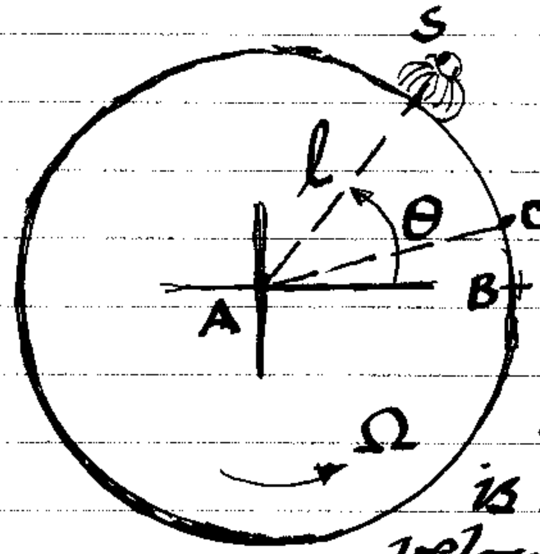
Hand in Problems for 5C1105
Spring 2004

Problem 1



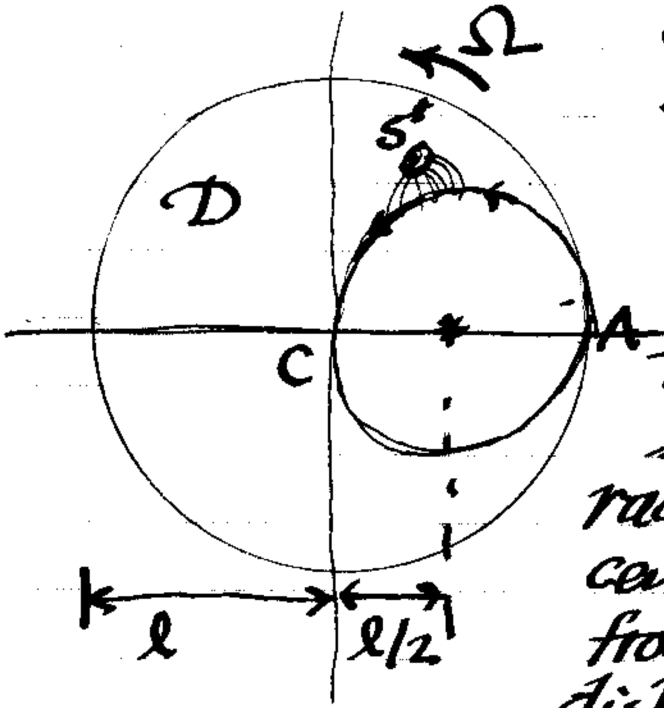
A spider is located at position S, given by $le^{i\theta}$ in complex notation, relative to the point "A" fixed in space. The spider moves along the rim with velocity V_s on the (relative to) the disk which is itself rotating with angular velocity Ω. At $t=0$ both the spider and the line AC (fixed on the disk) are at B, i.e. S, C, and B are coincident.

(a) Find the angular position θ of the spider in terms of Ω, V_s and l.

(b) Find the spiders complex velocity relative to A.

(c) Find the spiders complex acceleration.

Problem 2



A spider moves in a circular path on the surface of a rotating disk. The path is fixed with respect to the disk hence rotates with it! The disk radius is l and the path radius is $l/2$ with center at a distance $l/2$ from the center of the disk. Assume that at $t=0$ the point S (the spider) coincides with point A and the path is in the position shown.

(a) Find the complex position of the spider as a function of time.

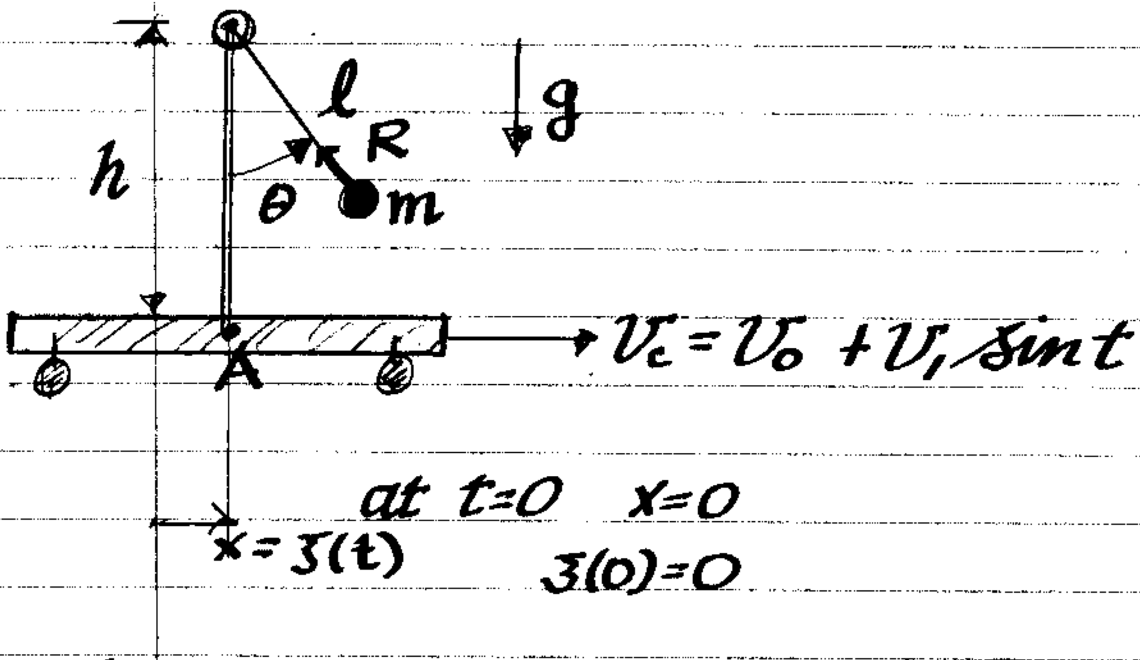
(b.) Find the complex velocity.

(c.) If the spider's velocity relative to the disk is $\Omega l/2$ show that the spiders path <u>in space</u> is given by

$$\left(x_s-\frac{l}{2}\right)^2+y_s^2=\left(\frac{l}{2}\right)^2$$

i.e. is the one indicated at $t=0$ on the disk!

Problem 3



The position of a mass in pendulum motion fixed to a pole on a moving cart is given by

$$r = \xi_{cart} + ih - ie^{i\theta}\ell$$

The force on the mass is the sum of the gravitational and constraint force.

$$F = ie^{i\theta}R - img.$$

(a) Find the constraint free equation of motion

(b) If $g/\ell = \omega_0^2 = 2$, $V_1 = 1$, $\theta(0) = \dot\theta(0) = 0$ find a solution for the case $|\theta| \ll 1$.

(c) Numerically find $\theta(t)$ for the case $\omega_0^2 = 1$ $\theta = \dot\theta = 0$ for the full equations (~~supp~~ Show a computer generated curve)

Problem 4

A linear oscillator obeys the equation

$$m\ddot{x} + kx = f_0\left(H(t) - H(t-1)\right)$$

<u>where</u> $H(t) = 0 \;, t<0,$
$\qquad\qquad = 1 \;, t>0,$

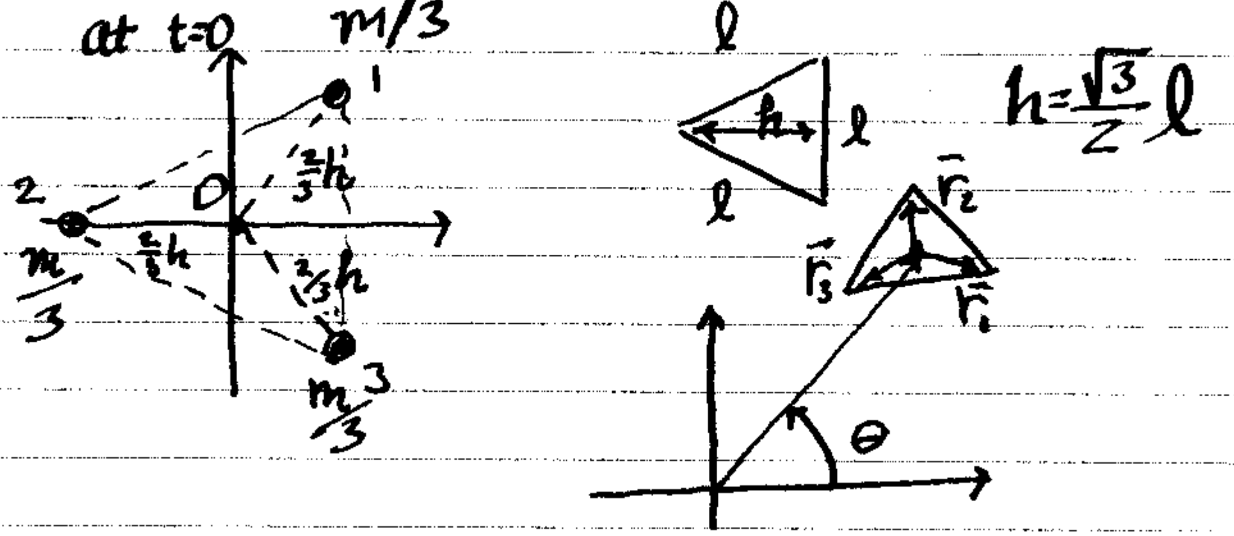
(a) Find an analytic solution for $t<0$
$X = \dot{X} = 0$ when $t<0$.

(b) Show a numerical solution for

$$m\ddot{x} + k_1\dot{x} + kx = f_0(H(t) - H(t-1))$$

use units where $m = k_1 = k = 1$
and take $f_0 = 10$ newtons.

Problem 5



Three point masses are constrained to be at the vertices of an equilateral triangle of side l. In terms of the height $h = (\sqrt{3}/2)\, l$ we chose a point 'O' equidistant from each of the three equal mass points. So the position of the three points are given at $t=0$ by.

$$\bar{r}_1 = \frac{2}{3} h e^{i\pi/3}$$

$$\bar{r}_2 = \frac{2}{3} h e^{i\pi}$$

$$\bar{r}_3 = \frac{2}{3} h e^{-i\pi/3}$$

(a) Show that $\bar{r}_1 + \bar{r}_2 + \bar{r}_3 = 0$

(b) Show that $|\bar{r}_1 - \bar{r}_2| = |\bar{r}_1 - \bar{r}_3| = |\bar{r}_2 - \bar{r}_3| = l$

Problem 5 continuation

As the mass points move about they retain their relative positions. Let $re^{i\theta}$ be the position of the point O at an arbitrary time. Show that:

(c) $r_1 = re^{i\theta} + \frac{\sqrt{3}}{3} l e^{i(\theta + \frac{\pi}{3})}$

(d) find r_2 and r_3

(e) find V the velocity matrix column

$$V = \begin{bmatrix} \dot{r}_1 \\ \dot{r}_2 \\ \dot{r}_3 \end{bmatrix}$$

(f) find $Q_r = \frac{\partial V}{\partial \dot{r}}$ and $Q_\theta = \frac{\partial V}{\partial \dot{\theta}}$

the direction or tangents to the motion.

(g) form the acceleration column

$$A = \begin{bmatrix} \ddot{r}_1 \\ \ddot{r}_2 \\ \ddot{r}_3 \end{bmatrix}$$

(h) form the inertia force column.

$$\dot{p} = \begin{bmatrix} \frac{m}{3}\ddot{r}_1 \\ \frac{m}{3}\ddot{r}_2 \\ \frac{m}{3}\ddot{r}_3 \end{bmatrix} = \frac{1}{3}m \begin{bmatrix} \ddot{r}_1 \\ \ddot{r}_2 \\ \ddot{r}_3 \end{bmatrix}$$

(i) The gravity force is given by

$$F_a = \begin{bmatrix} +1 \\ +1 \\ +1 \end{bmatrix} i\frac{mg}{3}$$

(j) The constraint forces can be ignored. Using Q_r and Q_θ we obtain the equations of motion

$$Q_r^* \dot{p} = Q_r^* F_a$$

$$Q_\theta^* \dot{p} = Q_\theta^* F_a$$

<u>where</u> $Q_r^* = e^{-i\theta}[1, 1, 1]$

$$Q_\theta^* = -i e^{-i\theta}[r_1^*, r_2^*, r_3^*]$$

Show the final equations!

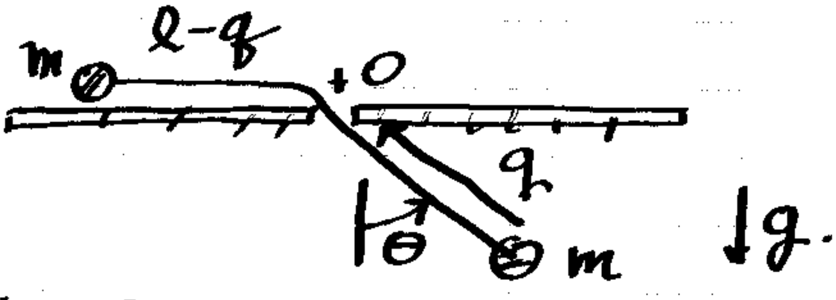
Problem 6

With the notation used in the online notes show that the total energy of the orbit of a body of mass m about a primary with Gravitational strength parameter $K = GM$ is given by.

$$E = -\frac{1}{2}(1-\varepsilon)\, m v_c^{2} = -\frac{1}{2}\frac{r_0}{a}\, m v_c^{2}$$

where $v_c = \sqrt{K/r_0}$, r_0 is the minimum distance from the primary, a is the semi-major axis of an ellipse — the second form is only valid for the ellipse.

Problem 7



The configuration of a system of two mass points connected by a massless string that passes through a hole in a table top (See figure) can be described by the coordinates q and θ, the length and angle of the string between the bottom mass and the hole.

a) find the Kinetic energy of the system

b) find the potential energy

c) from $L = KE - PE$ find the equations of motion.

Problem 8

Problem 11.6 in Acheson is about the so called logistic map. Give a similar discussion of the 'tent' map:

$$x_{n+1} = \lambda f(x_n)$$

- $0 \leq x_n \leq 1$
  $0 < \lambda \leq 1$

- $$f = 2\lambda x \quad \text{if} \quad 0 \leq x < \tfrac{1}{2}$$
  $$= 2\lambda(1-x) \quad \text{if} \quad \tfrac{1}{2} \leq x < 1$$

Problem 9

Use eithier Kane or Lagrange method to derive Acheson's equation 12.15

Problem 10

Discuss the bifurcations of the equalibrium solutions to

$$\dot{x} = 1 + rx + x^2$$

as r is varied from ~~r~~ large negative to large positive values.