LARGE-EDDY SIMULATION OF WALL-BOUNDED HIGH-REYNOLDS NUMBERS FLOWS

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Abstract

Large-eddy simulation (LES) of turbulent channel flow is performed with the recently developed Explicit Algebraic sub-grid scale model (EASSM) and the Dynamic Smagorinsky model (DSM). The LES with EASSM gave a good prediction of the turbulent flow using coarse resolutions and involving moderately high Reynolds numbers, while LES with DSM gives much less accurate predictions at coarser resolutions.

1 Introduction

Wall-resolved LES of wall-bounded flows at high Reynolds numbers is computationally very costly since the near-wall structures need to be approximately resolved. To reduce computational costs, common alternatives used in engineering circumvent the need of resolving wall structures or even all boundary layer structures. These alternatives use a Reynolds-averaged Navier-Stokes (RANS) model for these structures, as in hybrid RANS/LES and detached-eddy simulation (DES), or solving simplified governing equations on a fine grid near the wall, as in zonal methods or wall-modelled LES (WMLES). However, these methods have disadvantages. For instance, coupling the wall-modelled and LES parts in these hybrid and zonal methods with the interface within the boundary layer poses problems and occasionally there is a mismatch between the mean velocity in these two parts ([9], [3]).

Wall-resolved LES does not have these problems, and is therefore still appealing. The aim of this study is to find out if resolution requirements and thus the computational costs of wall-resolved LES can be substantially reduced by using better SGS models. To this purpose, we perform LES of channel flow at coarse resolution with the EASSM and the DSM.

2 SGS model

The EASSM is non-linear and derived from the modelled transport equations of SGS stress anisotropy [5]. The expression for the modelled stress tensor reads:

$$\tau_{ij} = \frac{2}{3}\delta_{ij}K_{SGS} + \underbrace{\beta_1 K_{SGS}}_{eddy-viscosity}\ \widetilde{S}^*_{ij} + \underbrace{\beta_4 K_{SGS}(\widetilde{S}^*_{ik}\widetilde{\Omega}^*_{kj} - \widetilde{\Omega}^*_{ik}\widetilde{S}^*_{kj})}_{anisotropic\ contribution}. \tag{1}$$

where τ_{ij} is the SGS stress tensor, and $\widetilde{S}^*_{ij}$ and $\widetilde{\Omega}^*_{ij}$ are the resolved strain and rotations rate tensors, respectively, normalized by the SGS time scale τ^*. K_{SGS} is the SGS kinetic energy, modelled as

$$K_{SGS} = c\widetilde{\Delta}^2|\widetilde{S}_{ij}|^2, \tag{2}$$

$\widetilde{\Delta}$ is the filter scale, and model coefficient c is dynamically computed using a test filter and the Germano identity. β_1 and β_4 are model coefficients and depend on $\widetilde{S}_{ij}$ and $\widetilde{\Omega}_{ij}$. The second term on the right-hand-side of (1) is an eddy-viscosity term while the third non-linear term aims to improve the modelling of τ_{ij} in regions of strong anisotropy. From previous tests, EASSM has been proven to significantly improve LES of rotating and non-rotating wall-bounded turbulent flows ([5],[6], [7]). LES with EASSM is more accurate, especially at coarse resolutions, than the eddy viscosity SGS models like the dynamic Smagorinsky model. The better performance of the EASSM can be attributed to the third term on the right-hand-side of (1), which gives a significant contribution near the wall.

3 Numerical setup

We have performed LESs of turbulent plane channel flow at three Reynolds numbers corresponding to the DNSs of ([1],[4]), with $Re_\tau = 550, 2000$ and $Re_\tau = 5200$, based on friction velocity and channel half-width. The LESs are performed with a pseudo-spectral code with a constant mass flux constraint. The bulk Reynolds number is the same in the LES and DNS.

The LESs are performed with two resolutions; a coarse resolution with $\Delta x^+ \approx 160$ and $\Delta z^+ \approx 60$, and a very coarse resolution with $\Delta x^+ \approx 250$ and $\Delta z^+ \approx 100$, in the streamwise and spanwise directions, respectively, in wall units. Such resolutions are very coarse compared to the DNS resolutions, which are about 12 and 6 in the streamwise and spanwise direction respectively in wall units, and substantially lower than the resolutions for LES suggested by [8]: $(\Delta x^+, \Delta z^+) \approx (50, 15)$ in the streamwise and spanwise directions, respectively.

4 Results and discussion

Mean velocity profiles

Figure 1 (a) shows the mean velocity profiles at the three Reynolds numbers, with a coarse resolution for the LES with DSM at $Re_\tau = 550$, a coarse resolution and a very coarse for EASSM and DSM for $Re_\tau = 2000$ and $Re_\tau = 5200$.

LESs with the DSM deviate significantly from the DNS for all Reynolds numbers and come closer to the DNS only when the resolution is sufficiently high (not shown here). On the other hand, LESs with the EASSM shows much closer agreement with DNS at all Reynolds numbers, especially in the outer region. It has also been shown that LESs with the EASSM are much more resolution-independent, according to [6].

Since the bulk Reynolds number is the same in the LESs and DNS, the friction Reynolds number Re_τ and thus the skin friction (and, therefore, the mean pressure gradient) are different. Figure 1 (b) shows the skin friction coefficient ratio between LES and DNS as a function of the friction Reynolds number. For the very coarse resolution the skin friction is strongly underpredicted by the DSM, while the EASSM predicts skin friction coefficient with an error of less than 10% for the whole range studied here, which decreases with increasing Re_τ. For the coarse resolution the error for the DSM predictions is still large while the EASSM gives quite accurate predictions.

Spectral analyses show that, although near wall structures are not well captured by LES at these coarse resolutions, the large-scale structures, typically found in the outer region, are sufficiently well-computed by LESs. The intensity of the outer peak estimated by LES with EASSM comes even closer to DNS than the LES with DSM. The better results of the LESs with EASSM can be attributed to a better prediction of the SGS dissipation and anisotropy near the wall [6].

Reynolds stresses

Using an eddy-viscosity-based model like the DSM, the only significant SGS contribution to the Reynolds stress tensor is the one related to the shear stress component. The EASSM ensures a more physically-correct reproduction of the Reynolds stress tensor, and its estimation of the flow anisotropy helps LESs to achieve closer results to DNS. Figure 2 shows the streamwise Reynolds stress component, along the wall normal direction, for all Reynolds numbers investigated and using the coarse resolution. LES with EASSM come closer to DNS at every Reynolds number, and its accuracy - around the inner peak and in the outer region in particular - substantially increases with increasing Reynolds number. The SGS anisotropy contribution, in fact, counteracts the Reynolds number-dependent growth of the eddy viscosity SGS part.

The gap between the LES with DSM and DNS gets considerably larger at high Reynolds numbers.

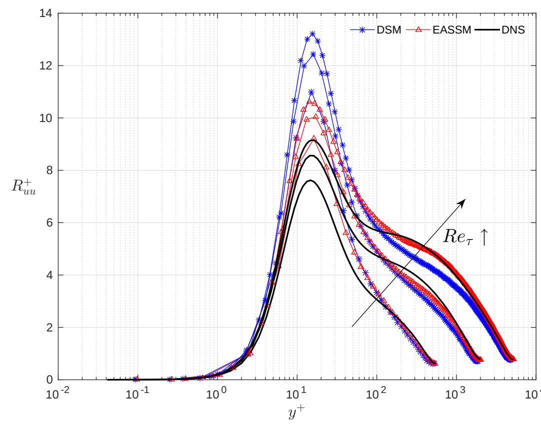


Figure 2: Streamwise Reynolds stress component in wall units, as a function of the inner units-scaled wall-normal direction, using a coarse resolution and $Re_\tau = 550, 2000$ and 5200. Arrow point at increasing Reynolds number direction. Red line refers to EASSM, blue one to DSM, black one to DNS.

Anisotropy maps

In order to check the realizability of the models and quantify the ability to capture anisotropy, anisotropy maps are considered using a coarse resolution and considering all Reynolds numbers. We define the anisotropy as

$$a_{ij} = \frac{< \widetilde{u}_i \widetilde{u}_j > + < \tau_{ij} >}{K_{RES} + < K_{SGS} >} - \frac{2}{3}\delta_{ij} \qquad (3)$$

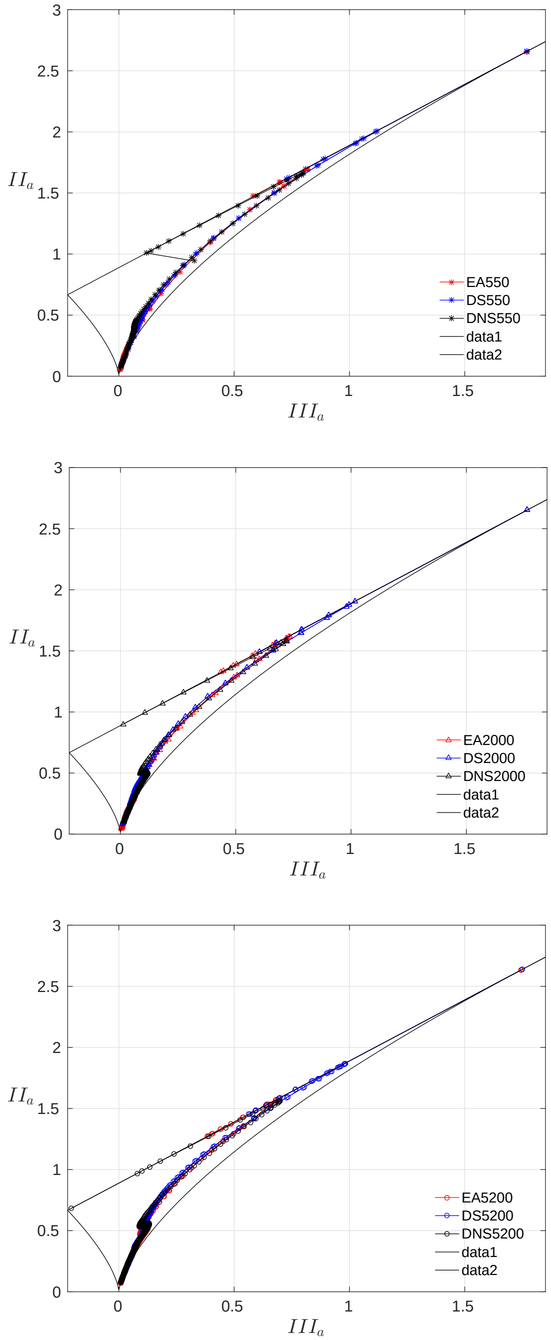
where $K_{SGS} = \tau_{kk}/2$ and $K_{RES} = < \widetilde{u_k}\widetilde{u_k} > /2$ are the SGS and resolved parts of the turbulence kinetic energy, respectively, and $< . >$ is the plane and time average. In order to quantify the anisotropy, we focus on the magnitude given by the second invariant of anisotropy,

$$II_a = a_{ij} a_{ji}. \qquad (4)$$

Figure 3 shows that both models are found to be realizable since all the anisotropy maps stay inside the Lumley triangle. However, a comparison between the LESs shows that LES with EASSM comes substantially closer to DNS than LES with DSM.

Vortical structures

In a LES, the quality of a SGS model strongly depends on the range of captured scales; the larger the range, the more turbulent structures will be computed. In this work turbulent structures are identified using the λ_2 criterion [2]. Figure 4 shows the probability



a)

b)

c)

Figure 3: Second anisotropy invariant, II_a as a function of the third anisotropy invariant, III_a. Red line refers to EASSM, blue one to DSM, black one to DNS. data 1 and data 2 denote the Lumley triangle.

density function (*p.d.f.*) of the λ_2 structures in the entire computational box, for the LES with EASSM and the LES with DSM. *P.d.f.* of the LES with EASSM presents a wider tale than *p.d.f.* of the LES with DSM. It can be concluded that LES with EASSM captures more of the structures with high λ_2-amplitudes than the LES with DSM. In particular LES with EASSM captures more small-scale anisotropic turbulent structures, which are generated close to the wall.

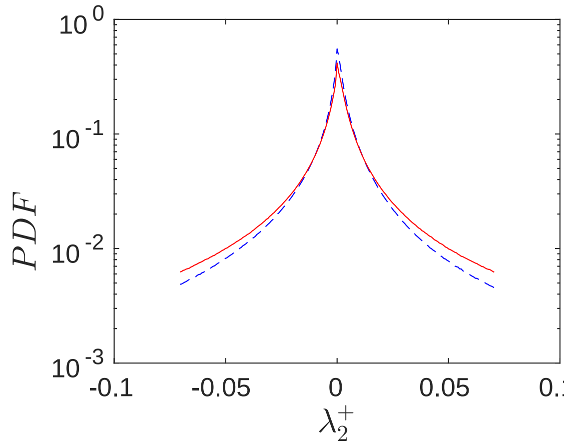


Figure 4: Probability density function of λ_2 structures in inner units, for the entire computational domain. Blue line refers to DSM, red one to EASSM.

5 Conclusions

LESs with the EASSM are in reasonably good agreement with DNS of turbulent channel flow up to $Re_\tau = 5200$ even at very coarse resolutions while LESs with DSM substantially deviate from DNS at the same resolutions. In order to obtain comparably good results for the skin friction and velocity profiles the LESs with the DSM need one order of magnitude more grid points than with the EASSM. We conclude that eddy viscosity-based SGS stress models like the DSM do not seem to be appropriate for LES of high Reynolds number wall-bounded turbulent flows at coarse resolution. More advanced models like the EASSM with a better description of the near wall anisotropy appear to be necessary to retain acceptable accuracy while keeping the computational costs acceptable for LES of such flows.

Acknowledgments

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References

[1] Hoyas, S. and Jiménez, J. (2006). Scaling of the velocity fluctuations in turbulent channels up to $Re_\tau = 2003$. *Phys. Fluids*,18(1):011702.

[2] Jeong, J.and Hussain, F. (1995) On the Identification of a Vortex. *J. Fluid Mech.*, Vol. 285, pp. 69-94.

[3] Larsson, J., Kawai, S., Bodart,J. and Bermejo-Moreno,I. (2015). Large eddy simulation with modeled wall-stress: recent progress and future directions. *Mech. Eng. Rev.*,(0).

[4] Lee, M. and Moser, R.D. (2015). Direct numerical simulation of turbulent channel flow up to $Re_\tau \approx 5200$. *J. Fluid Mech.* Vol. 774, pp. 395-415.

[5] Marstorp, L. and Brethouwer, G. and Grundestam, O. and Johansson, A. V. (2009). Explicit algebraic subgrid stress

models with application to rotating channel flow. *J. Fluid Mech.* Vol.639, pp.403-432.
[6] Rasam, A. and Brethouwer, G. and Schlatter, P. and Li, Q. and Johansson, A. V. (2011). Effects of modelling, resolution and anisotropy of subgrid-scales on large eddy simulations of channel flow. *J. Turb.*, (12).
[7] Rasam, A. and Wallin, S. and Brethouwer, G. and Johansson, A. V. (2014). Large eddy simulation of channel flow with and without periodic constrictions using the explicit algebraic subgrid-scale model. *J. Turb.*, 15(11):752-775.
[8] Sagaut, P. (2006) Large eddy simulation for incompressible flows: an introduction. Springer Science & Business Media.
[9] Spalart, P.R. (2009). Detached-eddy simulation. *Annu. Rev. Fluid Mech.*, 44:181-202.

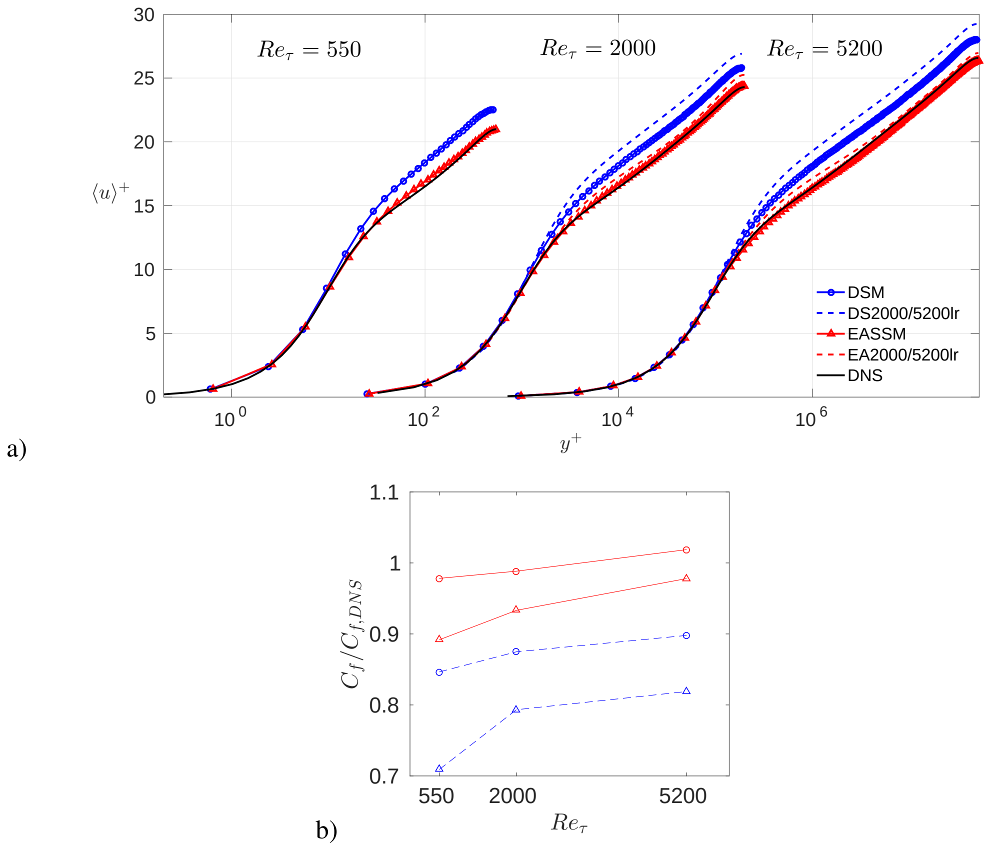

Figure 1: (a) Mean streamwise velocity in wall units, as a function of the inner units-scaled wall-normal direction: full line refers to coarse resolution LES, while dashed line to the very coarse. The three Re_τ cases are shifted by a factor of 10^2 viscous units, along the wall-normal coordinate. (b) Skin friction coefficient ratio, as a function of Re_τ: DSM(dashed blue lines), EASSM(full red lines). $\triangle$:very coarse resolution, $\bigcirc$: coarse resolution.