Introduction
Recent developments in cellular materials allow for the consideration of heat sink designs previously not possible. Cellular materials have properties similar to conventional materials on the macroscopic level but possess several advantages at the microscopic level, including the ability to be manufactured down to a very small scale. Their superior properties make them suitable for a wide range of applications, where size, weight, and efficiency are important.

This project aimed to take advantage of the benefits of cellular materials for use in heat sink design. The goal for this project was to develop a methodology which would allow one to develop an optimal heat sink based on geometrical considerations independent of working fluid choice and heat sink material choice. For a fixed volume heat sink with a given percentage of the volume available for cooling channels, two questions were posed:
- How should one layout the channels?
- What shape should the channels be?

To answer these questions, a purely theoretical approach was employed, with testing and analysis performed using computational fluid dynamics (CFD) software.

Problem Formulation
To determine an optimal design for a heat sink, the principle of superposition was used to reduce the design area to the smallest repeating cell. This allows one to determine the characteristics of the complex heat sink from a simple repeating cell. Within the cell, a number of design variables are to be considered including:
- Number of channels
- Placement of channels within the normal face of the metallic block
- Channel geometry (circle, square, triangle,...)
- Channel shape (straight versus coiled tube)

Another goal of this study was to ensure that the minimum amount of working fluid would be needed to provide the desired cooling.

Performance Curves
This work was done to gain insight into the relationship between input parameters such as the mass flow rate and number of holes, and output parameters such as the temperature at critical locations and pumping power required.

Performance metric
For optimization purposes, it is useful to have a single quantitative performance measure which can be minimized or maximized as an objective function subject to specific performance constraints. Four goals were sought for an optimal heat sink:
- Removes the greatest amount of heat
- The fluid just reaches a fully hot state as it exits
- The bottom wall temperature is as close to as possible while still below a critical temperature
- Uses the smallest amount of pumping power

Each of the above goals was written in equation form and then non-dimensionalized by a problem relevant quantity. The beneficial aspects were emphasized while the disadvantageous one was penalized. The following metric was obtained:

\[
score = \min \left( \frac{Q}{\dot{Q}_0}, \frac{T_{\text{core}}}{T_{\text{hot}}}, \frac{T_{\text{bottom}}}{T_{\text{critical}}} \right) \left( \frac{P}{P_0} \right)
\]

\[
P_0 = \frac{\mu (\min / \Delta A_{\text{in}})^2 LA_{\text{in}}}{H^2}
\]

\[
\dot{Q}_0 = k(T_{\text{hot}} - T_{\text{in}})LW / H
\]

The best designs were single channels located as close to the hot surface as possible. The additional pumping power requirements outweighed the increased heat flux benefit from more holes. There was not enough variation in the fluid core and bottom wall temperatures between the different designs to make any of the performance constraints significant. The fixed temperature boundary condition for the hot surface was not a good choice for the problem. The heat flux on the side of the cooling channel to the surface results in the lack of a baseline problem. Without such a baseline case it is not possible to conduct a rational optimization study. A better choice would be to use a constant heat flux boundary condition. A constant heat flux prevents designs with channels located close to the top surface from drawing upon an infinite reservoir of heat.

Thermal Network Analysis
Valdevit et al. used a thermal resistance network analysis with an algorithm to determine the temperature profile in a square channel and obtained good results. The network being developed in this project aims to further generalize their results by being able to obtain reasonable answers for designs where lateral conduction also plays a significant role and 2-D effects need to be considered.