Development and validation of Explicit Algebraic Reynolds stress modelling for hybrid RANS-LES computations

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Abstract

A new model for hybrid RANS-LES computations based on the improved-delayed-detached eddy simulation (IDDES) approach is proposed. The model combines the Explicit Algebraic Reynolds Stress model for RANS and the Explicit Algebraic sub-grid scale stress model. Several tests using the present model have been conducted for a turbulent plane channel flow, periodic hill and 3-D diffuser flow adopting the general-purpose finite-volume code OpenFOAM. For each geometry two different Reynolds numbers are investigated. For all the cases considered, hybrid computations using the EARSM-IDDES model have shown a good agreement with the established $k-\omega$ SST-IDDES model, reasonably predicting first- and second-order statistics. In 3-D diffuser flow, EARSM-IDDES has shown a better agreement to DNS data in the estimation of skin friction, especially at the inlet duct.

1 Introduction

A new anisotropy-resolving hybrid RANS-LES model is proposed, where Explicit Algebraic Reynolds stress modelling [10] is applied to both RANS and LES formulations. Introduced by Shur *et al.* [9], IDDES methodology has an extended RANS-LES switching function which enables wall-modelled LES for sufficiently fine meshes. The novel EARSM-IDDES is an extension of the $k-\omega$ SST IDDES [2], where the RANS modelling part of the EARSM-IDDES allows for a reasonable estimation of flows characterised by swirl, curvature and three-dimensional effects.

2 Mathematical formulation

The 2-D formulation of the EARSM-IDDES is non-linear and derived from the modelled transport equations of modelled stress anisotropy. The expression for the modelled LES SGS stress tensor reads:

$$\tau_{ij} = \frac{2}{3}\delta_{ij}k + \underbrace{\beta_1 k \widetilde{S}^*_{ij}}_{\text{eddy-viscosity}} + \underbrace{\beta_4 k(\widetilde{S}^*_{ik}\widetilde{\Omega}^*_{kj} - \widetilde{\Omega}^*_{ik}\widetilde{S}^*_{kj})}_{\substack{\text{anisotropy of}\\ \text{modelled stresses}}}. \tag{1}$$

where $\widetilde{S}^*_{ij}$ and $\widetilde{\Omega}^*_{ij}$ are the resolved strain and rotations rate tensors, respectively, normalized by the modelled time scale τ^*. Differently from the Explicit Algebraic SGS model where the SGS kinetic energy was estimated by a dynamic procedure ([3],[6]) the LES part of EARSM-IDDES employs the $k-\omega$ transport model equations for the evaluation of subgrid-scale kinetic energy k.

β_1 and β_4 are model coefficients depend on $\widetilde{S}_{ij}$ and $\widetilde{\Omega}_{ij}$. The second term on the right-hand-side of (1) is an eddy-viscosity term while the third non-linear term aims to improve the modelling of τ_{ij} in regions of strong anisotropy.

The RANS stress tensor is modelled according to [10] with the five-term 3D formulation. The blending between LES and RANS stress is made by the use of the $\widetilde{f}_d$ blending function as defined in [2].

3 Results

The model is initially validated for turbulent channel flow on a computational domain of $(L_x, L_z) = (2\pi, \pi)\delta$, where δ is the channel half-height. Friction Reynolds numbers of 934 and 5200 are considered, where grid resolutions in wall units of $(\Delta x^+, \Delta z^+) = (98, 49)$ and $(544, 272)$ are used, respectively, resulting in less than a half million grid points for all the cases. The wall-normal resolution is set such that $y^+ \sim 1$ at the first cell. The mean velocity profiles are reasonably computed by EARSM-IDDES and the $k-\omega$ SST IDDES, shown in figure 1 a) for $Re_\tau \approx 5200$. EARSM is found to substantially reduce the RANS-LES transitional region, resulting in a substantially larger amount of resolved turbulence near the wall, see the relation between resolved and modelled Reynolds shear stress in figure 1 b). We believe that this can be of importance in mitigating the so called grey-area problems resulting in a more rapid transition to resolved turbulence in more complex cases.

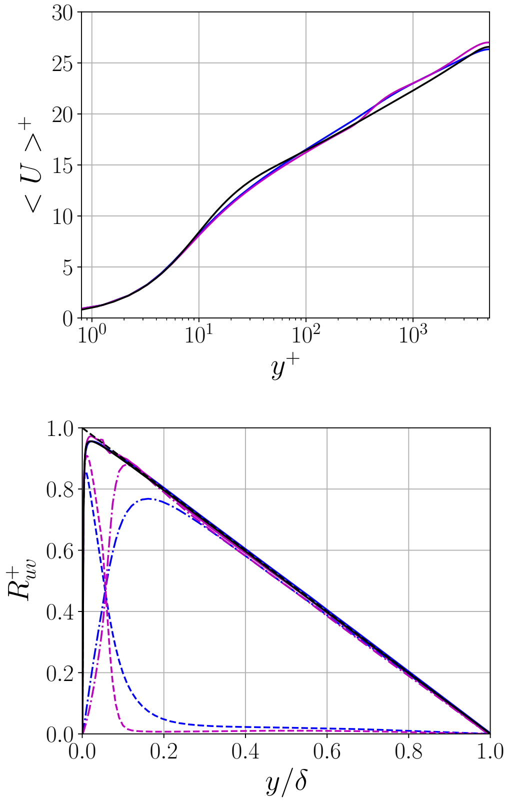


Figure 1: a) Inner-scaled mean velocity profile and b) Reynolds shear stress along the wall-normal direction, at $Re_\tau \approx 5200$, —— : $k - \omega$ SST-IDDES, —— : EARSM-IDDES, —— : DNS. The turbulence shear stress is divided into modelled $(- - -)$ and resolved $(- \cdot -)$ contributions.

In the second part of the study the EARSM-IDDES methodology has been tested on a periodic hill flow [4] at bulk Reynolds numbers of $Re_b = 10595$ and $Re_b = 37000$. The wall-normal resolution is $y^+ \sim 1$ at the first cell, while the average $(\Delta x^+, \Delta z^+)$ along the streamwise direction are about $(40, 25)$ and $(120, 100)$, at $Re_b = 10595$ and 37000 respectively. The grids for both cases are very coarse with less than half million grid points to be able to observe influence of modelling. For both the Reynolds numbers, the skin friction in figure 2 is reasonably predicted by both the models. The separated region is found to be better captured by $k - \omega$ SST, while the EARSM gives a better estimation of the reattachment. The streamlines plot, shown in figure 3, shows that EARSM predicts a larger extent of the separation bubble compared to $k - \omega$ SST. Both the models are capable of capturing the essential Reynolds number dependency, but found to perform better for the highest Reynolds number with a more pronounced separation between RANS and LES regions.

In the third part of the study the EARSM-IDDES

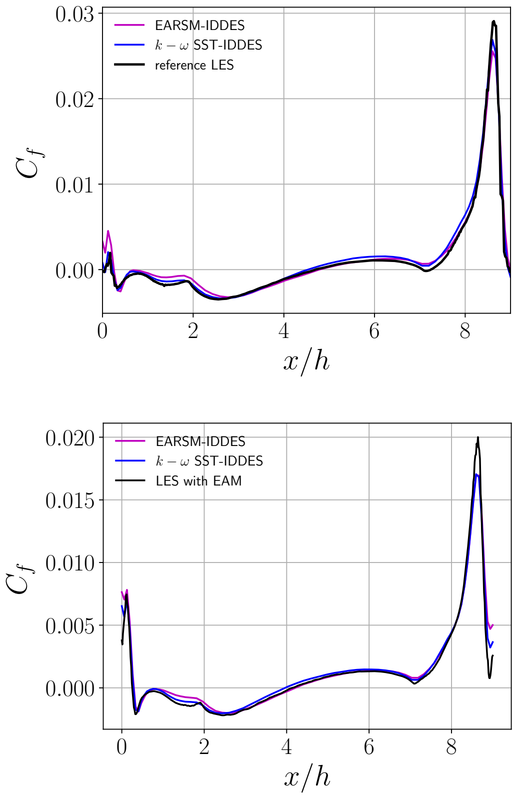


Figure 2: Skin friction coefficient in the streamwise direction at a) $Re_b = 10595$ and b) $Re_b = 37000$, —— : $k - \omega$ SST-IDDES, —— : EARSM-IDDES, —— : LES from Breuer *et al.* on a), LES with EAM on b).

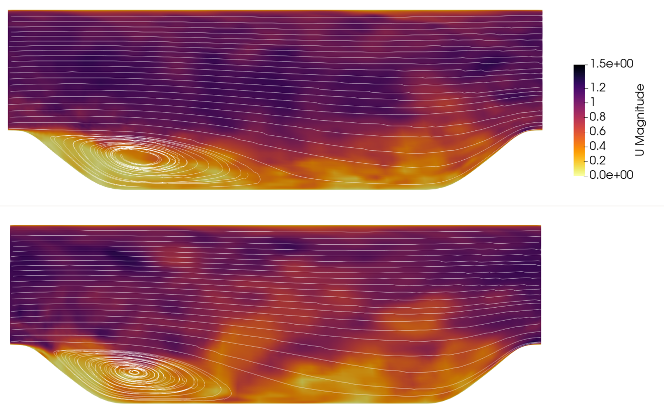


Figure 3: Streamlines plot with contours of the instantaneous velocity at $Re_b = 10595$, EARSM-IDDES (top) $k - \omega$ SST-IDDES (bottom)

is used to compute the flow on a 3-D diffuser configuration ([8], [1]). The incoming fully developed duct flow including the corner vortices is critical for accurate prediction of the 3D separation and is computed in a periodic domain. As shown in figure 4, EARSM-IDDES is able to properly capture the mean secondary flow. Note that the magnitude of the instantaneous velocity fluctuations is about ten times larger. Additionally, the amplitude of the secondary motion is about 2% of the axial bulk velocity, and agrees reasonably well with the values previously found in the 3-D diffuser flow by Cherry *et al.* [1]. For the present study, two different grids are used, a ”coarse” one of 1.4 and a ”fine” one of 2.8 million grid points. In figure 6 a) and c) pressure coefficient at $Re_h = 10000$ and $Re_h = 30000$ is shown along the streamwise direction. For both the resolutions, both the models give a similar prediction of the pressure coefficient at the lower Reynolds number, while the EARSM-IDDES gives a substantial improvement compared to $k-\omega$ SST-IDDES using the ”fine” mesh at the higher Reynolds number. At the lower Reynolds number the friction coefficient is better predicted by using EARSM-IDDES and both the grids, the largest improvements are visible towards the inlet duct (see figure 6 b) . Looking at the contours of the mean streamwise velocity along vertical planes at different x-coordinates (figure 7), one can confirm that both the models lead to similar results that reasonably agree to DNS data.

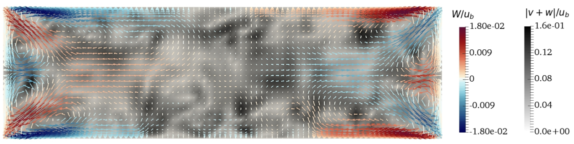


Figure 4: Time-averaged velocity vectors in the y-z plane computed with EARSM-IDDES for $Re_h = 10000$, colored with the mean spanwise velocity. Colorplot of the instantaneous secondary velocity magnitude in background.

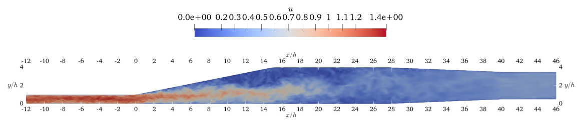


Figure 5: Colorplot of the streamwise velocity along the mid-span plane at $Re_h = 10000$.

All the simulations described here are performed by using the general-purpose code OpenFOAM, with a second-order accurate spatial discretization. A second-order backward time integration has been used with a fixed timestep, corresponding to a Courant number ($CN = u_b\Delta t/\Delta x$) of 0.1. Artificial dissipation resulting from the use of the Rhie and Chow (R&C) interpolation is here minimized by manipulating a scaling factor in the pressure correction step of the standard `pimpleFoam` solver, details can be found in [7].

4 Conclusions

A new formulation based on the explicit algebraic Reynolds stress model (EARSM) has been proposed for IDDES formulations in both the RANS

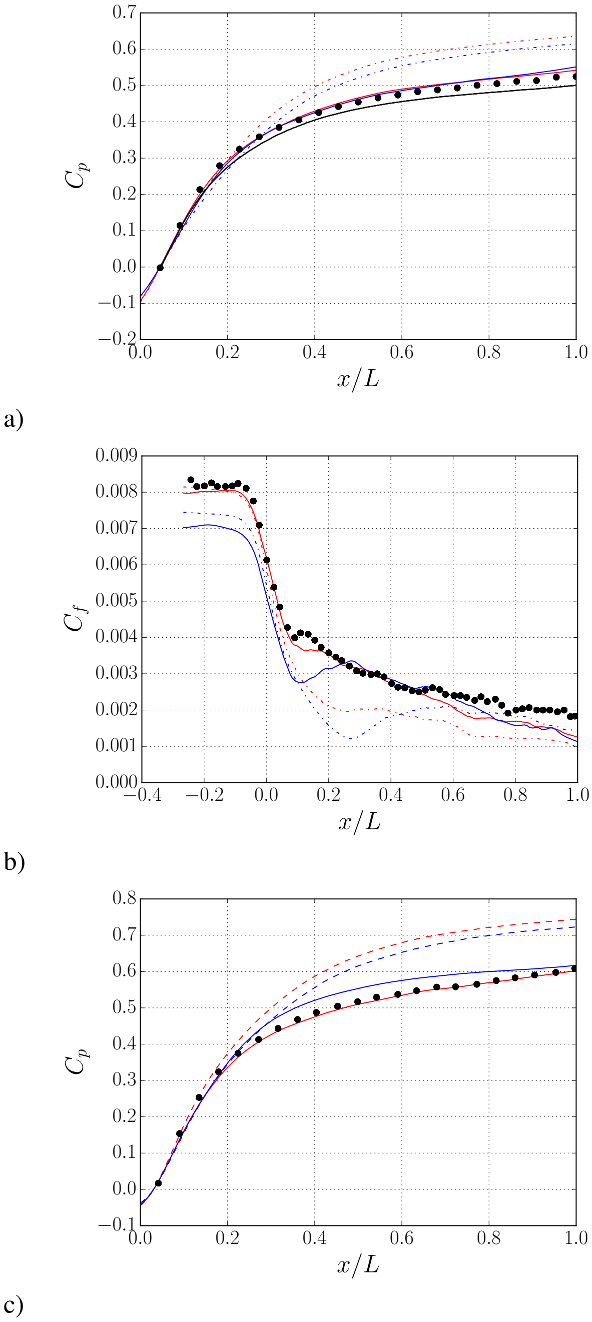
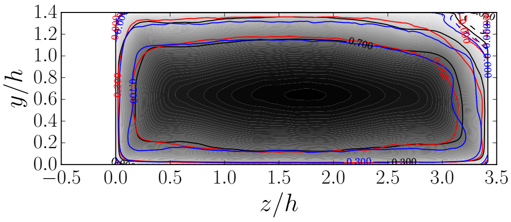
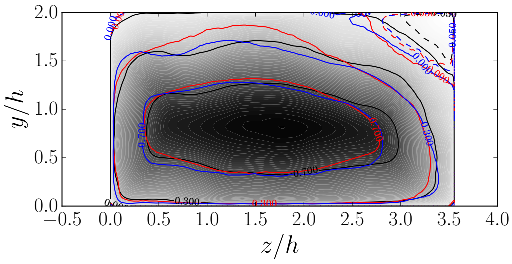


Figure 6: a,c) Pressure and b) friction coefficients along the streamwise direction at a,b) $Re_h = 10000$ and c) 30000. Lines and symbols in black refer to reference data ([8], [1]), red to EARSM-IDDES, blue to $k-\omega$ SST-IDDES. Solid red and blue lines correspond to fine mesh, dashed-dotted to coarse.

and LES parts. Simulations of a turbulent channel flow at $Re_\tau \approx 950$ and $Re_\tau \approx 5200$ have shown that the EARSM-IDDES implemented in OpenFOAM give reasonable predictions of the mean quantities and Reynolds stresses, comparable to the ones with the $k-\omega$ SST-IDDES model. Due to the non-linear formulation, the model was able to properly capture the anisotropy close to the wall, resulting in a more correct prediction of the wall-normal Reynolds stress compo-



a)



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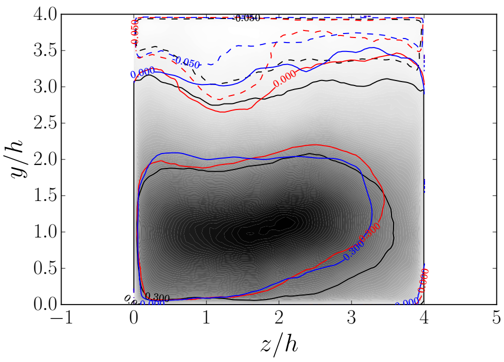

c)

Figure 7: Contours of the mean streamwise velocity in the y-z plane at a) $x/h = 2.0$, b) $x/h = 5.0$, c) $x/h = 15.0$, at $Re_h = 10000$. DNS values in solid black line and contours in background. Solid lines refer to fine resolution, dashed lines to coarse resolution. Blue color refers to $k-\omega$ SST-IDDES, red one to EARSM-IDDES.

nent. In periodic hill flow at $Re_b = 10595$ and 37000, the proposed model gives reasonable results, which are comparable with the $k-\omega$ SST-IDDES. In 3-D diffuser flow at $Re_h = 10000$, EARSM-IDDES is able to predict more accurately the skin friction coefficient than $k-\omega$ SST-IDDES, especially at the inlet duct.

Acknowledgments

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References

[1] E. M. Cherry, C. J. Elkins, and J. K. Eaton. Geometric sensitivity of three-dimensional separated flows. *International Journal of Heat and Fluid Flow*, 29(3):803–811, 2008.

[2] M. S. Gritskevich, A. V. Garbaruk, J. Schütze, and F. R. Menter. Development of DDES and IDDES formulations for the k-ω shear stress transport model. *Flow, turbulence and combustion*, 88(3):431–449, 2012.

[3] L. Marstorp, G. Brethouwer, O. Grundestam, and A. V. Johansson. Explicit algebraic subgrid stress models with application to rotating channel flow. *Journal of Fluid Mechanics*, 639:403–432, 2009.

[4] C. Mellen, J. Fröhlich, and W. Rodi. Large eddy simulation of the flow over periodic hills. In *16th IMACS world congress*, pages 21–25, 2000.

[5] C. Mockett, W. Haase, and D. Schwamborn. *Go4Hybrid: Grey Area Mitigation for Hybrid RANS-LES Methods*, volume 134. Springer, 2018.

[6] M. Montecchia. *Numerical and modelling aspects of large-eddy and hybrid simulations of turbulent flows*. PhD thesis, KTH Royal Institute of Technology, 2019.

[7] M. Montecchia, G. Brethouwer, S. Wallin, A. V. Johansson, and T. Knacke. Improving LES with OpenFOAM by minimising numerical dissipation and use of explicit algebraic SGS stress model. *Journal of Turbulence*, pages 1–26, 2019.

[8] J. Ohlsson, P. Schlatter, P. F. Fischer, and D. S. Henningson. Direct numerical simulation of separated flow in a three-dimensional diffuser. *Journal of Fluid Mechanics*, 650:307–318, 2010.

[9] M. L. Shur, P. R. Spalart, M. K. Strelets, and A. K. Travin. A hybrid RANS-LES approach with delayed-DES and wall-modelled LES capabilities. *International Journal of Heat and Fluid Flow*, 29(6):1638–1649, 2008.

[10] S. Wallin and A. V. Johansson. Modelling streamline curvature effects in explicit algebraic reynolds stress turbulence models. *International Journal of Heat and Fluid Flow*, 23(5):721–730, 2002.