Problem Assignment Solutions
5C1105 Spring 2004

Problem 1

From the information given the angular velocity of the spider is V_s/ℓ hence.

$$\dot{\theta} = \Omega + V_s/\ell$$

$$\Rightarrow \quad \theta = (\Omega + V_s/\ell)t \qquad (a)$$

so that $r = \ell \exp(i(\Omega + V_s/\ell)t)$

The spiders complex velocity is thus.

$$v = \dot{r} = \ell(\Omega + V_s/\ell)\, i\, e^{i(\Omega + \frac{V_s}{\ell})t} \qquad (b)$$

and the acceleration is

$$a = \dot{v} = -\ell(\Omega + \tfrac{V_s}{\ell})^2 e^{i(\Omega + \frac{V_s}{\ell})t} \qquad (c)$$

Problem 2

The angular velocity of the spider <u>relative</u> to the rotating disk is $\Omega_s = 2V_s/\ell$ so

$$r_s = \frac{\ell}{2} e^{i\Omega t} + \frac{\ell}{2} e^{i\Omega t} e^{i\frac{2V_s}{\ell}t}$$

which is the sum of the distance to the center of the inscribed circle and the distance from the center to the spider.

$$\text{(a)} \quad r_s = \frac{\ell}{2} e^{i\Omega t}\left(1 + e^{i\frac{2V_s}{\ell}t}\right)$$

check when $t = 0$ $\quad r_s = \frac{\ell}{2}(1+1) = \ell$ ✓

Now $v_s = \dot{r}_s$

(b) $v_s = \frac{i\Omega l}{2} e^{i\Omega t}(1+e^{i\frac{2V_s}{l}t}) + iV_s e^{i(\Omega t+\frac{2V_s}{l}t)}$

The last part of the problem is to find the path in space when the velocity relative to the moving disk is $\Omega l/2$

from (a) we have taking Real and Imaginary parts.

$$x_s = l/2\left(\cos\Omega t + \cos(\Omega - \tfrac{2V_s}{l})t\right)$$
$$y_s = l/2\left(\sin\Omega t + \sin(\Omega - \tfrac{2V_s}{l})t\right)$$

now if $\Omega = 2V_s/l$

$$x_s = \frac{l}{2}(\cos\Omega t + 1)$$
$$y_s = \frac{l}{2}(\sin\Omega t)$$

hence $\frac{4y_s^2}{l^2} = \sin^2\Omega t = 1 - \cos^2\Omega t$

so $\cos^2\Omega t = 1 - \frac{4y_s^2}{l^2}$

so $x_s = \frac{l}{2}\left(\pm\sqrt{1-\frac{4y_s^2}{l^2}} + 1\right)$

$$\left(x_s - \frac{l}{2}\right)^2 = \frac{l^2}{4}\left(1 - \frac{4y_s^2}{l^2}\right) = \frac{l^2}{4} - y_s^2$$

complete the square of this to get

$$x_s^2 - \frac{2x_s l}{2} + \frac{l^2}{4} = \frac{l^2}{4} - y_s^2$$

as $\left(x_s - \frac{l}{2}\right)^2 = x_s^2 - x_s l + \frac{l^2}{4}$

$$(x_s - l/2)^2 + y_s^2 = (l/2)^2$$ ✓

Problem 3

$r = \xi_{cart} + ih - il e^{i\theta}$ given

$\dot{\xi}_{cart} = V_c = V_0 + V_1 \sin t$

(a). $\dot{r} = \dot{\xi} - i^2 l \dot{\theta} e^{i\theta} = \dot{\xi} + l\dot{\theta}e^{i\theta}$

$= V_0 + V_1 \sin t + l\dot{\theta}e^{i\theta}$

$\ddot{r} = V_1 \cos t + l\ddot{\theta}e^{i\theta} + il\dot{\theta}^2 e^{i\theta}$

$m\ddot{r} = F = ie^{i\theta}R - img.$

so.

$m(V_1\cos t + l\ddot{\theta}e^{i\theta} + il\dot{\theta}^2 e^{i\theta}) = ie^{i\theta}R - img.$

The direction number $Q_\theta = \dfrac{\partial \dot{r}}{\partial \dot{\theta}} = l e^{i\theta}$

$\therefore \quad Q_\theta^* = l e^{-i\theta}$ and

$m(V_1 \cos t\, e^{-i\theta} l + l^2\ddot{\theta} + il^2\dot{\theta}^2) = ilR - imgl e^{-i\theta}$

Take the real part of this to obtain the equation of motion.

$l m V_1 \cos t \cos\theta + ml^2\ddot{\theta} = -mgl\sin\theta$

(a) $\boxed{\ddot{\theta} + \left(\frac{g}{l}\right)\sin\theta + \frac{V_1}{l}(\cos t)\cos\theta = 0}$

for part (b) $g/l = 2$, $v_1 = l = 1$

$$\ddot{\theta} + 2\sin\theta + \cos t \cos\theta = 0$$

as $\sin\theta \sim \theta + O(\theta^3)$, $\cos\theta \sim 1 + O(\theta^2)$
for $|\theta| \ll 1$ we have.

$$\ddot{\theta} + 2\theta = -\cos t, \quad \theta(0) = \dot{\theta}(0) = 0$$

$$\theta_{comp.} = A\cos\sqrt{2}\,\theta + B\sin\sqrt{2}\,\theta$$

and it is easy to see that a particular soln is $\theta_p = -\cos t$ as
$\ddot{\theta}_p + 2\theta = \cos t - 2\cos t = -\cos t$ ✓

$$\therefore \theta = A\cos\sqrt{2}t + B\sin\sqrt{2}t - \cos t$$
$$\dot{\theta} = -\sqrt{2}A\sin\sqrt{2}t + \sqrt{2}B\cos\sqrt{2}t + \sin t$$

$\therefore$ $\theta(0) = 0 \rightarrow A - 1 = 0$
$\dot{\theta}(0) = 0 \rightarrow \sqrt{2}B = 0$
$\therefore$ $B = 0$, $A = 1$

$$\boxed{\theta = \cos\sqrt{2}t - \cos t} \text{ for } |\theta| \ll 1$$

(c) Numerical Solution when $\omega_0^2 = 1$; $v_1 = 1$, $l = 1$

$$\ddot{\theta} + \sin\theta + \cos t\cos\theta = 0$$

$\theta(0) = \dot{\theta}(0) = 0$

Problem 4

Work in units where $m = k = 1$ and note that if $\ddot{x}_1 + x_1 = f_0 H(t)$ then by superposition $x = x_1(t) - x_1(t-1)$, hence all we need is

$$\ddot{x}_1 + x_1 = f_0 H(t)$$

with $z = x + iv$, $v = \dot{x}$ we have from the notes

$$x_1 = \text{Real Part } i \int_0^t H(t') e^{-i(t-t')} dt'$$

The integral is $\int_0^t H(t') e^{-i(t-t')} dt' = \int_0^t e^{-i(t-t')} dt$

$$= -i(1 - e^{it}) \quad \text{hence}$$

$$x_1 = f_0(1 - \cos t) H(t) \qquad \text{as} \quad x_1 = 0 \text{ for } t < 0$$

hence the solution is

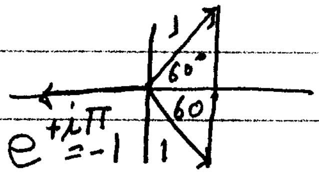
$$\boxed{x = f_0(1-\cos t)H(t) - f_0\left(1-\cos(t-1)\right)H(t-1)}$$

Problem 5

$$\bar{r}_1 = \frac{2}{3} h e^{i\pi/3}, \quad \bar{r}_2 = \frac{2}{3} h e^{i\pi}, \quad \bar{r}_3 = \frac{2}{3} h e^{-i\pi/3}$$

(a) $\bar{r}_1 + \bar{r}_2 + \bar{r}_3 = \frac{2}{3} h \left(e^{i\pi/3} + e^{i\pi} + e^{-i\pi/3} \right)$

note $\pi/3 \to 60^\circ$

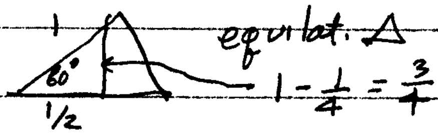


$\to e^{i\pi/3} + e^{-i\frac{\pi}{3}}$

show that $e^{i\pi/3} + e^{-i\pi/3} = +1$ proves the assertion

now $e^{i\pi/3} + e^{-i\pi/3} = 2\cos\frac{\pi}{3}$

~~but~~ $\cos\frac{\pi}{3} = \frac{1}{2}$ q.e.d.

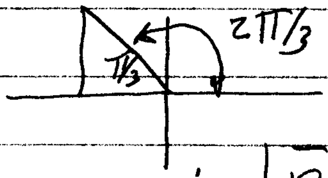


(b) $\bar{r}_1 - \bar{r}_2 = \frac{2}{3} h \left(e^{i\pi/3} - e^{i\pi} \right)$

$$|\bar{r}_1 - \bar{r}_2|^2 = \frac{4}{9} h^2 \left(e^{i\pi/3} - e^{i\pi} \right) \left(e^{-i\frac{\pi}{3}} - e^{-i\pi} \right)$$

$$= \frac{4}{9} h^2 \left(\cancel{1} + \cancel{1} - e^{i\frac{2\pi}{3}} - e^{-i\frac{2\pi}{3}} \right)$$

$$= +\frac{4}{9} h^2 \left(2 - 2\cos\frac{2\pi}{3} \right) = \frac{8}{9} h^2 \left(1 - \cos\frac{2\pi}{3} \right)$$



$\cos\frac{2\pi}{3} = -\frac{1}{2}$

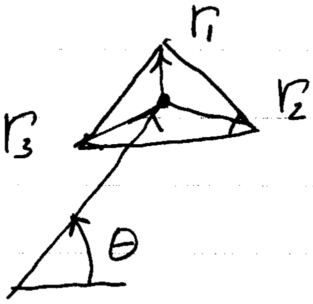
$$\therefore \; |\bar{r}_1 - \bar{r}_2|^2 = \frac{8}{9} \cdot \frac{3}{2} h^2 = \frac{4}{3} h^2$$

$$|\bar{r}_1 - \bar{r}_2| = \frac{2}{\sqrt{3}} h = l$$

The other cases follow in a similar manner.

Problem 5 continued.

(c) follows from the geometry



The entire config. turns thru an angle θ.

(d) $r_1 = re^{i\theta} + \frac{\sqrt{3}}{3} l e^{i(\theta + \frac{\pi}{3})}$

$r_2 = re^{i\theta} + \frac{\sqrt{3}}{3} l e^{i(\theta + \pi)}$

$r_3 = re^{i\theta} + \frac{\sqrt{3}}{3} l e^{i(\theta - \frac{\pi}{3})}$

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note

$$r_1 = e^{i\theta}\left(r + \frac{\sqrt{3}}{3} l e^{i\pi/3}\right)$$

let $a_1 = \frac{\sqrt{3}}{3} l e^{i\pi/3}$, $a_2 = \frac{\sqrt{3}}{3} l e^{i\pi}$, $a_3 = \frac{\sqrt{3}}{3} l e^{-\frac{i\pi}{3}}$

$$r_n = e^{i\theta}(r + a_n) \qquad n = 1, 2, 3$$

(e)

$$v = \begin{bmatrix} \dot{r}e^{i\theta} + i\dot{\theta}(r + a_1)e^{i\theta} \\ \dot{r}e^{i\theta} + i\dot{\theta}(r + a_2)e^{i\theta} \\ \dot{r}e^{i\theta} + i\dot{\theta}(r + a_3)e^{i\theta} \end{bmatrix}$$

or $$V_n = \dot{r}e^{i\theta} + i\dot{\theta}(r+a_n)e^{i\theta}$$

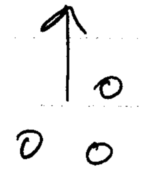
(f) ∴ $(Q_r)_n = e^{i\theta}$, $(Q_\theta)_n = i(r+a_n)e^{i\theta}$

(g)
$$\dot{V}_n = \ddot{r}e^{i\theta} + \dot{r}i\dot{\theta}e^{i\theta} + \dot{r}i\dot{\theta}e^{i\theta} + i\ddot{\theta}re^{i\theta} - r\dot{\theta}^2e^{i\theta} + i\ddot{\theta}a_ne^{i\theta} - \dot{\theta}^2a_ne^{i\theta}$$

$$\dot{V}_n = e^{i\theta}\left\{\ddot{r} + 2i\dot{r}\dot{\theta} - r\dot{\theta}^2 + i\ddot{\theta}(r+a_n) - a_n\dot{\theta}^2\right\}$$

$$\boxed{\dot{V}_n = e^{i\theta}\left\{\ddot{r} + 2i\dot{r}\dot{\theta} - r\dot{\theta}^2 + ir\ddot{\theta} + a_n(i\ddot{\theta} - \dot{\theta}^2)\right\}}$$

(h) $$\dot{P}_n = \frac{1}{3}m\dot{V}_n$$



(i) $$(F_a)_n = \frac{img}{3}$$

gravity. up or
coord. down +
as you wish!

$$F_a Q_r^* = \frac{i\,mg}{3}(1,1,1)\begin{pmatrix}1\\1\\1\end{pmatrix} e^{-i\theta}$$

$$\boxed{F_a Q_r^* = i\,mg\,e^{-i\theta}}$$

$$F_a Q_\theta^* = \frac{i\,mg}{3}(1,1,1)\begin{pmatrix}-i(r+a_1)\\-i(r+a_2)\\-i(r+a_3)\end{pmatrix} e^{-i\theta}$$

$$= \frac{i\,mg}{3}\left(-3ir - \underbrace{(a_1+a_2+a_3)}_{\to 0}i\right) e^{-i\theta}$$

$$\boxed{F_a Q_\theta^* = mg\,r\,e^{-i\theta}}$$

$$\cancel{\dot{P}\,Q_r^*} = \frac{1}{3}m$$

$$Q_r^* \dot{P} = e^{-i\theta}(1,1,1)\begin{pmatrix}\dot{V}_1\\\dot{V}_2\\\dot{V}_3\end{pmatrix}\frac{1}{3}m$$

$$= \frac{1}{3}m e^{-i\theta}\left(\dot{V}_1 + \dot{V}_2 + \dot{V}_3\right)$$

$$Q_r^* \dot{P} = \cancel{\tfrac{1}{3}}m\cancel{e^{-i\theta}}\left(\ddot{r} + 2i\dot{r}\dot{\theta} - r\dot{\theta}^2 + i r\ddot{\theta}\right)\cancel{e^{i\theta}}$$

where we used the fact that $a_1 + a_2 + a_3 = 0$

$$Q_r^* \dot{P} = m\left(\ddot{r} + 2i\dot{r}\dot{\theta} - r\dot{\theta}^2 + i r\ddot{\theta}\right)$$

$$Q_\theta^* \dot{P} = -ie^{-i\theta}(r+a_1^*, r+a_2^*, r+a_3^*)\begin{pmatrix}\dot{V}_1\\ \dot{V}_2\\ \dot{V}_3\end{pmatrix}\frac{1}{3}m$$

$$= -ime^{-i\theta}r(\ddot{r}+2i\dot{r}\dot{\theta}-r\dot{\theta}^2+ir\ddot{\theta})e^{i\theta}$$

$$-i\frac{m}{3}e^{-i\theta}(a_1^*\dot{V}_1 + a_2^*\dot{V}_2 + a_3^*\dot{V}_3)$$

now. $$a_n^*\dot{V}_n e^{-i\theta} = a_n^*(\ddot{r}+2i\dot{r}\dot{\theta}-r\dot{\theta}^2+ir\ddot{\theta} + a_n(i\ddot{\theta}-\dot{\theta}^2))$$

$$= (\ddot{r}+2i\dot{r}\dot{\theta}-r\dot{\theta}^2+ir\ddot{\theta})a_n^* + a_n a_n^*(i\ddot{\theta}-\dot{\theta}^2)$$

$$\therefore Q_\theta^*\dot{P} = -i\frac{m}{3}(\ddot{r}+2i\dot{r}\dot{\theta}-r\dot{\theta}^2+ir\ddot{\theta})\overbrace{(a_1^*+a_2^*+a_3^*)}^{0}$$

$$-i\frac{m}{3}\cancel{e^{-i\theta}}\sum(a_n a_n^*)(i\ddot{\theta}-\dot{\theta}^2) - im\cancel{e}r(\ddot{r}+2i\dot{r}\dot{\theta}-r\dot{\theta}^2+ir\ddot{\theta})$$

$\therefore$ taking real part we find

(1) $$m(\ddot{r}-r\dot{\theta}^2) = mg\sin\theta$$

(2) $$\frac{m}{3}\sum|a_n|^2\ddot{\theta} + mr(\dot{r}\dot{\theta}+r\ddot{\theta})$$

$$= mgr\cos\theta$$

note $r\dot{r}\dot{\theta} + r^2\ddot{\theta} = \frac{1}{2}\frac{d}{dt} r^2\dot{\theta}$

hence.

$$\frac{m\Sigma}{3}|a_u|^2\ddot{\theta} + \frac{d}{dt}\left(\frac{m}{2}r^2\dot{\theta}\right)$$

$$= mg.r\cos\theta$$

or

$$\frac{d}{dt}\left(\frac{m\Sigma}{3}|a_u|^2\dot{\theta} + \frac{1}{2}mr^2\dot{\theta}\right) = mgr\cos\theta$$

let $\frac{m}{3}\Sigma|a_u|^2 = I$

$$\frac{d}{dt}\underbrace{\left(I\dot{\theta} + \frac{1}{2}mr^2\dot{\theta}\right)}_{\text{Rate of change of Total angular momentum}} = mgr\cos\theta$$

= Moment of force around orig

$$\Sigma|a_u|^2 = \frac{1}{3}l^2 + \frac{1}{3}l^2 + \frac{1}{3}l^2 = l^2$$

$$\therefore \boxed{I = \frac{1}{3}ml^2}$$

This is called the Moment of Inertia about the center of mass

The moment of Inertia about the origin is $\boxed{I + \frac{1}{2}mr^2}$ of the 3 particle system.

This problem is oversimplified in that we have no rotation about the center of mass— we have a fixed orientation.

Problem 6

$$E = V + K = \text{Potential} + \text{Kinetic Energy}$$

$$\therefore\ V = -\frac{km}{r^2} \qquad \text{where } K = MG$$

$$K = \frac{1}{2} m |v|^2 = \frac{1}{2} m \dot{z}^* \dot{z}$$

$$\therefore\ E = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 - \frac{km}{r^2}$$

in the ~~notion~~ notation of the notes. we fix the Constant energy as. (since $\dot{r} = 0$ at perihelion)

$$E_T = \frac{1}{2} m r_0^2 \dot{\theta}_0^2 - \frac{mk}{r_0} = \frac{1}{2} m v_m^2 - m v_c^2$$

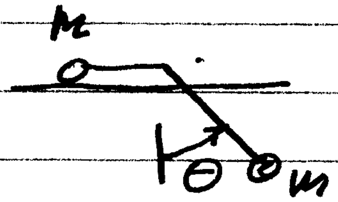
$$E_T = m\left(\frac{1}{2} v_m^2 - v_c^2\right)$$

but $v_m^2 = (1+\varepsilon) v_c^2$

$$\therefore\ E_T = \frac{\varepsilon - 1}{2} m v_c^2 = -\frac{1}{2}(1-\varepsilon) m v_c^2$$

or $\quad E = -\frac{1}{2}\left(\frac{r_0}{a}\right) m v_c^2$

Problem 7



$$z_1 = -(l-q)$$
$$z_2 = -q\,i\,e^{i\theta}$$

$$\dot{z}_1 = \dot{q}\,, \quad \dot{z}_2 = -i\dot{q}e^{i\theta} + q\dot{\theta}e^{i\theta}$$

$$\therefore \quad KE = m\dot{q}^2 + \frac{1}{2} m q^2 \dot{\theta}^2$$

$$PE = +\int_{\frac{\pi}{2}}^{\theta} mgq \sin\theta \, d\theta = -mgq\cos\theta$$

hence $\boxed{L = m\dot{q}^2 + \frac{1}{2}mq^2\dot{\theta}^2 + mgq\cos\theta}$

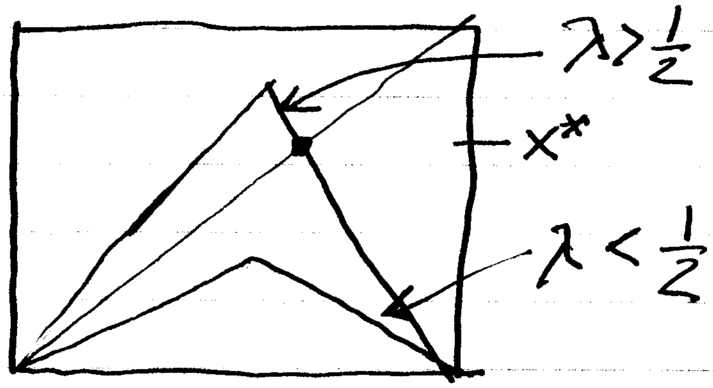
$$\frac{\partial L}{\partial \dot{q}} = 2m\dot{q}\,, \quad \frac{\partial L}{\partial \dot{\theta}} = mq^2\dot{\theta}$$

$$\frac{\partial L}{\partial q} = mq\dot{\theta}^2 + mg\cos\theta$$

$$\frac{\partial L}{\partial \theta} = -mgq\sin\theta$$

$$\therefore \quad \boxed{\begin{aligned} &\frac{d}{dt}(2m\dot{q}) - mq\dot{\theta}^2 - mg\cos\theta = 0 \\ &\frac{d}{dt}(mq^2\dot{\theta}) + mgq\sin\theta = 0 \end{aligned}}$$

Problem 8 Tent Map

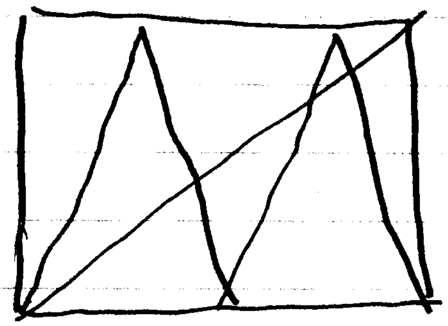


$\lambda < \frac{1}{2}$ only root is 0 which is stable.

$\lambda > \frac{1}{2}$ two roots, 0 & and $\bar{x} > \frac{1}{2}$

both are unstable. but larger root has eigenvalue < 0 generates cycles!

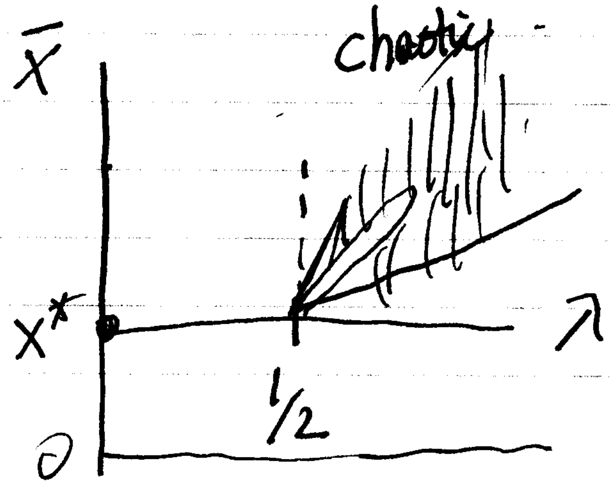
at $\lambda = \frac{1}{2}$ all $0 < x < \frac{1}{2}$ are neutral stable



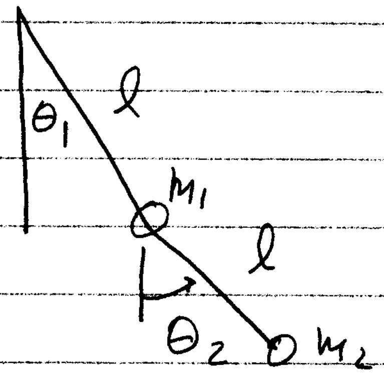
$f \circ f$ at $\lambda = 1$

Shows how multiple periods arise ∴

Bifurcat. diagram



problem 9



$\downarrow g$,

choose units where $l = 1$, $m_2 = 1$, $g = 1$

$$z_1 = -lie^{i\theta_1}, \quad z_2 = -lie^{i\theta_1} - lie^{i\theta_2}$$

or in normalized units

$$z_1 = -ie^{i\theta_1}, \quad z_2 = -ie^{i\theta_1} - ie^{i\theta_2}$$

$$\dot z_1 = \dot\theta_1 e^{i\theta_1}, \quad \dot z_2 = \dot\theta_1 e^{i\theta_1} + \dot\theta_2 e^{i\theta_2}$$

so

$$V = \begin{bmatrix} \dot\theta_1 e^{i\theta_1} \\ \dot\theta_1 e^{i\theta_1} + \dot\theta_2 e^{i\theta_2} \end{bmatrix} = Q_1 \dot\theta_1 + Q_2 \dot\theta_2$$

$$Q_1 = \begin{pmatrix} e^{i\theta_1} \\ e^{i\theta_1} \end{pmatrix} \quad Q_2 = \begin{pmatrix} 0 \\ e^{i\theta_2} \end{pmatrix}$$

$$P = \begin{bmatrix} m_1 \dot\theta_1 e^{i\theta_1} \\ \dot\theta_1 e^{i\theta_1} + \dot\theta_2 e^{i\theta_2} \end{bmatrix}$$

$$\dot P = \begin{bmatrix} m_1 \ddot\theta_1 e^{i\theta_1} + m_1 \dot\theta_1^2 i e^{i\theta_1} \\ \ddot\theta_1 e^{i\theta_1} + \ddot\theta_2 e^{i\theta_2} + i\dot\theta_1^2 e^{i\theta_1} + i\dot\theta_2^2 e^{i\theta_2} \end{bmatrix}$$

$$F = \begin{bmatrix} -i m_1 g \\ \\ \cancel{i m_2} -i g \end{bmatrix}$$ where $m_2 = 1$, $g = 1$, $l = 1$

so $$F = \begin{bmatrix} -i m_1 \\ -i \end{bmatrix}$$

$$Q_1^* \dot{P} = (e^{-i\theta_1}, e^{-i\theta_1}) \dot{P}$$

$$= m_1 \ddot{\theta}_1 + m_1 i \dot{\theta}_1^2 + \ddot{\theta}_1 + i \dot{\theta}_1^2 + \ddot{\theta}_2 e^{i(\theta_2 - \theta_1)} + i \dot{\theta}_2^2 e^{i(\theta_2-\theta_1)}$$

$$Q^* F = (e^{-i\theta_1}, e^{-i\theta_1}) \begin{pmatrix} -i m_1 \\ -i \end{pmatrix}$$

$$= -i m_1 e^{-i\theta_1} - i e^{-i\theta_1}$$

taking real part and equating the results.

$$m_1 \ddot{\theta}_1 + \ddot{\theta}_1 + \ddot{\theta}_2 \cos(\theta_2 - \theta_1) - \dot{\theta}_2^2 \sin(\theta_2 - \theta_1)$$

$$= -m_1 \sin\theta_1 - \sin\theta_1$$

or

$$(1+m_1)\ddot{\theta}_1 + \ddot{\theta}_2 \cos(\theta_2 - \theta_1) - \dot{\theta}_2^2 \sin(\theta_2 - \theta_1)$$

$$= -(1+m_1) \sin\theta_1$$

$$\ddot{\theta}_1 + \frac{1}{1+m_1}\ddot{\theta}_2 \cos(\theta_2 - \theta_1) - \frac{1}{1+m_1}\dot{\theta}_2^2 \sin(\theta_2 - \theta_1)$$
$$+ \sin\theta_1 = 0$$

to find the coef. in normal units recall we used $m_2 = 1$

$$\therefore \frac{1}{1+m_1} \to \frac{1}{1+\frac{m_1}{m_2}} = \frac{m_2}{m_1+m_2} = M$$

the time unit is $\sqrt{\frac{l}{g}}$ therefore

the $\frac{d\theta_1^2}{dt^2}$ shows each term must have units of $\frac{1}{(\text{time})^2}$ $\therefore \sin\theta_1 \to \frac{g}{l}\sin\theta_1$

or

$$0 = \ddot{\theta}_1 + M\ddot{\theta}_2 \cos(\theta_2 - \theta_1) - M\dot{\theta}_2^2 \sin(\theta_2 - \theta_1) + \frac{g}{l}\sin\theta_1$$

as in Ackeson eq. 12.15(a)

(12.15 b) is derived by evaluating

$$\text{Real}.\left(Q_2^* \dot{P} - Q_2^* F\right) = 0$$

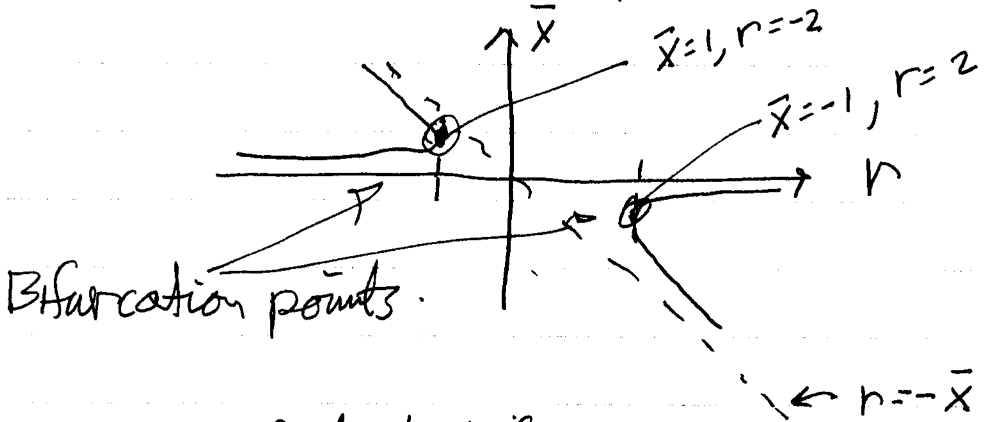
The work involved is far less than with Lagrange's equations.

Problem 10

$$\dot{x} = 1 + rx + x^2 = f(r, x)$$

we can solve $1 + r\bar{x} + \bar{x}^2 = 0$ for $r = -1/x - x$ and see what happens as $x \to 0$ with $x > 0$, $x \to 0$, with $x < 0$, $x \to \infty$ with $x > 0$, $x \to \infty$ with $x < 0$

This leads to the sketch below



we can find the bifurcation points by computing $\frac{dr}{d\bar{x}}$

$$\cancel{\frac{dr}{dx}} \quad \frac{d}{d\bar{x}}\left(1 + r\bar{x} + \bar{x}^2\right) = 0$$

$$\therefore \quad \frac{dr}{d\bar{x}}\bar{x} + r + 2\bar{x} = 0$$

or $$\frac{dr}{d\bar{x}} = -\frac{2\bar{x} + r}{\bar{x}}$$

and $\frac{dr}{d\bar{x}} = 0$ when $r = -2\bar{x}$ thus from $1 + r\bar{x} + \bar{x}^2 = 0$ we see $\bar{x} = \pm 1$, $r = \mp 2$

stability is found by noting that $f(0,0) = +1 > 0$