Optimization of the Geometry of a Heat Sink

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Introduction

Recent developments in cellular materials allow for the consideration of heat sink designs previously not possible. Cellular materials have properties similar to conventional materials on the macroscopic level but possess several advantages at the microscopic level, including the ability to be manufactured down to a very small scale.[1] Their superior properties make them suitable for a wide range of applications where size, weight, and efficiency are important.

This project aimed to take advantage of the benefits of cellular materials for use in heat sink design. The goal for this project was to develop a methodology which would allow one to develop an optimal heat sink based on geometrical considerations independent of working fluid choice and heat sink material choice. For a fixed volume heat sink with a given percentage of the volume available for cooling channels, two questions were posed:

- How should one layout the channels?
- What shape should the channels be?

To answer these questions, a purely theoretical approach was employed, with testing and analysis performed using computational fluid dynamics (CFD) software.

Problem Formulation

To determine an optimal design for a heat sink, the principle of superposition was used to reduce the design area to the smallest repeating cell. This allows one to determine the characteristics of the complex heat sink from a simple repeating cell. Within the cell, a number of design variables are able to be considered including:

- Number of channels
- Placement of channels within the normal face of the metallic block
- Channel geometry (circle, square, triangle,…)
- Channel shape (straight versus coiled tube)
- Fluid mass flow rate

Another goal of this study was to ensure that the minimum amount of working fluid would be needed to provide the desired cooling.

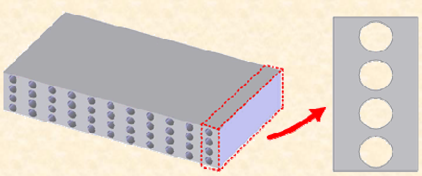

Figure 1. Repeating cell concept.

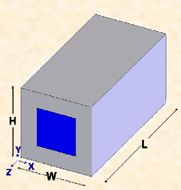

Figure 2. Cell dimensions.

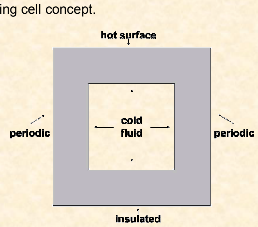

Figure 3. Boundary conditions.

2-D Conduction

2-D conduction in a square with cooling channels was examined to get an idea of the role that conduction would play in this problem.

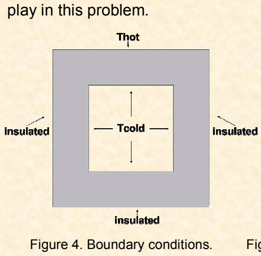

Figure 4. Boundary conditions.

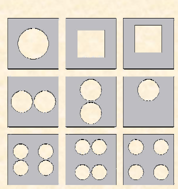

Figure 5. Layouts considered.

Designs with one large hole located near the top performed much better than their competitors. This is because the pumping power penalty increases dramatically with the number of holes. The driving force for the heat flux is the distance between the hot surface and the cooling channel.

$$\dot{Q} = kA_n \frac{dT}{dx}$$

This value is maximized by minimizing the distance between the two different temperature surfaces. The improved heat transfer from multiple holes is outweighed by the pumping penalty.

Performance Curves

This work was done to gain insight into the relationship between input parameters such as the mass flow rate and number of holes, and output parameters such as the temperature at critical locations and pumping power required.

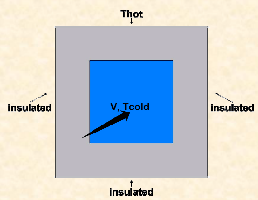

Figure 6. Boundary conditions.

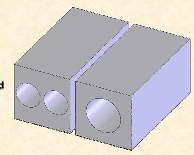

Figure 7. Shapes considered.

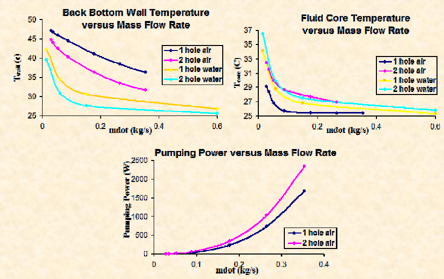

Figure 8. Performance curves.

All of the performance curves generated showed monotonically increasing or decreasing values. Increasing the mass flow rate of the cooling fluid increases the pumping power required. It also lowers the back bottom wall temperature and the fluid core temperature at the exit.

Performance metric

For optimization purposes, it is useful to have a single quantitative performance measure which can be minimized or maximized as an objective function subject to specific performance constraints. Four goals were sought for an optimal heat sink:

- Removes the greatest amount of heat
- The fluid just reaches a fully hot state as it exits
- The bottom wall temperature is as close to as possible while still below a critical temperature
- Uses the smallest amount of pumping power

Each of the above goals was written in equation form and then non-dimensionalized by a problem relevant quantity. The beneficial aspects were emphasized while the disadvantageous one was penalized. The following metric was obtained:

$$score = \frac{[\dot{Q}/\dot{Q}_0][T_{core}/T_{hot}][T_{bottom}/T_{critical}]}{[P/P_0]}$$

$$P_0 = \frac{\mu(\dot{m}/\rho A_{hole})^2 L A_{hole}}{H^2}$$

$$\dot{Q}_0 = \frac{k(T_{hot}-T_{in})LW}{H}$$

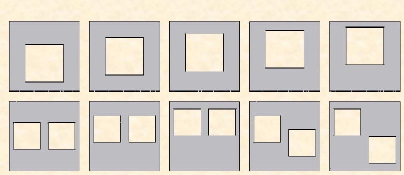

Figure 9. Shapes considered. *Note that models are 3D but are shown in 2D for ease of viewing.*

The best designs were single channels located as close to the hot surface as possible. The additional pumping power requirements outweighed the increased heat flux benefit from more holes. There was not enough variation in the fluid core and back bottom wall temperatures between the different designs to make much of a difference in the final score.

The fixed temperature boundary condition for the hot surface is not a good choice for the problem. Such implicit dependence of the heat flux on the proximity of the cooling channel to the surface results in the lack of a baseline problem. Without such a baseline case it is not possible to conduct a rational optimization study. A better choice would be to use a constant heat flux boundary condition. A constant heat flux prevents designs with channels located close to the top surface from drawing upon an infinite reservoir of heat.

Thermal Network Analysis

Valdevit et al. used a thermal resistance network analysis with a fin analogy to determine the temperature profile in a square channel and obtained good results.[2] The network being developed in this project aims to further generalize their results by being able to obtain reasonable answers for designs where lateral conduction also plays a significant role and 2-D effects need to be considered.

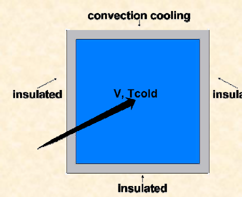

Figure 10.Boundary conditions.

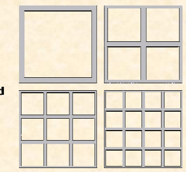

Figure 11. Possible geometries. *Designs with n^2 holes are being examined.*

Table 1. Problem Parameters

Inputs
Material properties
Model dimensions
Fluid inlet flow rate
Fluid inlet temperature
Hot temperature value
Outputs
Node temperatures
Node heat fluxes
Fluid exit temperature

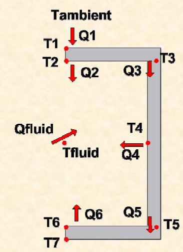

Figure 12. Simplified thermal network with temperatures and heat fluxes shown.

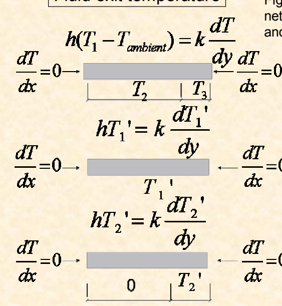

Figure 13. Heat conduction in the upper right corner. *The top square shows the problem to be solved. The problem is broken down into two-sub problems and their solutions are superposed to form the desired solution.*

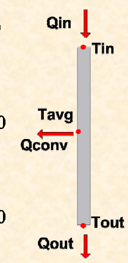

Figure 14. Side element analysis.

After relating all the temperatures and the heat fluxes in the solid, the conservation of energy is applied to relate the fluid and solid temperatures. The resulting system of equations is solved for the exit fluid temperature and from that all other values of interest are derived.

Conclusions

Initial attempts at developing a systematic method to determine an optimal layout for a heat sink have proven unsuccessful. Both framing the problem and determining a valid performance metric for ranking different designs have proven significantly more difficult than anticipated. The majority of this work has been done with a constant surface temperature boundary condition which has consistently led to designs with one large square channel located close to the hot surface being shown as the best solution. This result is in contrast to current designs and leads one to believe that the problem to be solved has not been adequately formulated.

The greatest challenge remaining is how to frame the problem so that appropriate optimization techniques can be applied. As the skin cooling of a hypersonic vehicle application has shown, this research does have practical applications and will continue to become more important in the near future.

Future Work

Research should continue in the two following areas:

- Investigation of optimal geometries using a constant surface heat flux approach
- Complete and apply a thermal network approach

Acknowledgments

This work was supported by the Virginia Space Grant Consortium through an Undergraduate Aerospace Research Scholarship.

References

[1]Gibson, L. J., and Ashby, M. F., *Cellular Solids: Structure and properties*, 2nd ed., Cambridge University Press, United Kingdom, 1997, Chap. 1.

[2]Valdevit, L., Vermaak, N., Hsu, K., Zok, F. W., and Evans, A. G., "Design of Actively Cooled Panels for Scramjets," *Proceedings of the 14th AIAA/AHI Space Planes and Hypersonic Systems and Technologies Conference*, AIAA, Washington, DC 2006-8069, 2006.