Introduction course in particle image velocimetry

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Introduction

Particle image velocimetry (PIV) is a technique which enables instantaneous measurement of the flow velocity at several positions in a plane. The working principle is quite simple (c.f. figure 1): the flow is seeded with light reflecting particles (usually smoke is used in air and glass beads in water), a light sheet illuminates the particles in the measurement plane and a camera is used to take two exposures of the illuminated plane. The two exposures should be taken within a short interval, so that the same particles are caught in both exposures. The two exposures can be taken either as a double exposure of one image or as two different images. The double exposure technique is based on an auto-correlation and this results in an directional ambiguity (since it is impossible to tell whether an imaged particle was in the first or second exposure), thus the flow can only be in one direction when this method is used. Hence, the method with two images and cross-correlation is more commonly used and the remaining part of this introduction will deal with this technique. A good reference book which deals with most of the aspects of PIV is Raffel *et al.* (1998).
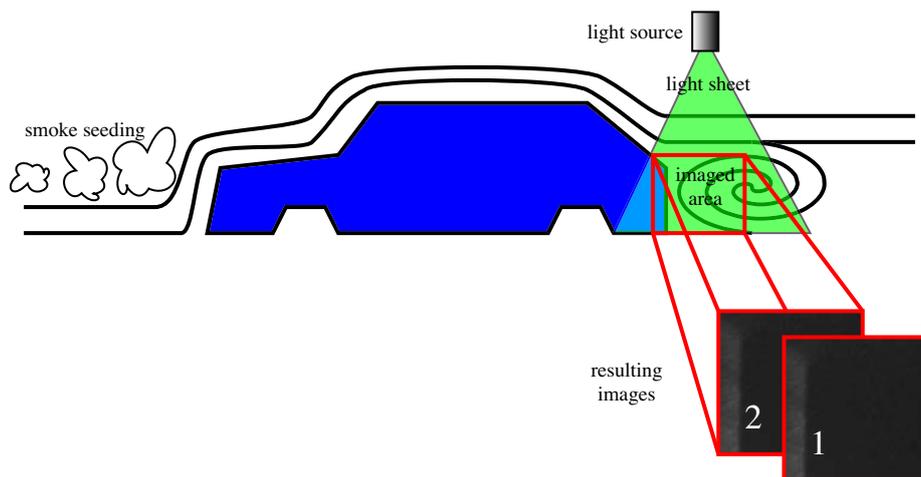

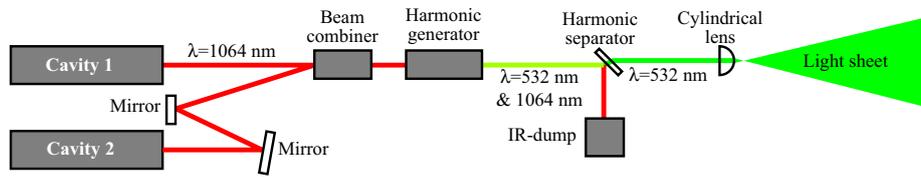Figure 1: Sketch of a typical setup for PIV measurements.

Figure 2: Double cavity Nd:YAG PIV-laser.

Measurement technique

Seeding

Seeding the flow with light reflecting particles is necessary in order to image the flow field. The particles should be small enough to follow the flow but large enough to reflect the required amount of light. In general one can say that PIV needs a higher seeding density than LDV (Laser Doppler Velocimetry). A good rule of thumb is that around ten particles should be correlated for each measured velocity vector.

The light sheet

Figure 1 shows a typical PIV setup and figure 2 shows the principal layout of a common PIV-laser. The plane where the measurements are to be taken should be illuminated by a light sheet. Commonly a pulsed Nd:YAG laser (Neodymium Yttrium Aluminum Garnet) is used as the light source because of its high light intensity. Pulsed lasers need some time to build up energy before they can deliver a new pulse and the two images in a PIV image pair has to be taken within a quite short period of time. Therefore, it is common in PIV to use a laser with two cavities. The laser pulses have a duration time of 5-10 ns and the energy in one pulse can be up to 400 mJ[1].

Nd:YAG lasers emit light with a wavelength of 1064 nm which is in the infrared range. For PIV-purposes light with this wavelength is not very useful since most cameras have their maximum sensitivity in the blue-green part of the spectrum. It is also disadvantageous not to be able to see the light sheet when positioning it in the measurement section. For these reasons the wavelength of the Nd:YAG laser is halved, using a device called a harmonic generator, so that it becomes 532 nm. The harmonic generator is not 100% efficient and therefore a separator and an IR-dump is needed to get rid of the remaining IR-light. The laser beam coming out from the laser has an axisymmetric shape which need to the shaped with a cylindrical lens in order to form a planar light sheet.

The camera

PIV puts special demands on the camera that is going to be used, especially if the flow velocity is high, the imaged area is small and if the particles are small. The first two circumstances requires the camera to be able to take two images

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[1]Note that this duration time and energy level make a power of more than 40 MW possible, so the laser light should be handled with care
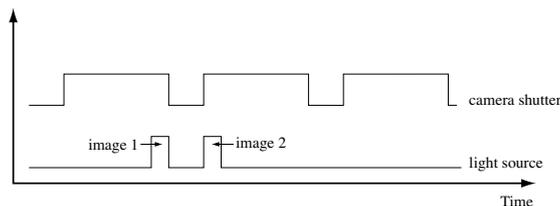
Figure 3: How short inter image times are achieved when using continuos high speed cameras.

within a short period of time in order for the same individual particles to appear in both images. Short inter exposure times can be achieved either by having a high speed camera which continuously records images at a rate of several kHz or by using a camera with a progressive scan architecture. With a high speed camera the shortest inter image time possible today is around 10 μs and this is achieved by letting the first laser pulse come at the end of the exposure of the first image and the second pulse in the beginning of the exposure of the second image, c.f. figure 3. Cameras with progressive scan architecture can take two images with less than 1 μs delay. Figure 4 shows the layout of such a sensor. Directly after the first image has been recorded the charge of each pixel is transferred to its designated position in the interline shift register, a new image can now be recorded by the pixels. The second image is exposed until the first image has been read out from the interline shift register and is then transferred to the image buffer in the same way. The second image will due to this procedure be exposed for a longer time than the first and to avoid it becoming overexposed by ambient light a filter which only pass the laser wavelength can be fitted on the camera lens.

If the particles are small or if the light intensity is low the camera needs to have a high sensitivity to incoming light. The sensitivity of a photon sensor such as a CCD-camera is measured in QE (quantum efficiency) which is the average number of electrons that are released from the sensor when it is hit by a photon. The QE is often wavelength dependent with a maximum efficiency in the blue-green part of the visible spectrum. The most sensitive cameras on the market today have Peltzier cooled (to reduce thermal noice) CCD-sensors (CCD=Charge coupled device) and a QE of around 70%.
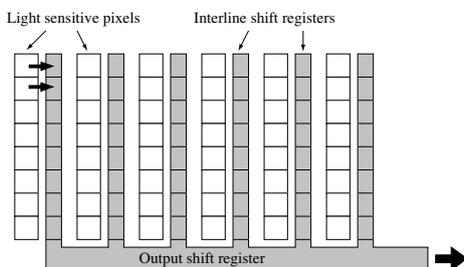

Figure 4: Architecture of an interline CCD.

Since the laser light is monochromatic there is no need to use a colour camera, this also makes the image files smaller.

Image evaluation

Cross-correlation of images

The aim of the cross-correlation is to find the distance that the particle pattern has moved during the inter image time and translate this into a velocity measure. The relation between velocities $\mathbf{u}$ and particle displacements $\mathbf{d}$ is simply

$$\mathbf{u} = \frac{\mathbf{d}}{M\Delta t},\tag{1}$$

where M is the magnification and Δt is the inter image time.

The cross-correlation function is not calculated on the whole images but on smaller parts of these called interrogation areas (IAs), see figure 6. The cross-correlation from one IA results in one velocity vector. The cross-correlation can be seen as finding which relative displacement of the IAs that gives the best pattern match. This displacement should be proportional to the average velocity in the IAs.

Cross-correlation functions can be calculated in a number of different ways (c.f. e.g. Bendat & Piersol (1986)), the direct procedure to compute an unbiased one dimensional sample cross-correlation function $R_{AB}(x,y)$, for the $M \times N$-point samples $A(m,n)$ and $b(m,n)$, with $x < M$ and $y < N$ is defined by

$$R_{A,B}(x,y) =$$
$$\begin{cases} \frac{1}{(M-|x|)(N-|y|)} \sum_{m=1}^{M-x} \sum_{n=1}^{N-y} A(m,n)B(m+x,n+y) & x,y \geq 0 \\ \frac{1}{(M-|x|)(N-|y|)} \sum_{m=1}^{M+x} \sum_{n=1}^{N-y} A(m-x,n)B(m,n+y) & x<0,\ y \geq 0 \\ \frac{1}{(M-|x|)(N-|y|)} \sum_{m=1}^{M-x} \sum_{n=1}^{N+y} A(m,n-y)B(m+x,n) & x \geq 0,\ y < 0 \\ \frac{1}{(M-|x|)(N-|y|)} \sum_{m=1}^{M+x} \sum_{n=1}^{N+y} A(m-x,n-y)B(m,n) & x,y < 0 \end{cases}\tag{2}$$

Cross-correlation function via finite Fourier transforms

The direct method to compute the cross-correlation quickly becomes very heavy to apply when larger data-sets are to be analysed. A more efficient way to estimate cross-correlation functions is use fast Fourier transforms (FFTs). This reduces the computation from $\mathcal{O}[N^4]$ operations to $\mathcal{O}[N^2 \log_2 N]$ operations in the case of a two-dimensional correlation. When Fourier transforms are used one takes advantage of the correlation theorem (see e.g. Bendat & Piersol (1986)) which states that the cross-correlation of two functions is equivalent to a complex conjugate multiplication of their Fourier transforms:

$$R_{AB} \Leftrightarrow \hat{A} \cdot \hat{B}^*\tag{3}$$

where $\hat{A}$ and $\hat{B}$ are the Fourier transforms of A and B, respectively and $\hat{B}^*$ represents the complex conjugate of $\hat{B}$.

Using FFTs means treating the data as if it is periodic. The periodicity can give rise to aliasing if the particles have moved a distance larger than half the

Sample weighting

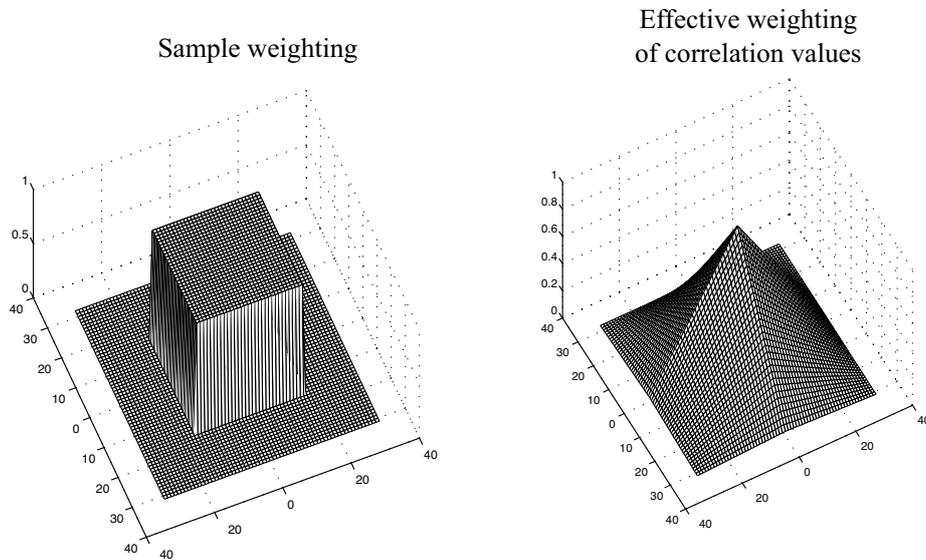Effective weighting
of correlation values

Figure 5: Effective correlation value weighting in FFT-based cross-correlation calculations. The effective weighting function to the right is the convolution of two sample weighting functions such as the one to the left.

size of the IA. The solution to aliasing problems is to either increase the IA size or reduce the inter image time Δt. A maybe more serious problem with the FFTs is that bias errors occur if these are not accounted for. Due to the finite size of the IAs the overlap of the images becomes smaller with increasing displacement. This bias results in an underestimation of the peak magnitude for all displacements other than zero. A weighting function should be applied to the cross-correlation function to avoid this bias. This weighting function is found by convoluting the sample weighting functions (which should be equal to one for all points in the image and zero elsewhere). The bias is removed by dividing the correlation function with the effective weighting function which will have a pyramid shape, see figure 5. An instructive example of convolution can be found on `http://www.jhu.edu/~signals/convolve/`.

It should be noted that many FFT implementations (e.g. the one in Matlab) shuffles the output data so that the DC-component is found at index 1 and the highest positive and negative frequency at index $N/2 - 1$. To get a monotonically increasing spectrum (and resulting dispacements) the data has to be reorganised (`fftshift` in Matlab).

Peak detection and subpixel interpolation

When the cross-correlation has been performed a measure of the displacement is found by detecting the location of the highest correlation peak. Just detecting the peak will result in an uncertainty of $\pm 1/2$ pixel in the peak location. However, the accuracy can be increased substantially by curve fitting and interpolation. This may sound as 'inventing' new information which hasn't been measured. But the procedure can be defended with the argument that the cor-
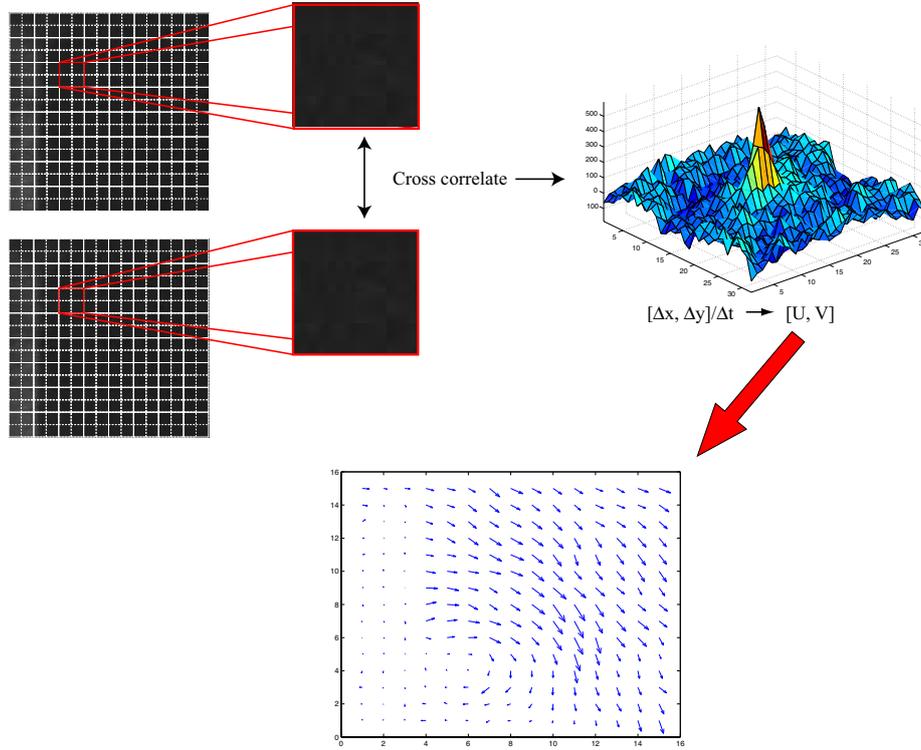
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Figure 6: Illustration of how velocity information is extracted from an image pair.

relation is based on the images of several particles. If, for example, an interrogation area pair contains ten particle images and eight particles has a displacement of 3 pixels and two a displacement of 2 pixels, the maximum correlation peak would be located at 3 pixels but a subpixel interpolation could predict the correct displacement of 2.8 pixels since the correlation at two pixels will be higher than the one at four pixels.

The most common way to perform the subpixel interpolation is to use a three-point estimator. When the maximum peak has been detected at $[i, j]$, the neighbouring values are used to fit a function to the peak. In the case of a Gaussian peak fit when the peak is assumed to have the shape $f(x) = C \exp[-(x_0 - x)^2/k]$, the displacements are found by:

$$
\begin{aligned}
x_0 &= i + \frac{\ln R_{(i-1,j)} - \ln R_{(i+1,j)}}{2 \ln R_{(i-1,j)} - 4 \ln R_{(i,j)} + 2 \ln R_{(i+1,j)}} \\
y_0 &= j + \frac{\ln R_{(i,j-1)} - \ln R_{(i,j+1)}}{2 \ln R_{(i,j-1)} - 4 \ln R_{(i,j)} + 2 \ln R_{(i,j+1)}}.
\end{aligned}
$$

Other ways to interpolate at the subpixel level are parabolic peak fit and peak centroid.

Overlapping IAs (oversampling) are often used, see figure 6, in order to maximise the use of available information and fulfil the Nyquist sampling theorem.

Homework assignment

Before the laboratory lesson at KTH you should prepare yourself by writing (preferably in Matlab) your own PIV-image evaluation routine. After you have collected your data in the laboratory lesson you should be able use your own routine to calculate the velocities.

To help you to develop a routine an image pair `Image18_A.tif` and `Image18_B.tif` are provided at the course web-site. The image pair was recorded, with an inter image time of 50 μs, behind a "notchback" car model as can be seen i figure 1. The images size is 256×256 pixels, the physical image size is 37.8×37.8 mm. For reference a velocity field computed by the PIV-system at Mechanics KTH is provided together with a Matlab script which reads and plots the data.

Suggested procedure

If you want you can start with the following procedure when designing your evaluation routine. Useful Matlab commands at the different stages are given.

- Read in the image data. (`imread`)

- Convert the data to double precision (native in Matlab) to avoid problems with certain functions. (`double`)

- Remove the dominating DC component in the images. (`A-mean(A)`)

- Select interrogation areas of 32×32 pixels with 50% overlap.

- Compute the FFTs of the IAs. (`fft2`)

- Compute the cross-spectral density function. (`A.*conj(B)`)

- Compute the cross-correlation by the inverse transformation. (`ifft2`)

- Shuffle the data so that the origin is at the center. (`ifftshift2`)

- Locate the highest correlation peak. (`[Y,I]=max(R)`)

- Convert the displacement to physical units and calculate the corresponding velocities.

- Plot the data. (`quiver`)

Compare your result with the provided reference field. Try to implement sub-pixel interpolation and see how the results improves. Spurious vectors can be avoided by applying a criterion for the minimum ratio between the highest peak and the second highest peak. A good value of the peak height ratio is 1.2.

Don't hesitate to contact me at telephone (08-790 6752) or e-mail (olle@mech.kth.se) if you have any questions regarding the assignment or PIV in general.

Acknowledgement

References

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RAFFEL, M., WILLERT, C. & KOMPENHANS, J. 1998 *Particle Image Velocimetry. A Practical Guide*. Springer.