SUBCRITICAL AND SUPERCRITICAL TRANSITION IN CURVED PIPES

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The flow through curved pipes has received increasing attention in the past decades, but several phenomena still miss an exhaustive explanation. Bent pipes are fundamental components in various industrial devices (for a review see Ref. [8]), and are also studied in the medical field, being an integral part of vascular and respiratory systems [2, 3]. The present work focuses on the flow inside a toroidal pipe, which constitutes the common asymptotic limit between two 'real' flow cases: the spatially developing and the helical pipes. The flow is determined by the Reynolds number Re and a single geometrical parameter: the curvature δ (the ratio between pipe and torus radii).

The transition to turbulence of this flow has received a considerable amount of interest in the past decades. The recent works by Canton *et al.* [4] and Kühnen *et al.* [6] determined that the flow is linearly unstable, and undergoes a Hopf bifurcation, for any curvature greater than zero and for $Re \approx 4000$. Different eigenmodes, in the shape of travelling waves, contribute to the neutral curve for the flow (see figure 5 in Ref. [4]). This behaviour is in contrast to the flow in a straight pipe which undergoes subcritical transition for $Re \gtrsim 2000$ [1].

The transition scenario is different for low curvatures: while for $\delta \geq 0.028$ direct numerical simulations (DNS) confirm the presence of a Hopf bifurcation [4], for $\delta < 0.028$ no clear boundary has been observed. For low curvatures a bent pipe appears to behave similarly to a straight pipe: the flow undergoes transition to turbulence despite being linearly stable to infinitesimal perturbations [6, 7]. We investigate this complex behaviour by means of nonlinear DNS performed with the spectral element code Nek5000 [5] and, in order to isolate the dominant structures in the flow, we analyse the flow by three-dimensional proper orthogonal decomposition (POD). We also analyse the transition process by quantifying the intermittency level and measuring the lifespan of turbulent events. Preliminary results indicate that, indeed, the flow does not abruptly transition from the steady to the unsteady regime. Instead, transition occurs over a range of curvatures and Reynolds numbers indicating a subcritical behaviour.

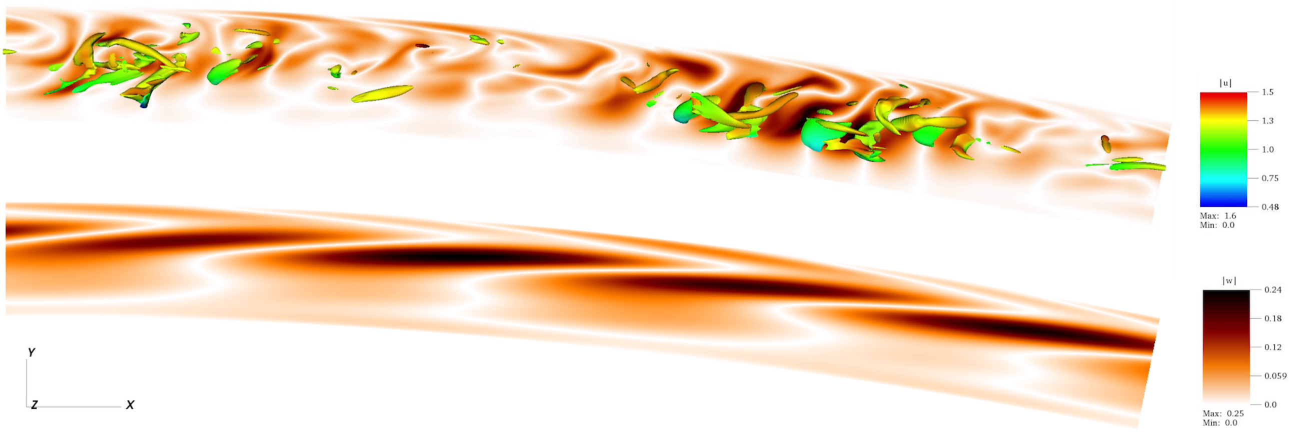

Figure 1. *(Top):* Instantaneous flow field for $\delta = 0.01$ and $Re = 3000$ represented by isocontours of negative λ_2 coloured by streamwise velocity magnitude (u) and azimuthal velocity magnitude (w) on a longitudinal section. *(Bottom):* Corresponding most energetic POD mode showing a travelling wave with a wavelength of about 5 pipe diameters.

References

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