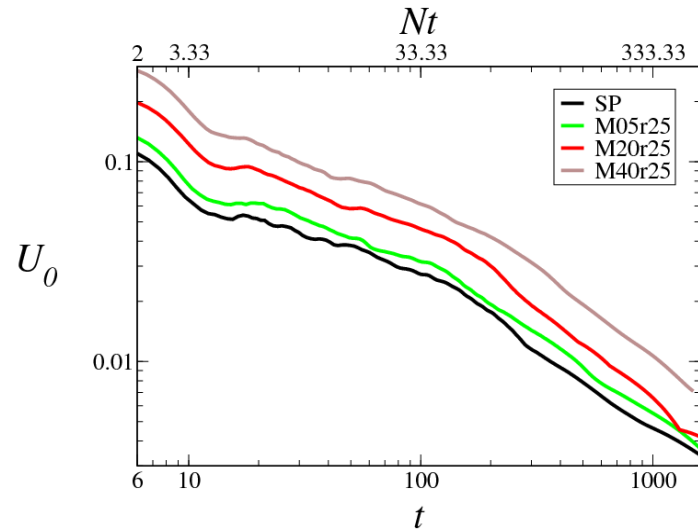


Greater excess momentum gives increased U_0 , MKE

Good qualitative agreement even in 40% momentum case

Three flow regimes are evident in all cases

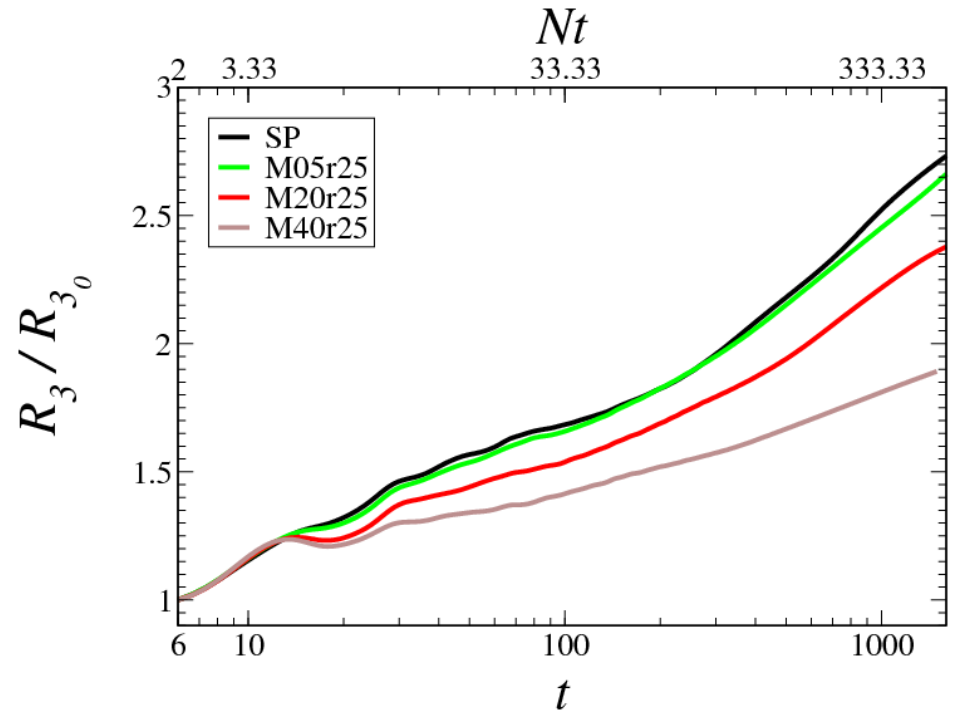
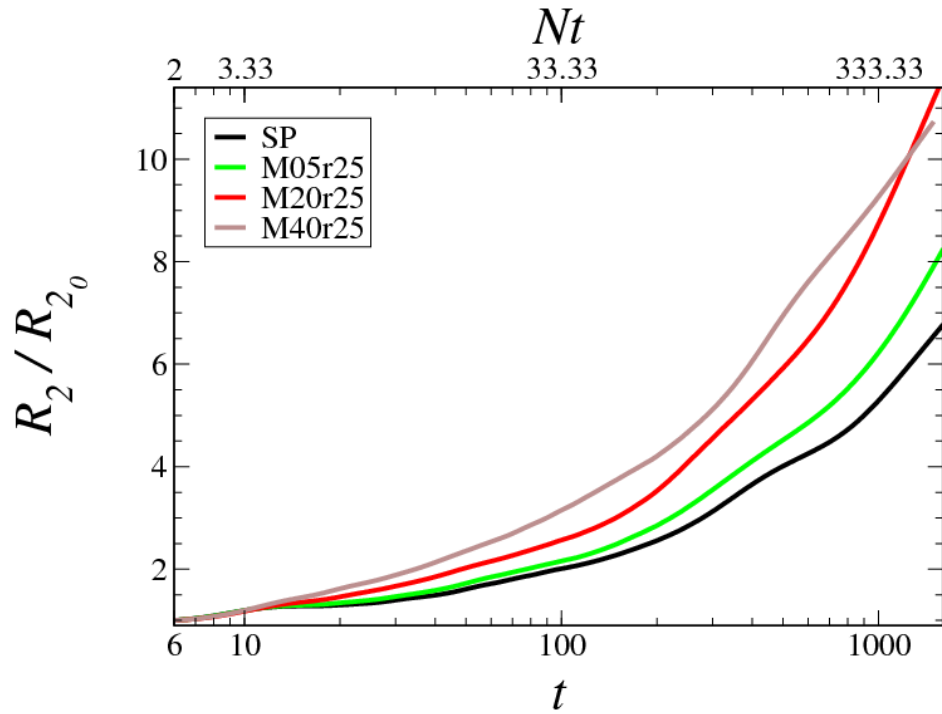
Defect Velocity and MKE



Data collapses when scaled by initial value

Collapse is better for MKE than U_0

Wake Dimensions



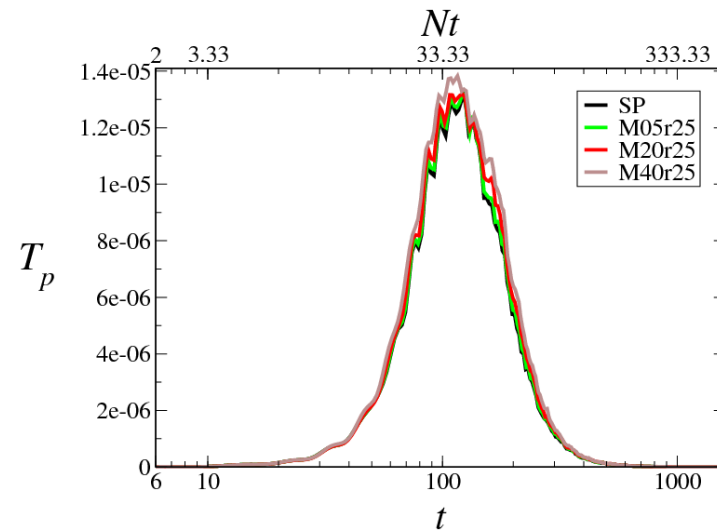
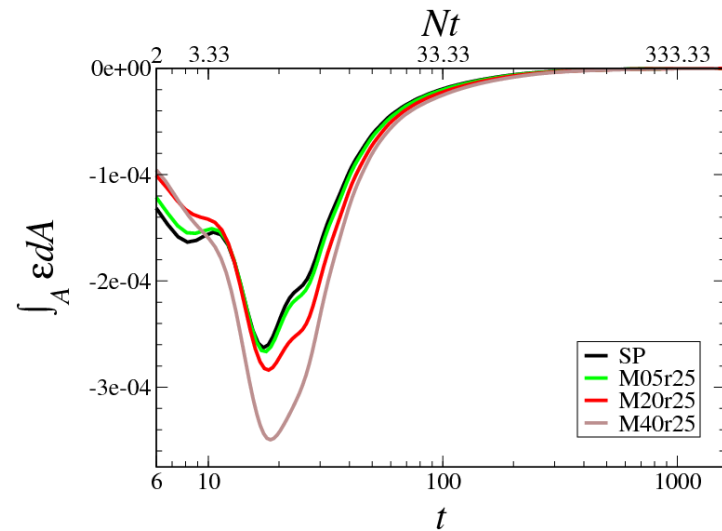
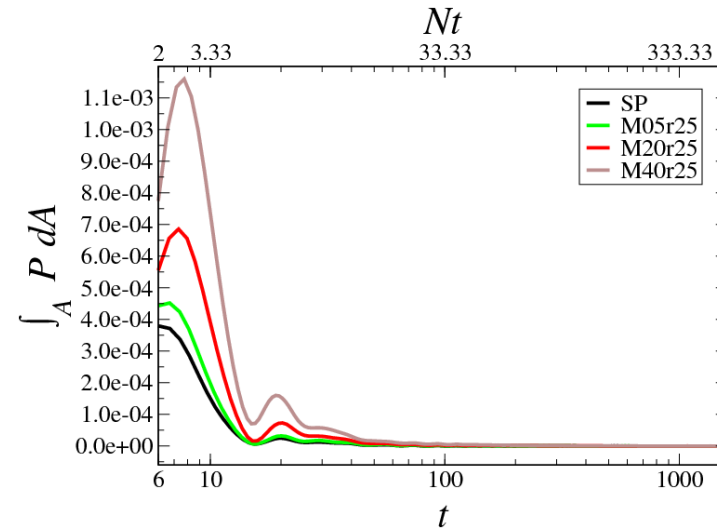
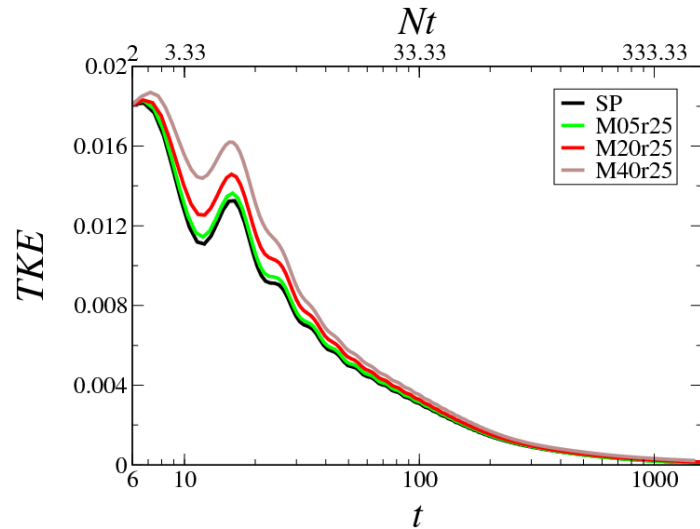
$$R_\alpha^2(t) = F \frac{\int_A (x_\alpha - x_\alpha^c)^2 \langle u_1 \rangle^2 dA}{\int_A \langle u_1 \rangle^2 dA}$$

$$x_\alpha^c(t) = \frac{\int_A x_\alpha \langle u_1 \rangle^2 dA}{\int_A \langle u_1 \rangle^2 dA}$$

Excess momentum diffuses horizontally but not vertically

Excess momentum remains close to vertical centerline

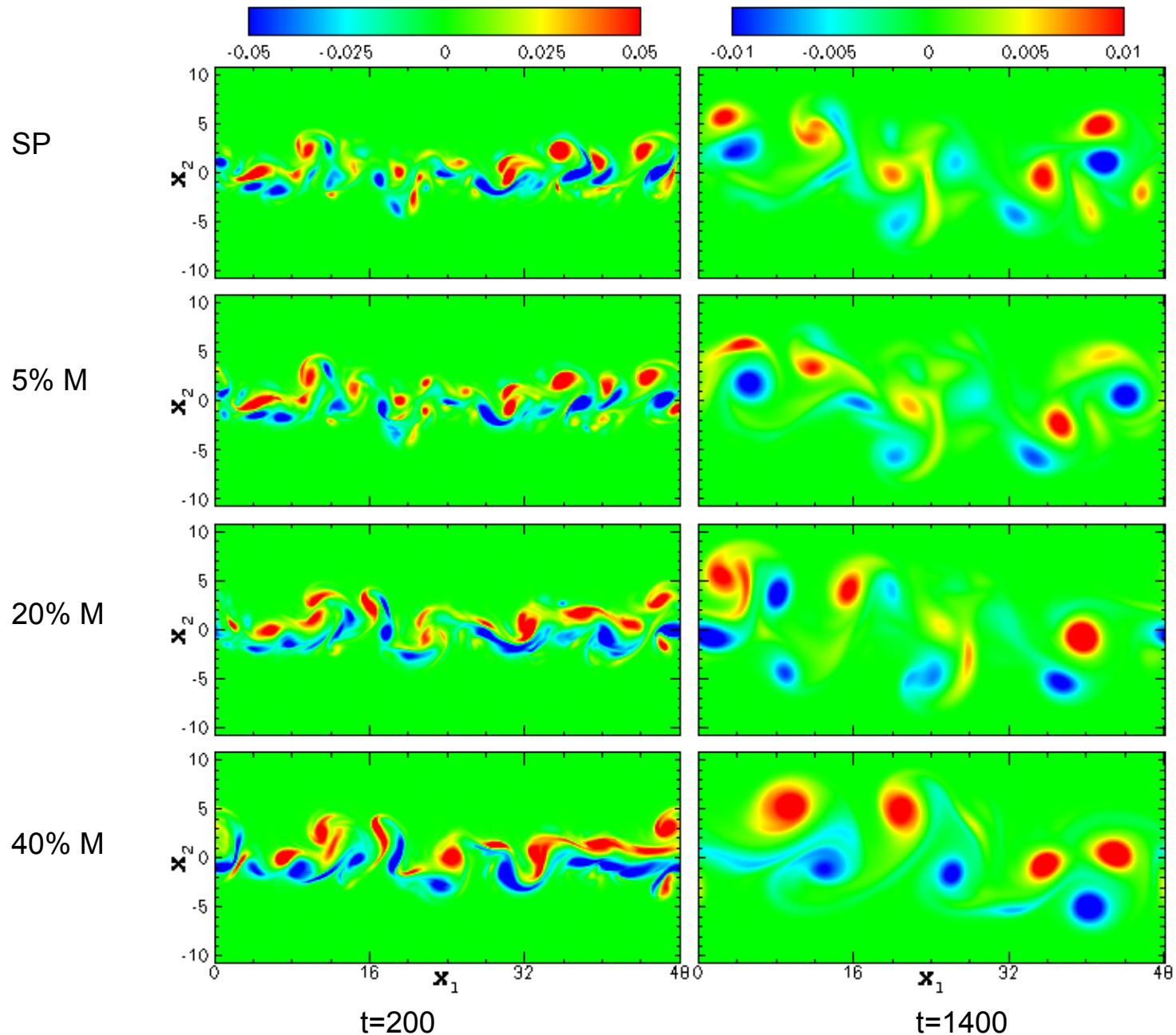
TKE and Wave Flux



Increased %M --> Sharper velocity gradient--> increased production --> increased TKE --> increased dissipation

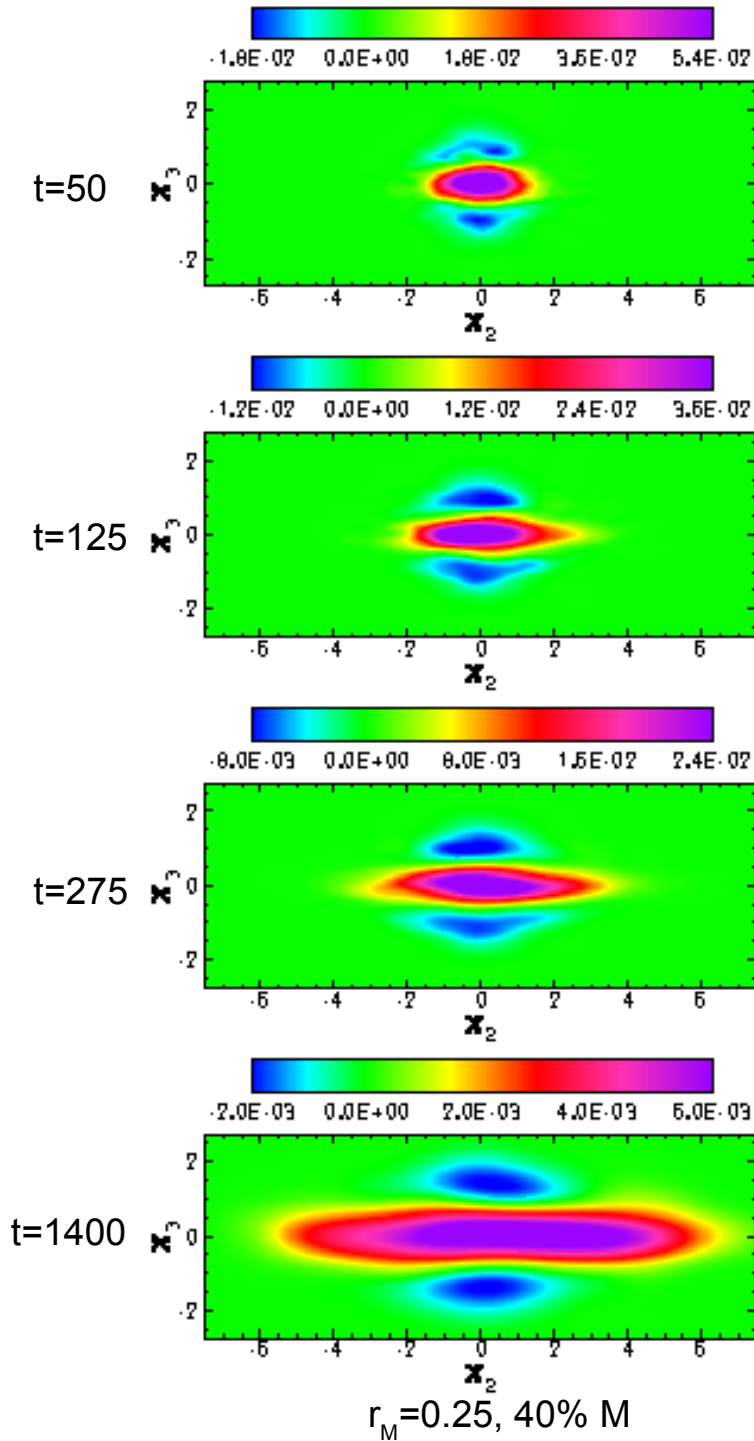
Despite relatively large quantitative differences, qualitative agreement is generally quite good

Wake Vorticity Field ω_3 at $x_3=0$



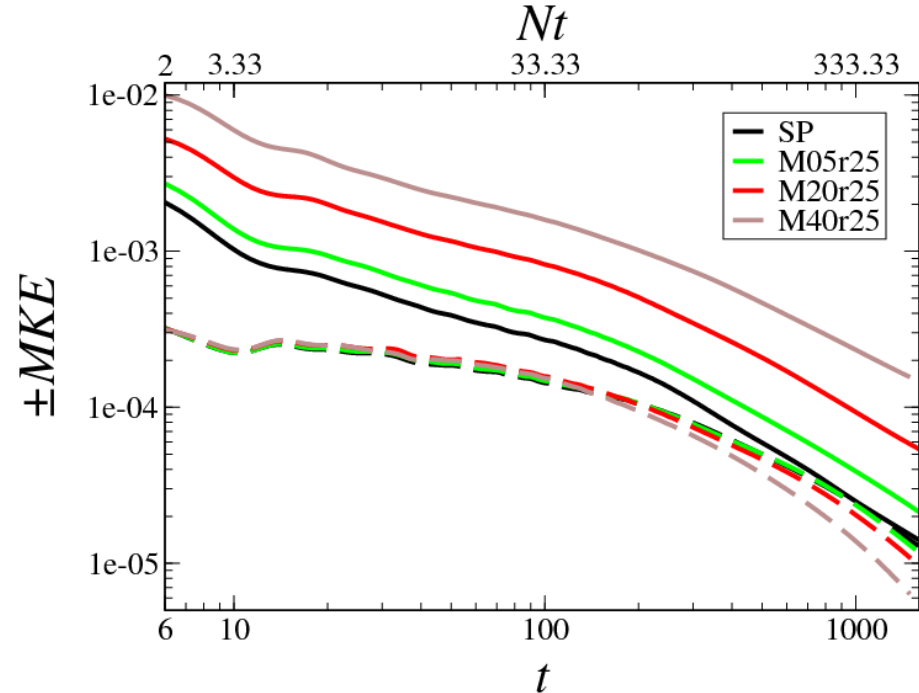
Larger structures formed with more excess momentum

Streamwise velocity structure and decay



Buoyancy preserves 3 lobe structure in late wake

Velocity decoupled



Solid line $\max(+U_1)$. Dashed line $\max(-U_1)$.

Small differences in drag lobe evolution with %M

Conclusions

Narrower shape and/or increased amount of excess momentum gives

Larger defect velocity, MKE, sharper velocity gradient

Increased production, TKE, and dissipation

Small differences in internal wave flux

Buoyancy traps excess momentum near the vertical centerline

Three lobed structure preserved in the vertical direction

Increasing excess momentum leads to larger structures

A small amount of excess momentum does not qualitatively change behavior

Numerical Method

Fully explicit finite volume staggered grid method

Second ordered centered differences for spatial terms

3rd order low storage RK3 time advancement

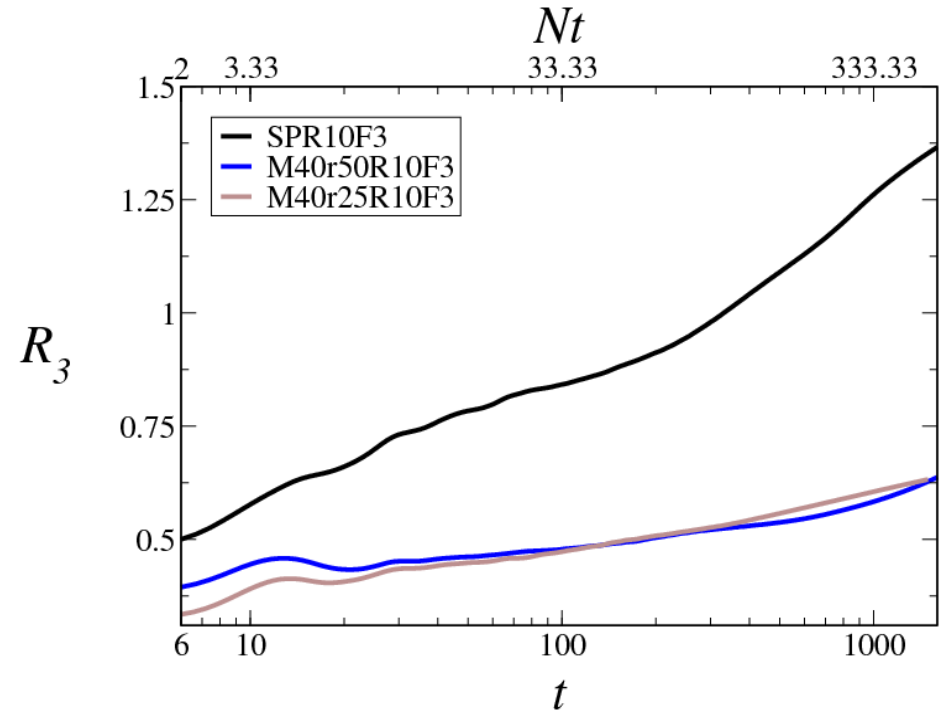
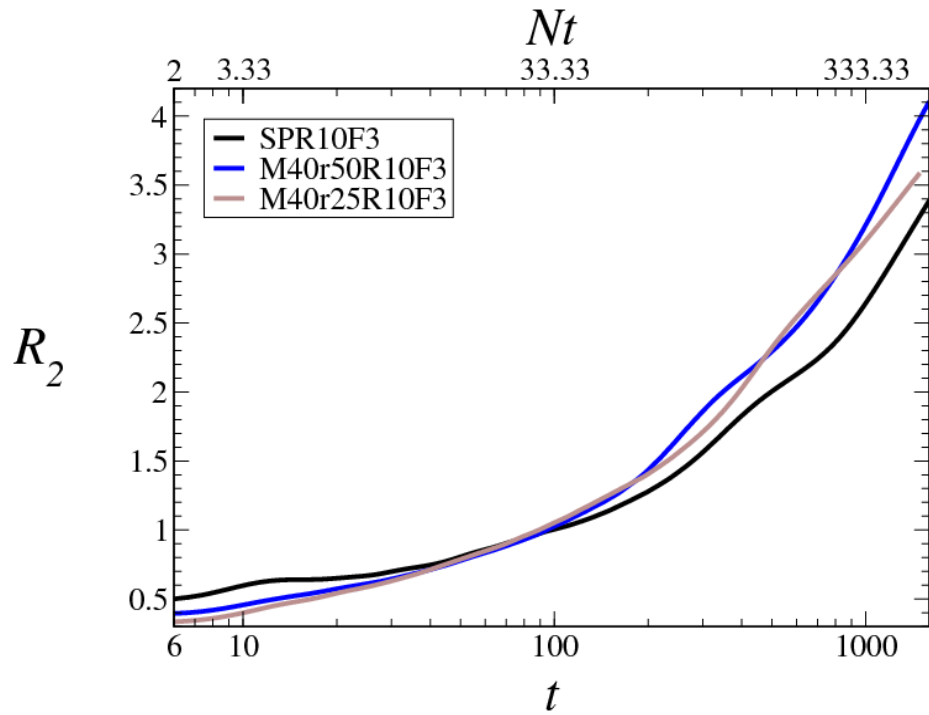
Parallel multigrid pressure solver

3-D domain decomposition using MPICH II

Sponge region at physical boundaries; streamwise periodicity

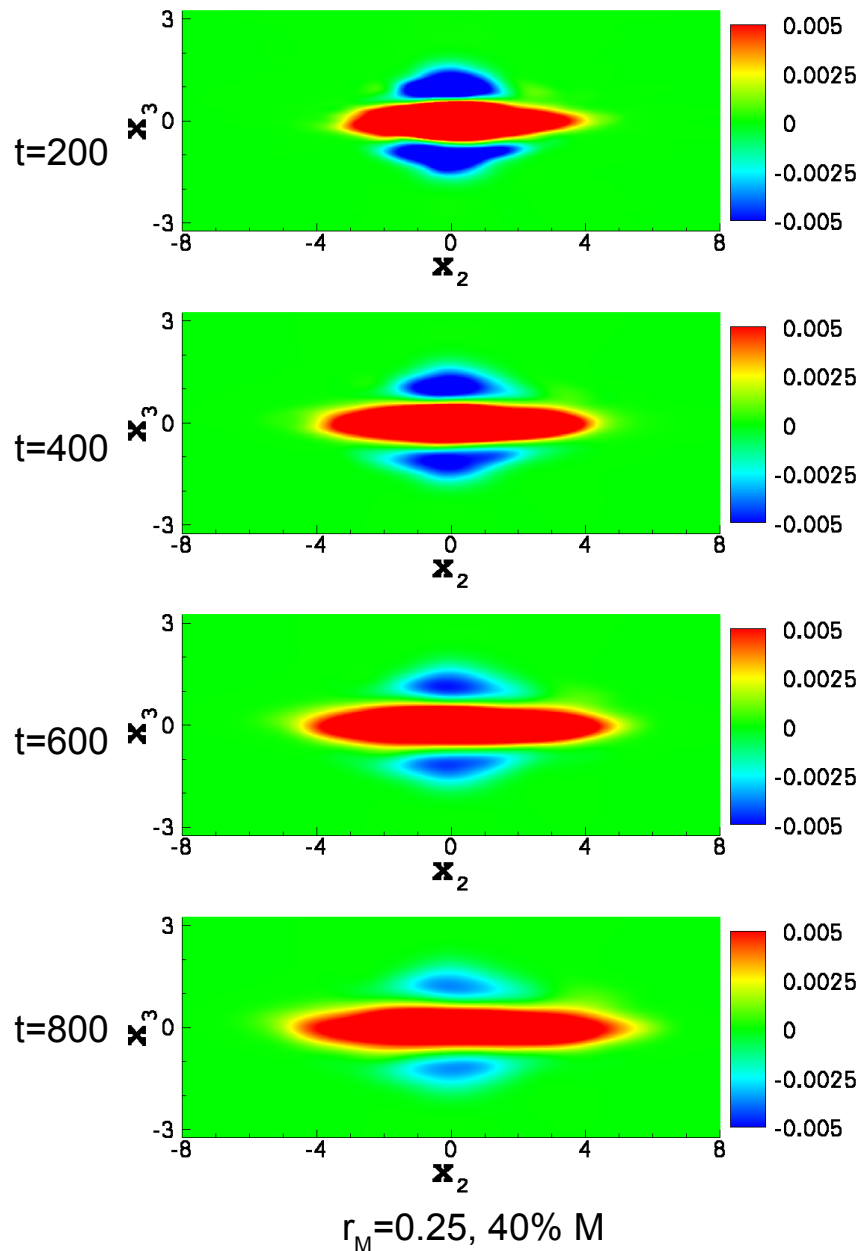
ICs: Idealized mean profile, broadband velocity fluctuations, no density fluctuations, adjustment period used to let correlations develop

Wake Dimensions



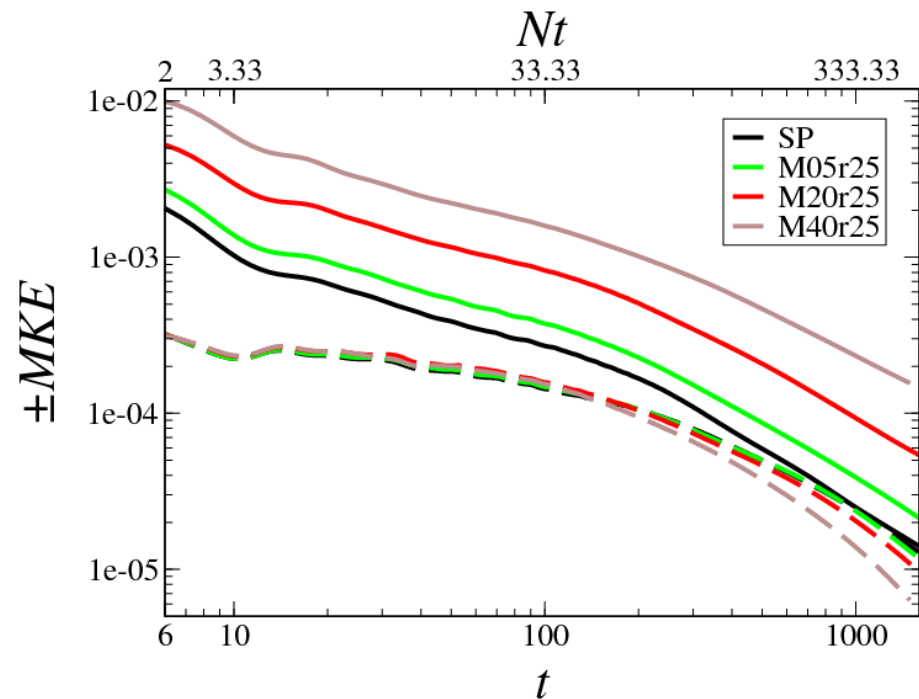
Excess momentum diffuses horizontally but not vertically
excess momentum remains close to vertical centerline

Streamwise velocity structure and decay



Buoyancy preserves 3 lobe structure in late wake

Velocity decoupled



Solid line $\max(+U_1)$. Dashed line $\max(-U_1)$.

Small differences in drag lobe evolution with %M

Motivation

Wake studies usually consider two cases:

Towed steady velocity

Self-propelled steady velocity

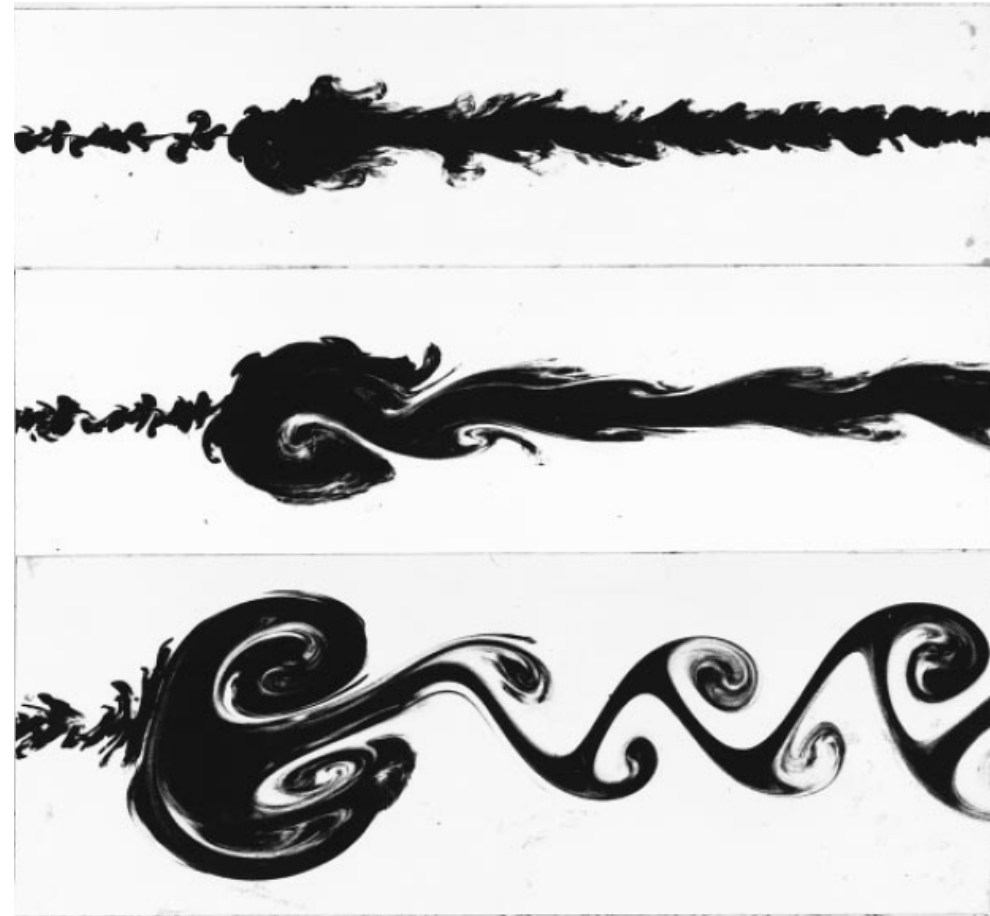
Neither case is realistic

Unsteady upstream conditions

Slight imbalances in thrust

Maneuvers

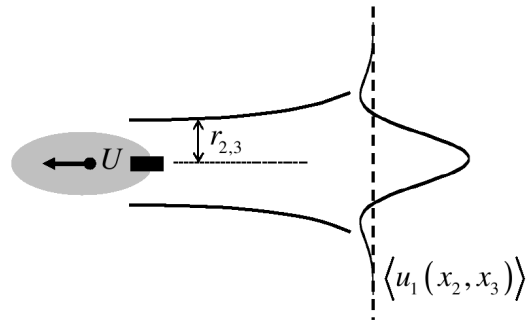
Large eddies observed
experimentally when
submersible accelerates



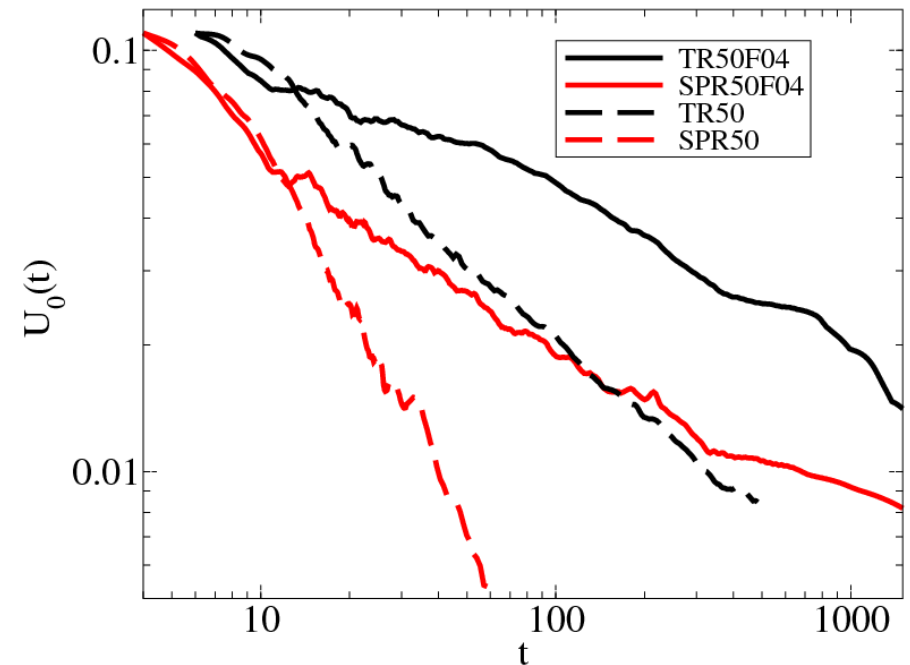
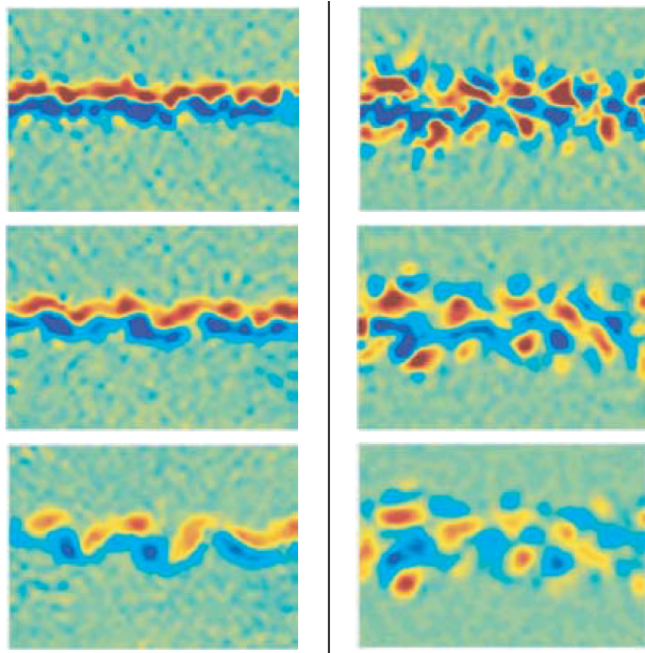
$Re=1274$, $Fr=26$

Voropayev et al. Physics of Fluids, 1999

Difference between self-propelled and towed wake



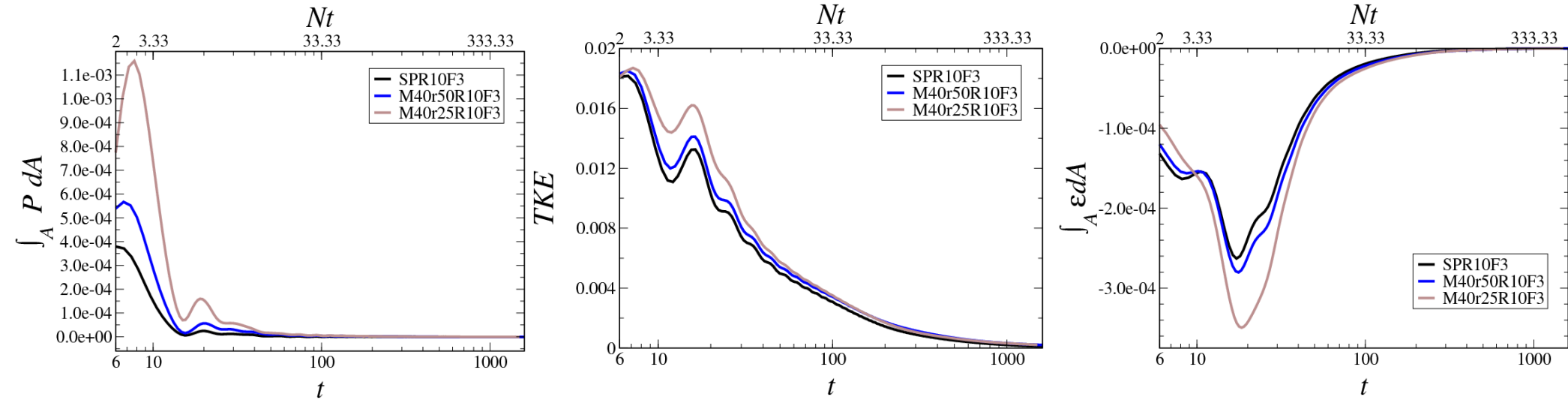
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Creator:fig2dev Version 3.2 Patchlevel 5
CreationDate:Sat Nov 20 12:44:15 2010



Vorticity data from Meunier and Spedding (2010) Data from Brucker and Sarkar (2010)

**Very different than
unstratified wake**

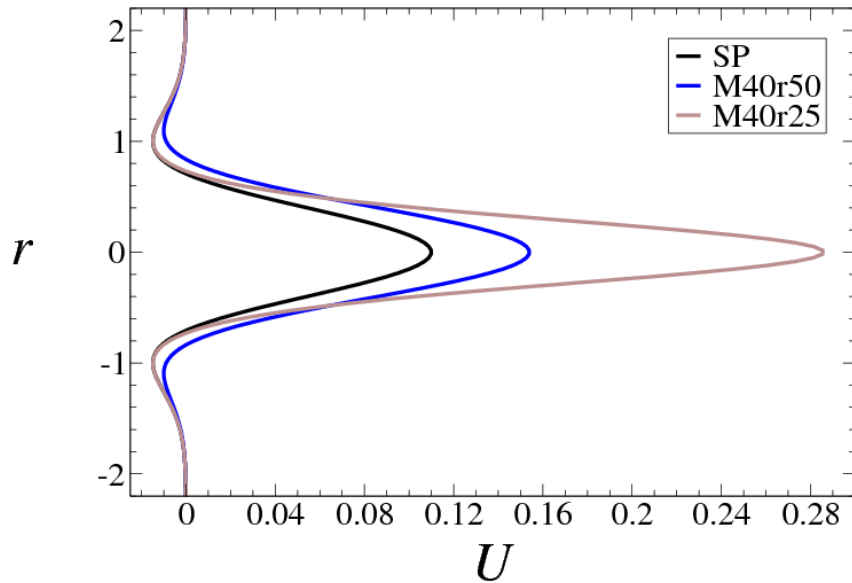
TKE and Wave Flux



Increased %M --> Sharper velocity gradient--> increased production --> increased TKE --> increased dissipation

Despite relatively large quantitative differences, qualitative agreement is generally quite good

Vary shape of excess momentum



Vary amount of excess momentum

Reducing radial extent of %M

Increases defect velocity

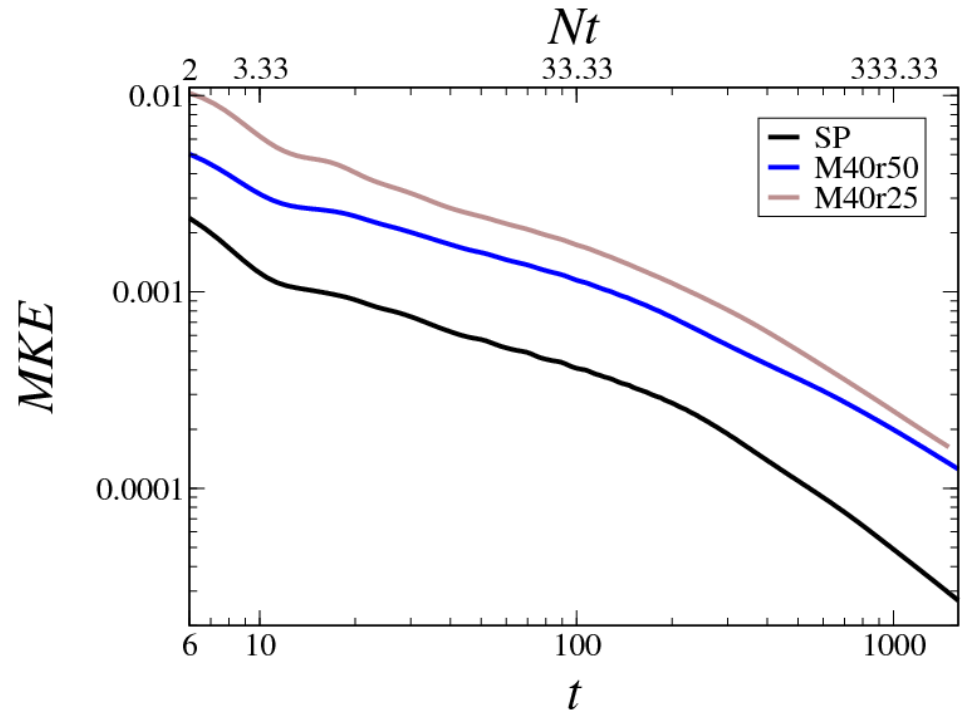
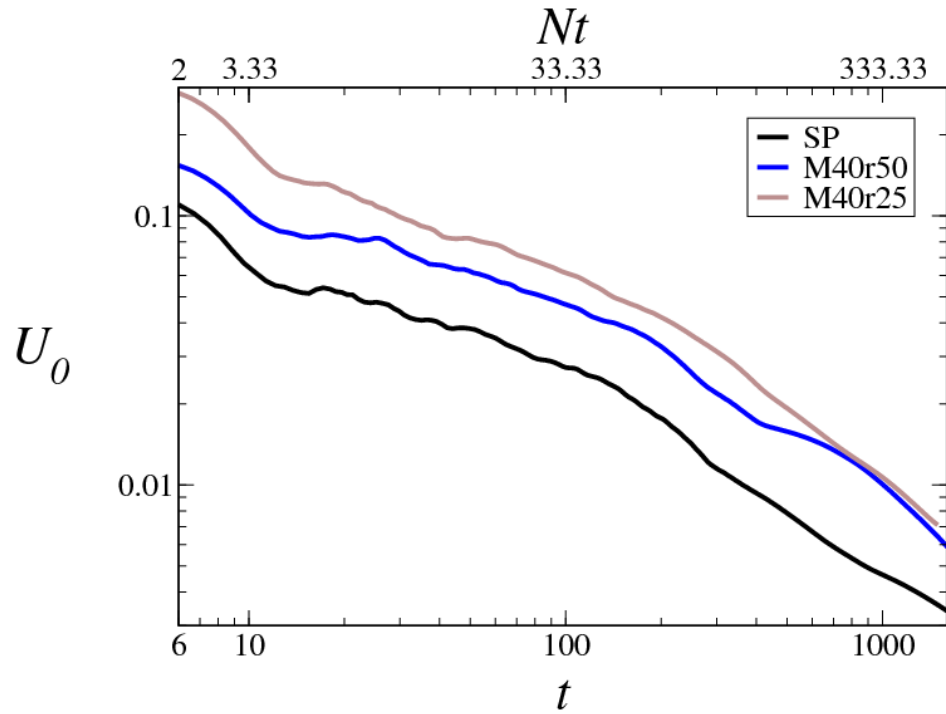
Increased MKE

Increases velocity gradient

r25 case does not disturb drag lobes

r50 case does

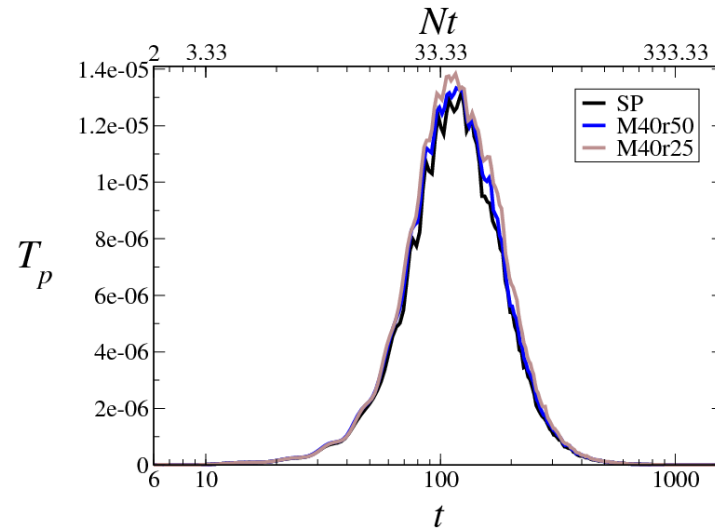
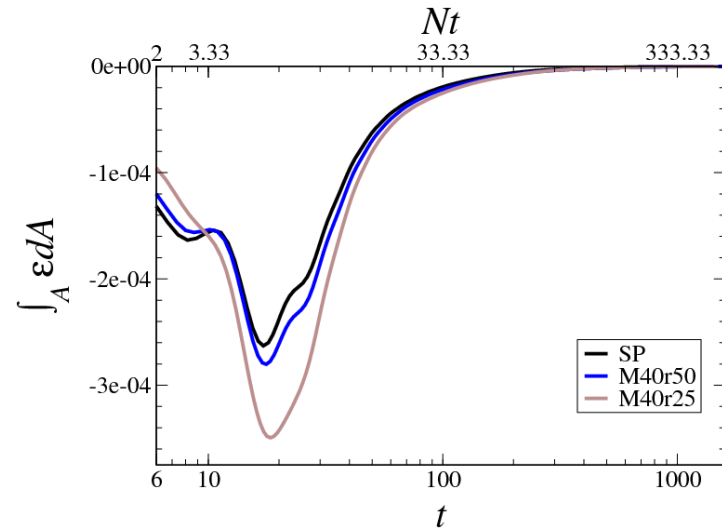
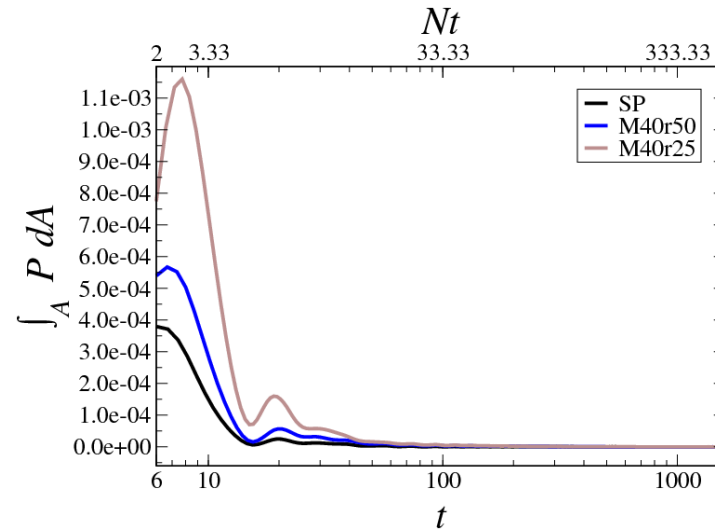
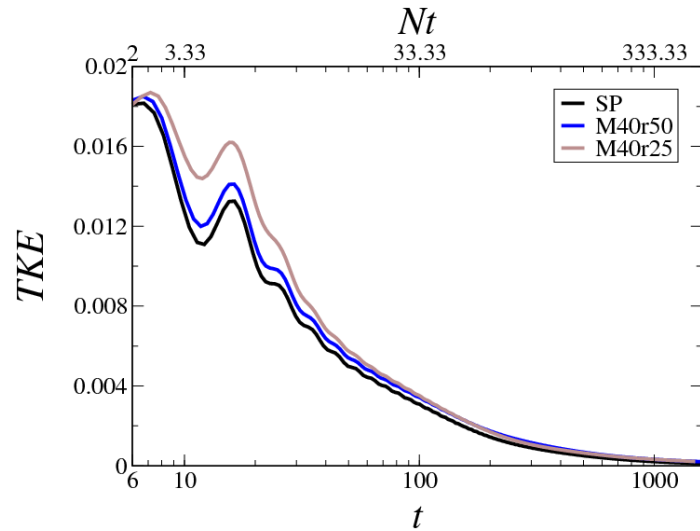
Defect Velocity and MKE



Narrower radial extent of excess momentum --> increased U_0 , MKE

Wake evolution follows similar trends compared to SP case for both cases

TKE and Wave Flux



Narrower radial extent --> Sharper velocity gradient--> increased production
--> increased TKE --> increased dissipation

Despite relatively large quantitative differences, qualitative agreement is generally quite good