Shear Layer Simulations Using DIABLO

Matt de Stadler
June 13, 2008

Introduction

From linear stability theory it is known that a shear layer is unstable to infinitesimal perturbations. Numerical simulations using the DIABLO code were used to observe the evolution of the shear layer once the perturbations are of finite amplitude. The effect of varying domain size, initial perturbation strength, and viscosity were all investigated. Testing of the order of convergence in space and time was also performed for the DIABLO code.

Governing Equations

The Navier-Stokes equations for a 2-dimensional parallel shear flow are given below. The buoyancy term is included to allow for the presence of a temperature stratification.

$$\nabla \cdot u_d = 0$$

$$\frac{\partial u_d}{\partial t_d} + u_d \cdot \nabla u_d = -\nabla p_d + \upsilon \nabla^2 u_d + \alpha g (T_d - T_{d0})$$

$$\frac{\partial T_d}{\partial t_d} + u_d \cdot \nabla T_d = \kappa \nabla^2 T_d$$

Where the subscript d indicates that quantities are dimensional. These equations can be non-dimensionalized to give the form below.

$$\nabla \cdot u = 0$$

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 u + Ri_b (T - T_0)$$

$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \frac{1}{\mathrm{Re}\,\mathrm{Pr}} \nabla^2 T$$

Where

$$\mathrm{Re} = \frac{\delta_{w,0} \Delta U}{\nu}$$

$$Ri_b = \frac{g \alpha \Delta T \delta_{w,0}}{(\Delta U)^2} = -\frac{g \Delta \rho \delta_{w,0}}{\rho_0 (\Delta U)^2}$$

$$\mathrm{Pr} = \frac{\nu}{\kappa}$$

Initial Conditions

A hyperbolic tangent profile was given for both the u velocity and the passive scalar, written here as T.

$$u = \frac{\Delta U}{2}\tanh\left(\frac{2y}{\delta}\right)$$

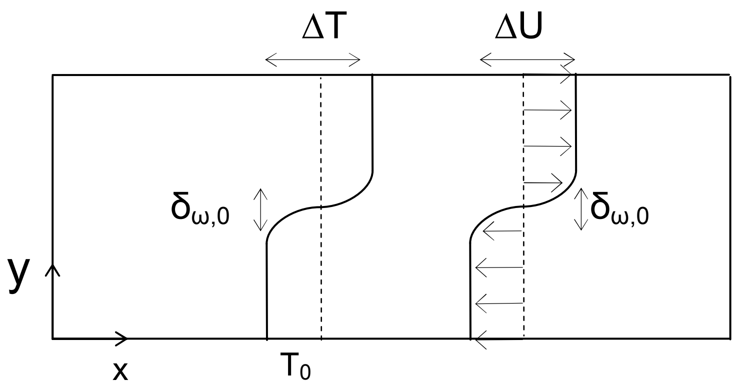
$$T = \frac{\Delta T}{2}\tanh\left(\frac{2y}{\delta}\right)$$

For the simulations in this paper, $\Delta U = \Delta T = 2$ and $\delta = 0.2$. Turbulent perturbations were added in the form

$$\hat{u} = (RNUM - 0.5) * kick * \exp((-20y)^2)$$

Where *RNUM* is a random number and *kick* is a scaling argument.

Figure 1 Velocity and Temperature Profiles



If stratification is present the pressure is hydrostatic, otherwise it is set to zero.

Boundary conditions

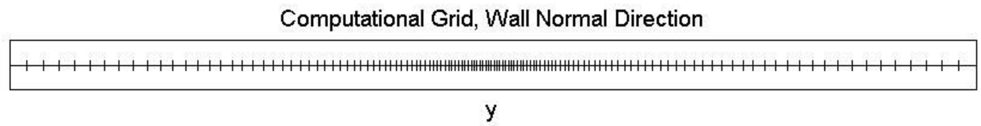
Neumann boundary conditions are imposed on the vertical velocity and horizontal velocity at the top and bottom boundaries. Dirichlet conditions are applied for the pressure. The grid is periodic in the x direction. The exact form of the boundary conditions used is given below for the top and bottom boundary.

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y} = 0, \frac{\partial T}{\partial y} = 0, p = 0$$

Grid Parameters

DIABLO is a mixed pseudospectral and finite difference code. In this problem, the grid is uniform in the x direction with spatial derivatives being done in Fourier Space. In the y direction finite differences are used with a stretched grid. An example grid in the y direction is shown. It is stretched so that more grid points are located in the shear layer.

Figure 2. Computational Grid



Qualitative behavior of the shear layer

After a short time wiggles become apparent in contour plots of the vorticity. One can also look at a passive scalar to observe the development of the shear layer and Kelvin-Helmholtz instability. These wiggles correspond to growing perturbations. They have a shape that comforms to the wavelength of a growing mode. It is known from linear stability theory that inviscid parallel shear flows are unconditionally unstable to infinitesimal perturbations.

Once the perturbations reach finite size they excite Kelvin-Helmholtz rollers. The number of these rollers that form is a function of the viscosity, domain size, Ri if used, and kick. Neighboring pairs of rollers will “pair” into one vortex and neighboring pairs will merge until one vortex is all that remains. This vortex will grow indefinitely, provided the horizontal and vertical domain size are large enough. For a given simulation, care should be taken not to use data that is gathered when the vortex is too close to the top wall.

Addition of temperature

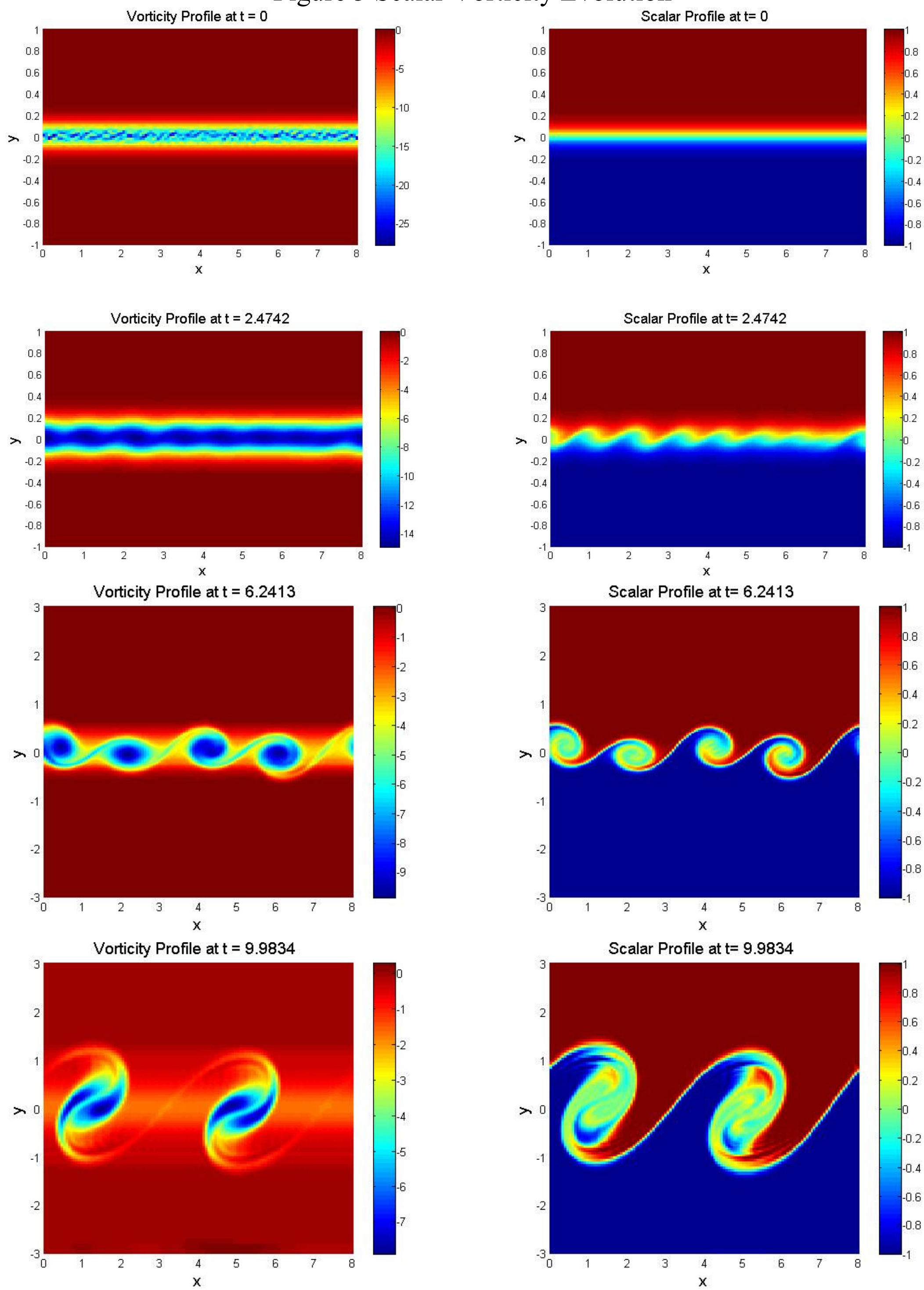
If temperature is considered as well, the problem behaves very differently. Now buoyancy must be considered which will cause a restoring force opposing the formation of the kh rollers. A necessary condition for instability is that somewhere in the flow the gradient richardson number is < 0.25. Unlike the unstratified case, a stratified shear layer will not continue to grow indefinitely. The shear layer will reach a maximum height, this occurs when there is a balance between the kinetic energy and the potential energy of the perturbations. The turbulence in the flow will cause the ordered vortex structures to break down as the time evolves. This results in smaller and smaller scales being formed which requires high grid resolution to capture.

Simulation Results

An example of some of the data that one can obtain from a shear layer simulation is given below. The variable u is used to refer to the horizontal velocity and v the vertical velocity. These simulations were all run with the following parameters:

Simulation parameters
LX = 8, LY = 6, NX = NY = 128, $\nu = 0.001$, Kick = 0.03, Number of timesteps 4000, Initial dt = 0.025. Data was recorded every 10 timesteps.

Figure 3 Scalar Vorticity Evolution



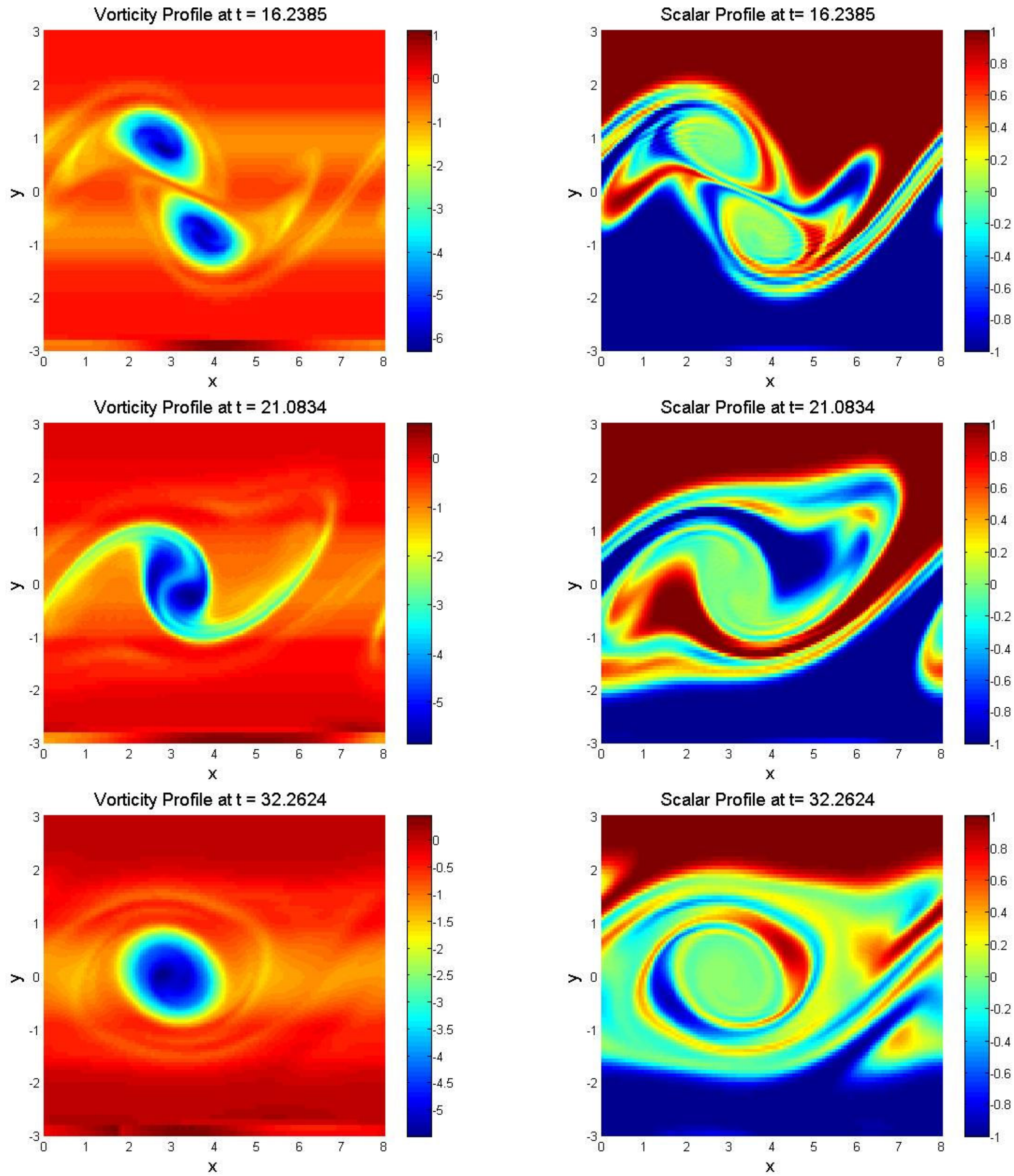
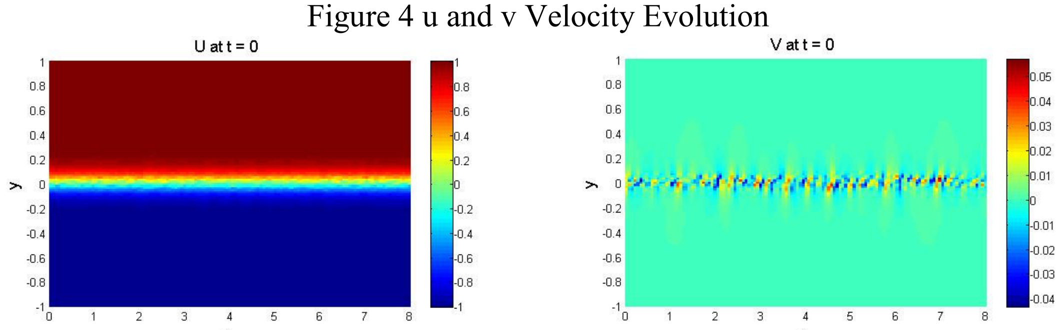
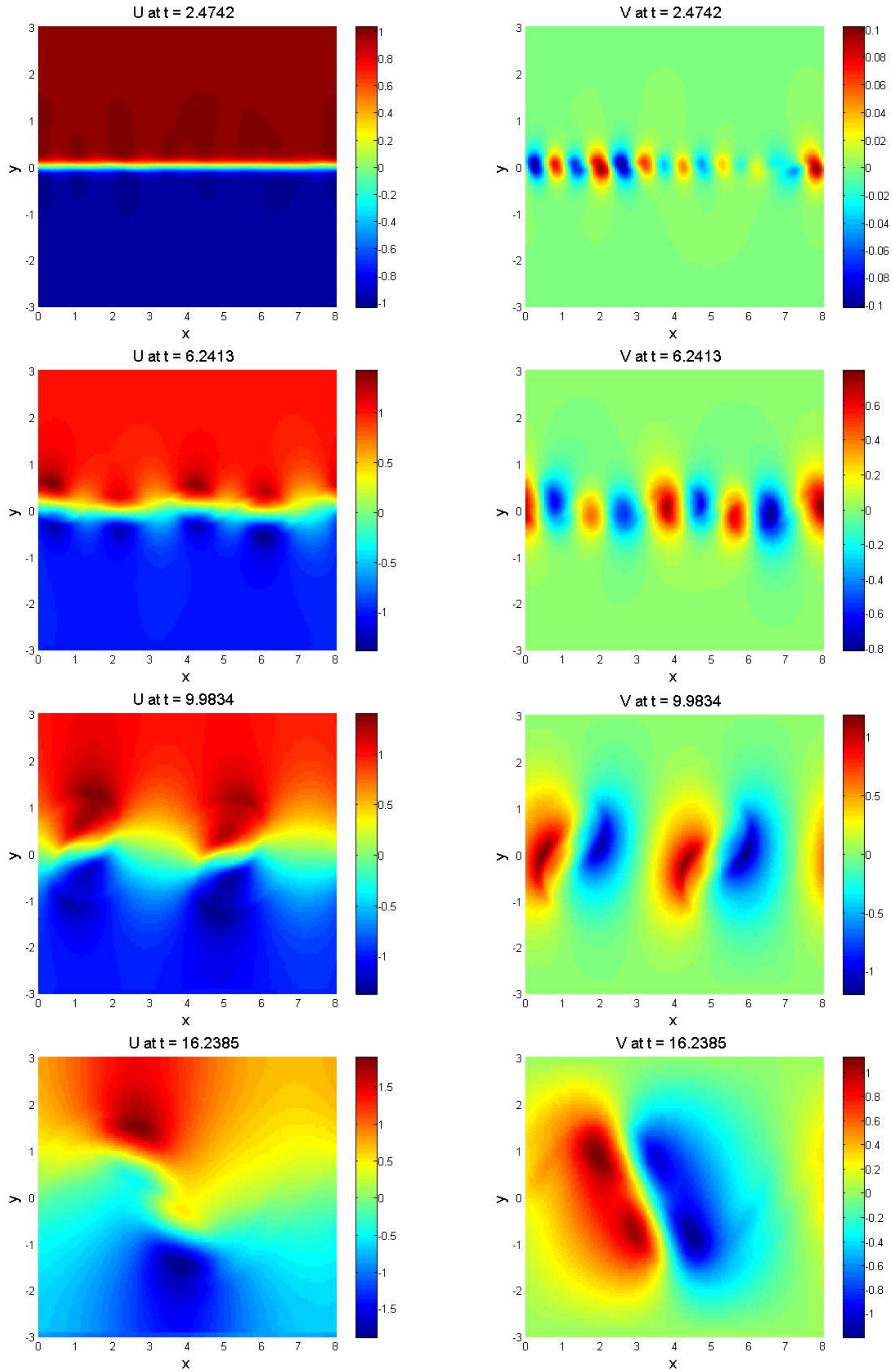


Figure 4 u and v Velocity Evolution





U at t = 2.4742
V at t = 2.4742
U at t = 6.2413
V at t = 6.2413
U at t = 9.9834
V at t = 9.9834
U at t = 16.2385
V at t = 16.2385
x
y

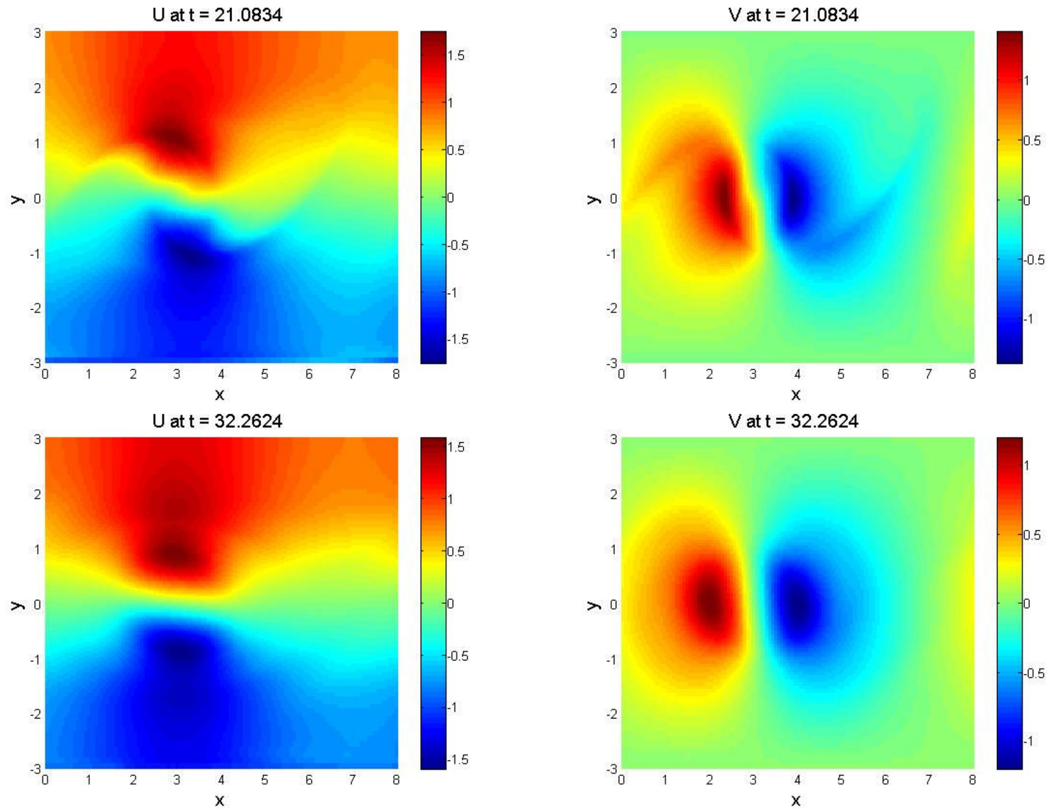


Figure 5 Evolution of <U>

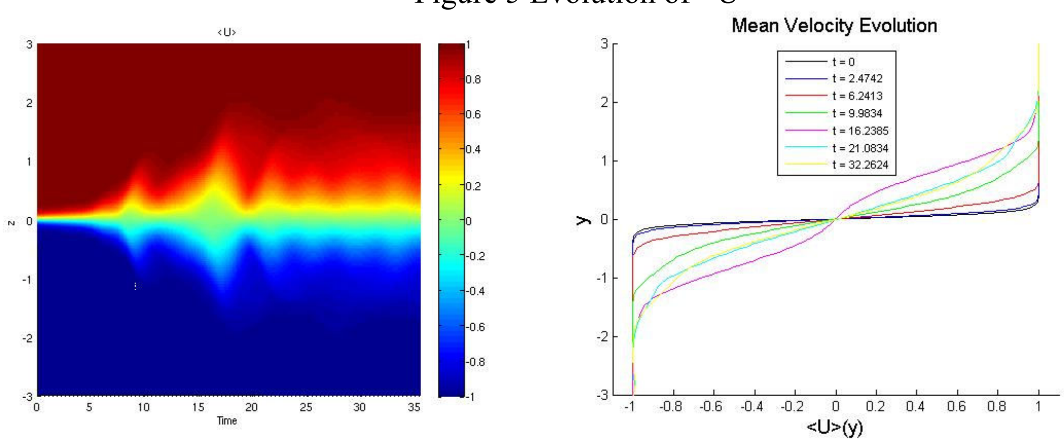


Figure 6 Streamlines

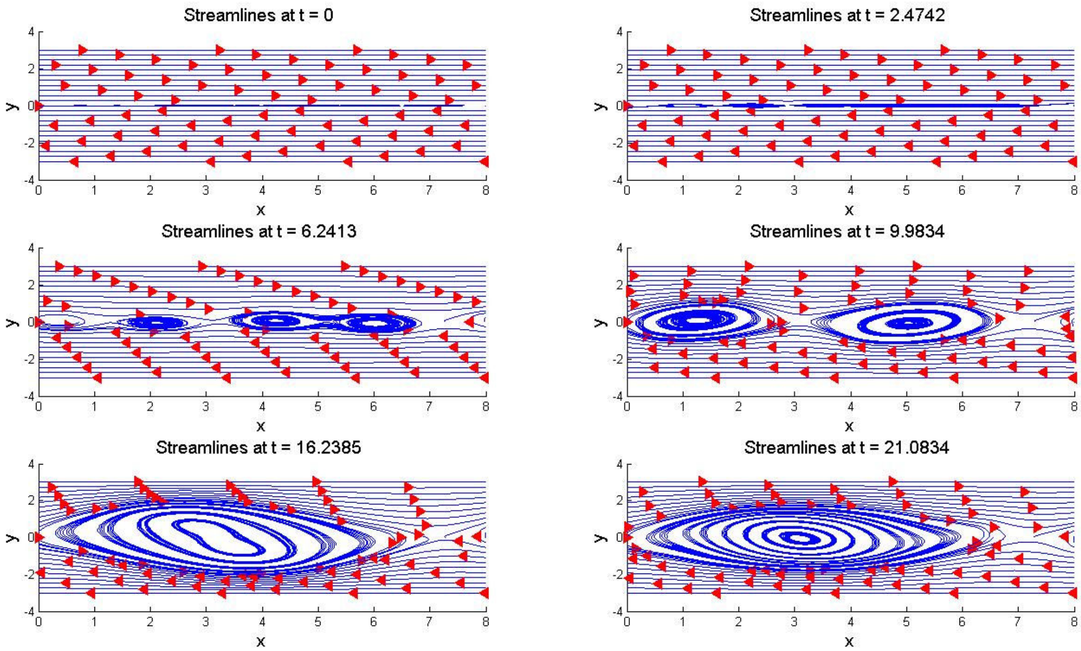


Figure 7 Vorticity Evolution

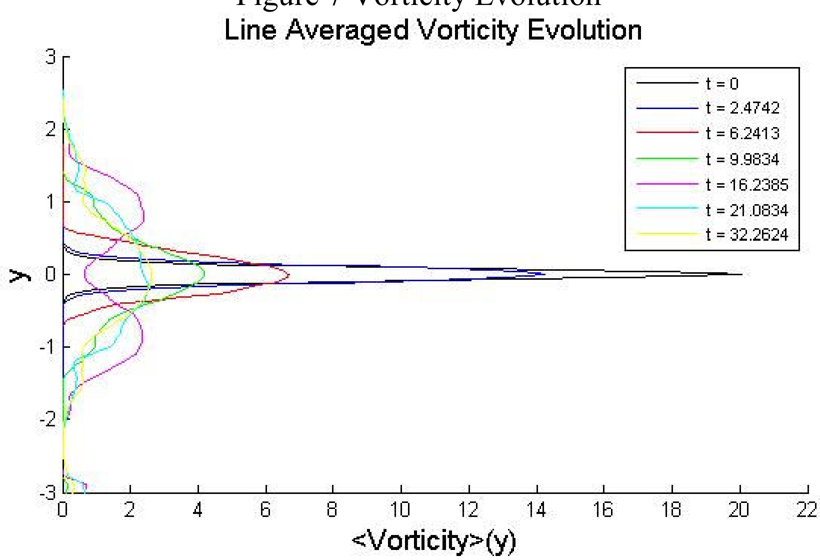
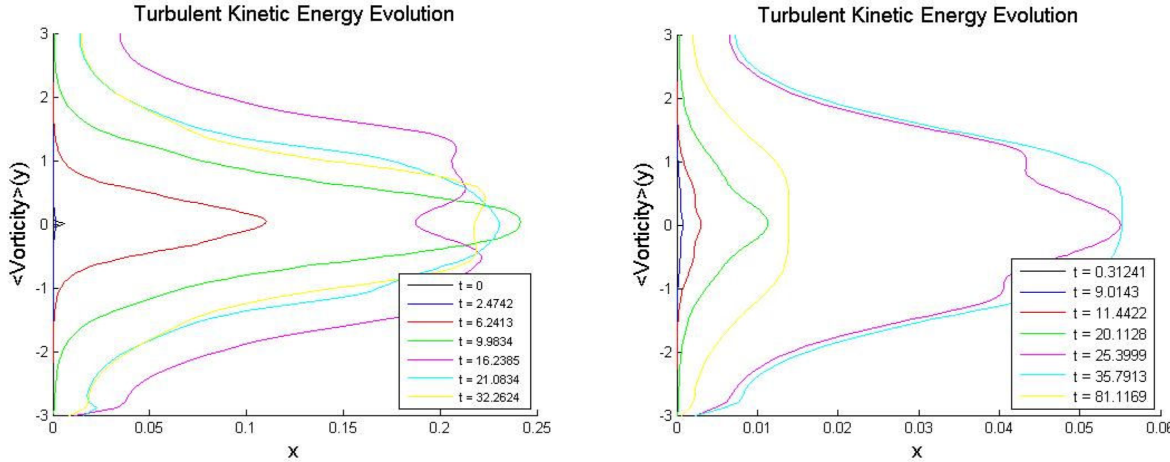


Figure 8 Turbulent Kinetic Energy Evolution



ν = 0.001 ν = 0.005

From the above images we can see about 8 Kelvin-Helmholtz rollers forming. This can be seen most clearly in the second scalar contour plot of Figure 3. We can later observe the process of pairing as the multiple rollers combine to form one large vortex. Viscosity causes the shear layer to grow, this can be seen most clearly by looking at <U> in Figure 5. Streamlines can be seen in Figure 6.

The maximum vorticity decreases over time and the thickness of the shear layer increases over time. The “sideways M” shape from time t= 16.2385 occurs because two vortices are at an early stage of pairing and each has high vorticity in their core. The two vortices merge into one and the vorticity is again “hump” shaped.

The streamlines in figure 6 show that away from the Kelvin-Helmholtz rollers the flow is parallel and in opposite directions. Inside the rollers the flow moves in an ovular path around the center.

The turbulent kinetic energy initially decreases but quickly increases as the Kelvin-Helmholtz Instability leads to turbulence. The increasing thickness of the shear layer can be observed from figure 8 as well as the existence of pairing. Pairing is shown by the double hump that occurs from the presence of two vortices on top of each other. At longer times one would observe the decrease in turbulent kinetic energy due to viscous dissipation. This is shown for the case of ν = 0.005 (the yellow line).

Another detail that can be observed from the numerical simulations is that there is an error forming at one of the boundaries. This can be seen at the bottom domain in the vorticity plots of figure 3 and also at the bottom at figure 7. One could remedy this error by imposing a dirichlet boundary condition, u = sign(1) at the top and bottom boundaries instead of using the neumann condition. One should not however that while this error is occurring, the effect appears to be localized and it does not affect the other results.

Numerical Aspects

Convergence in time

To get an estimate of the order of convergence in time, simulations were conducted to observe the difference between an exact value (taken to be the results from an extremely small timestep) and a value obtained from a given value of Δt. The values used as an estimate for the error were the mean horizontal velocity <U>(y) and the line averaged vorticity <ω>(y). The simulations were carried for small time to measure the error in the exponential growth phase of the perturbations. Another useful estimate would be to run this for a longer time so that a measurement could be taken when non-linear effects are strong. Unfortunately time constraints prevented this test from being performed.

<u>Testing parameters</u>
kick = 0.003, LX = 6, LY = 4, ν = 0.001, NX = NY = 128, t_{final} = 0.75,
exact values from dt = 0.0015625, 480 timesteps

dt	<U> error	<ω> error
0.025	0.2554*10^-5	0.5088*10^-4
0.0125	0.0631*10^-5	0.1258*10^-4
0.00625	0.0150*10^-5	0.0300*10^-4
0.003125	0.0030*10^-5	0.0060*10^-4

Order of accuracy **2.1308** **2.1287**

Convergence in x

A similar test as the one described above was performed for the spatial convergence in the x-direction.

<u>Testing parameters</u>
kick = 0.003, LX = 4, LY = 4, ν = 0.001, NY = 128, t_{final} = 0.75, dt = 0.003125, 240 timesteps
exact values from NX = 512, dx = 0.0078125

dx	<U> error	<ω> error
0.125	0.3636*10^-5	0.2044*10^-3
0.0625	0.3359*10^-5	0. 1819*10^-3
0.03125	0.2616*10^-5	0.1351*10^-3
0.015625	0.1696*10^-5	0.0447*10^-3

Order of accuracy **0.3661** **0.7011**

The data from this study does not have good agreement for an exponential relationship between the error and dx. The results were surprising because the code is intended to be spectral in the x-direction and higher accuracy was expected.

Convergence in y

For this case it was necessary to define a different measure for comparison since problem varies in the y-direction. The stretching of the grid creates further difficulty because for a different number of grid points, in general points are not collocated. The plane averaged vorticity was used to compare the effect of different values of dy.

Testing parameters
kick = 0.003, LX = 4, LY = 2, NX = 64, ν = 0.001, t_{final} = 0.75, dt = 0.003125, 240 timesteps
exact values from NY = 512, average dy = 0.00390625, mindy = 0.0014268

Average dy	Minimum dy	Plane averaged vorticity error
0.0625	0.02479	0.2480
0.03125	0.01182	0.1133
0.015625	0.00579	0.0480
0.0078125	0.00287	0.0159

Order of accuracy based on average dy: 1.3123
Order of accuracy based on minimum dy: 1.2647

Parameter Study

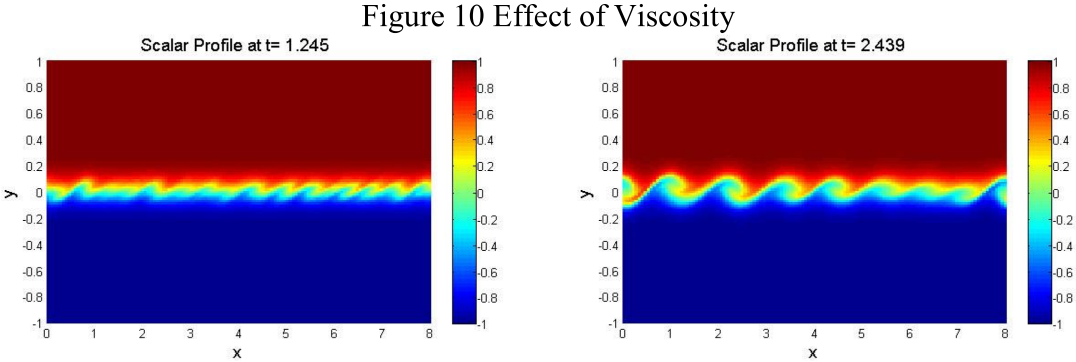
Unfortunately Diablo is built to run for a specified number of timesteps. This combined with the variable timestep makes it difficult to run longer time parameter studies. One way to get avoid this issue is to run with a very small time step but this has the disadvantage of requiring a long computational time. The following sections describe the effect of different problem parameters on the shear layer evolution and the Kelvin-Helmholtz Instability.

Varying the viscosity

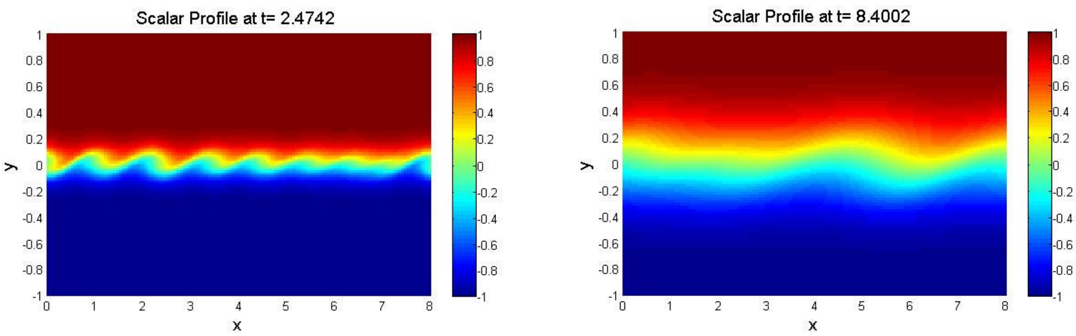
The effect of the viscosity is shown below in Figure 10. The first rollers to form are shown.

Simulation parameters
LX = 8, LY = 6, NX = NY = 128, ν = 0.001, Kick = 0.03, Number of timesteps 4000,
Initial dt = 0.025. Data was recorded every 10 timesteps.

Figure 10 Effect of Viscosity



ν = 0.0005　　ν = 0.0005



ν = 0.001 ν = 0.005

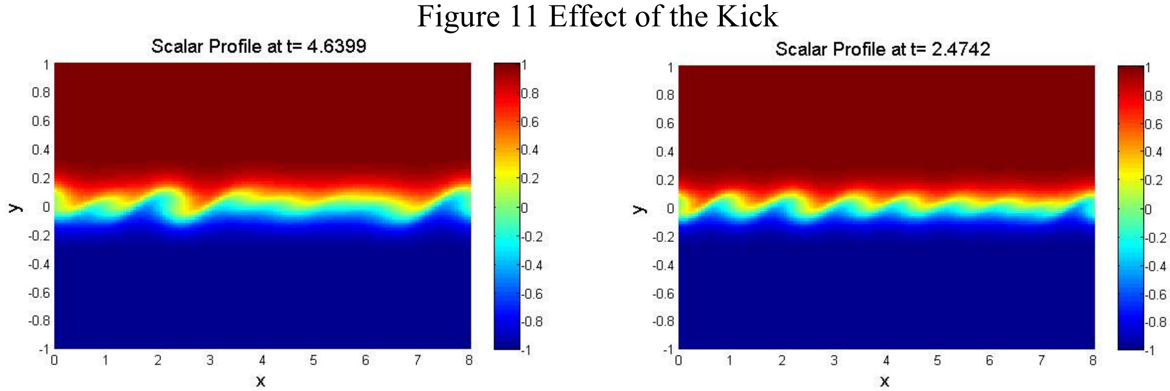
The above figures show clearly the strong influence that viscosity has in this problem. This is most dramatic in the ν = 0.005 case where the thickness is significantly larger and the number of rollers that form is significantly lower. For lower viscosities, the rollers will form faster and smaller wavelength rollers can form as shown in the first figure for the ν = 0.0005 case. Viscosity will damp out and inhibit the growth of higher frequency modes, this results in a slower growth rate and can also result in a different number of rollers forming.

Varying the kick

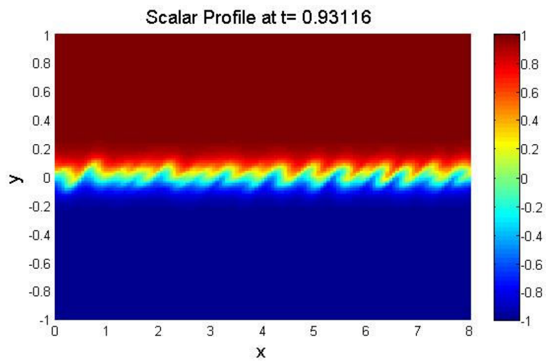
The effect of the kick is shown below in Figure 11. The first rollers to form are shown.

<u>Simulation parameters</u>
LX = 8, LY = 6, NX = NY = 128, ν = 0.001, Kick = 0.03, Number of timesteps 4000,
Initial dt = 0.025. Data was recorded every 10 timesteps.

Figure 11 Effect of the Kick



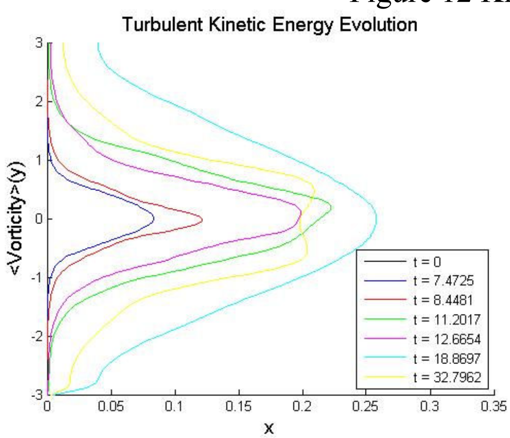
kick = 0.006 kick = 0.03



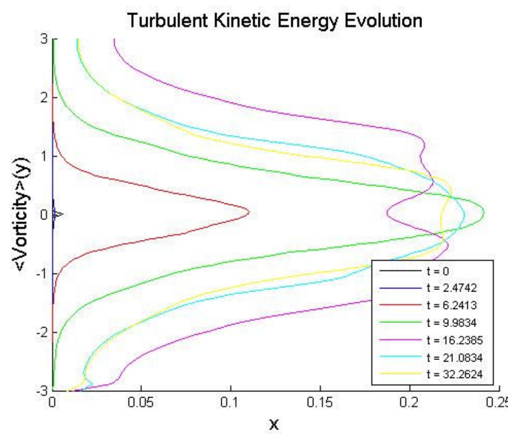
kick = 0.1

The kick gives a measure of the energy that will be present in the perturbations. Larger values result in more energy in all of the modes. This significantly increases the growth rate as shown above. More rollers will form for a larger value of the kick. If the kick is raised to a sufficiently large value, observable ordered vortex structure will not form and transition to turbulence will begin immediately.

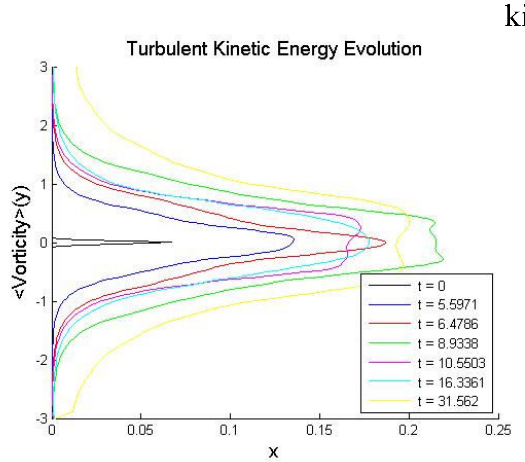
Figure 12 Kick effects on TKE



kick = 0.006



kick 0.03

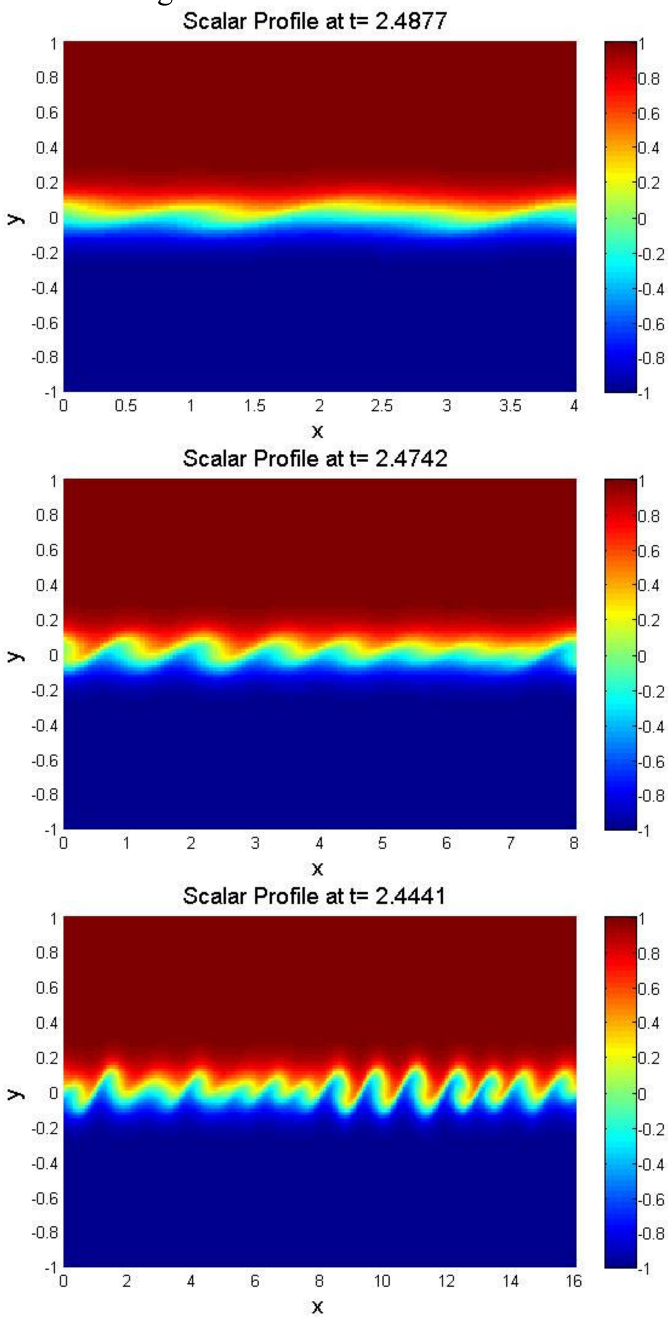


kick = 0.1

Varying the domain size

LY = 6, NY = 128, ν = 0.001, Kick = 0.03, Number of timesteps 4000, initial dt = 0.025. Data was recorded every 10 timesteps. NX was chosen so that dx is the same in all 3 cases.

Figure 13 Different Domain Sizes



In principle there should be no difference when the horizontal size of the domain is changed as the base state does not vary in the x-direction. In practice there will inevitably be small differences owing to the random perturbation structure and the non-linearity of the problem. With random initial conditions it is not possible to construct exactly the same simulation. For LX

= 4 four rollers form, for LX = 8, 8 form and for LX = 16, 15 rollers are observed. The time of plotting is slightly different but the qualitative behavior agrees very well.

Increasing the domain size in the vertical direction will not have an effect provided the domain size is sufficiently large that any vortices formed do not grow to the boundary. The vertical domain in reality is $-\infty$ to ∞ but by choosing appropriate boundary conditions one can model this using a finite boundary. It is best if the vertical boundaries are sufficiently far away that they have no effect on the flowfield.

Conclusions

Disturbances in a shear layer between two parallel, counterflowing streams evolve in a fixed manner. Initial perturbations grow and create Kelvin-Helmholtz rollers, these rollers then pair with their surrounding neighbors, which pair with their surrounding neighbors, until one large vortex is formed. This result was shown through the above simulations.

Two important results from this study are that the viscosity and the initial kick are important parameters which have a large impact on the evolution of disturbances in the shear layer. A larger viscosity will slow down the growth rate of rollers and prevent the formation of rollers with a small wavelength. A larger initial kick gives more energy to the perturbations and increases their growth rate and allows smaller wavelength rollers to form.