Simulation of a self-propelled wake with small excess momentum in a stratified fluid

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Features of Turbulent Wakes in Stratified Fluids

Stratification breaks radial symmetry: vertical motion inhibited

Transfer between kinetic and potential energy

Internal waves radiated

Late time quasi-2D flow

Very different than unstratified wake
Classes of wakes

Two fundamental classes of wake

Towed: Drag only

Propelled: Thrust and Drag

(a) Towed wake, only drag
(b) Self-propelled wake, thrust=drag, net momentum=0
(c) Excess momentum wake, thrust > drag
Difference between self-propelled and towed wake

Vorticity data from Meunier and Spedding (2006)
(Left) Momentum wake (right) SP wake

Data from Brucker and Sarkar (2010)
Why this problem?

A self-propelled (SP) wake with zero net momentum is unrealistic

- Unsteady upstream conditions
- Slight imbalances in thrust
- Maneuvers

The self-propelled wake with zero net momentum is qualitatively different from a towed (drag wake)

Assuming self-similarity, Tennekes and Lumley (1972) determined that an unstratified wake with any excess momentum at late time is dominated by the excess momentum component

Meunier and Spedding (2006) found qualitatively different results with > 2% excess momentum in the wake vorticity field

Brucker and Sarkar (2010) found that a self-propelled wake decayed much faster than a towed wake. SP wake has smaller vortical structures.

Voropayev et al. (1999) and Voropayev and Fernando (2010) show the formation of large coherent structures in the late wake after an accelerating maneuver
Study focus

How is the wake affected by excess momentum?

- Amount of excess momentum
- Shape of excess momentum

Do larger structures form in the late wake with small amounts of excess momentum?

Interested in the effect of excess momentum on

- Defect velocity
- Vorticity dynamics
- Mean and turbulent kinetic energy and associated budgets
- Wake dimensions
- Internal wave flux

Does a small amount of excess momentum qualitatively change the wake evolution?
Problem approach

Use DNS to solve the Boussinesq Navier-Stokes equations in a temporally evolving frame

First DNS of a self-propelled wake with excess momentum

Start with a self-propelled profile

Impulsively add momentum with a Gaussian profile

Linear density stratification

Simulation parameters: \( \text{Re} = 10,000, \quad \text{Fr}=3, \quad \text{Pr}= 1 \)

![Graph 1](image1)

Vary amount of excess momentum

![Graph 2](image2)

Vary shape of excess momentum
Vary amount of excess momentum

Increasing % M

- Increases defect velocity
- Increased MKE
- Increases velocity gradient
Defect Velocity and MKE

Greater excess momentum gives increased $U_0$, $MKE$

Good qualitative agreement even in 40% momentum case

Three flow regimes are evident in all cases
Defect Velocity and MKE

Data collapses when scaled by initial value

Collapse is better or $MKE$ than $U_0$
Wake Dimensions

Excess momentum diffuses horizontally but not vertically

Excess momentum remains close to vertical centerline
TKE and Wave Flux

Increased %M --> Sharper velocity gradient --> increased production --> increased TKE --> increased dissipation

Despite relatively large quantitative differences, qualitative agreement is generally quite good
Wake Vorticity Field $\omega_3$ at $x_3=0$

Larger structures formed with more excess momentum
Streamwise velocity structure and decay

Buoyancy preserves 3 lobe structure in late wake

Velocity decoupled

Small differences in drag lobe evolution with $\%M$
Conclusions

Narrower shape and/or increased amount of excess momentum gives
Larger defect velocity, MKE, sharper velocity gradient
Increased production, TKE, and dissipation
Small differences in internal wave flux

Buoyancy traps excess momentum near the vertical centerline
Three lobed structure preserved in the vertical direction

Increasing excess momentum leads to larger structures

A small amount of excess momentum does not qualitatively change behavior
Numerical Method

Fully explicit finite volume staggered grid method

Second ordered centered differences for spatial terms

3\textsuperscript{rd} order low storage RK3 time advancement

Parallel multigrid pressure solver

3-D domain decomposition using MPICH II

Sponge region at physical boundaries; streamwise periodicity

ICs: Idealized mean profile, broadband velocity fluctuations, no density fluctuations, adjustment period used to let correlations develop
Wake Dimensions

Excess momentum diffuses horizontally but not vertically
excess momentum remains close to vertical centerline
Streamwise velocity structure and decay

Buoyancy preserves 3 lobe structure in late wake

Velocity decoupled

Small differences in drag lobe evolution with %M
Motivation

Wake studies usually consider two cases:

- Towed steady velocity
- Self-propelled steady velocity

Neither case is realistic

- Unsteady upstream conditions
- Slight imbalances in thrust
- Maneuvers

Large eddies observed experimentally when submersible accelerates

Re=1274, Fr=26
Voropayev et al. Physics of Fluids, 1999
Difference between self-propelled and towed wake


Very different than unstratified wake
TKE and Wave Flux

Increased %M --> Sharper velocity gradient--> increased production --> increased TKE --> increased dissipation

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Vary shape of excess momentum

Reducing radial extent of %M
- Increases defect velocity
- Increased MKE
- Increases velocity gradient

Vary amount of excess momentum

r25 case does not disturb drag lobes
r50 case does
Defect Velocity and MKE

Narrower radial extent of excess momentum --> increased $U_0$, $MKE$

Wake evolution follows similar trends compared to SP case for both cases
Narrower radial extent --> Sharper velocity gradient --> increased production --> increased \( TKE \) --&gt; increased dissipation

Despite relatively large quantitative differences, qualitative agreement is generally quite good