LARGE-EDDY SIMULATION USING THE EXPLICIT ALGEBRAIC SUBGRID MODEL IN COMPLEX GEOMETRIES

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ABSTRACT

In Rasam *et al.* (2011) we compared the performance of the explicit algebraic subgrid-scale (SGS) model (EASSM) (Marstorp *et al.*, 2009) with that of the conventional dynamic Smagorinsky model (DSM) in large eddy simulation (LES) of channel flow using a pseudo-spectral Navier–Stokes solver. We showed that, due to the better prediction of the individual SGS stresses in a wide range of grid resolutions and due to the nonlinear SGS stress contribution by the model, the EASSM predictions were less resolution dependent and more accurate than those of the DSM. As the first step in this study towards LES in complex geometries, we extend our previous study and perform LES of turbulent channel flow at $Re_\tau = 590$ using the EASSM and the *code_Saturne*, which is an unstructured finite volume solver suitable for LES in complex geometries. The results are compared to those of the DSM and show that the EASSM predictions of the wall shear and the Reynolds stresses are more accurate. LES results using the EASSM obtained from the *code_Saturne* are also compared to those obtained using the pseudo-spectral solver obtained in our previous study (Rasam *et al.*, 2013). As the next step, we are performing LES of flow over periodic hill using the EASSM and the *code_Saturne*. The results will be compared with those of the DSM and the reference LES and will be presented at the conference.

Introduction

The importance of nonlinear SGS stress contribution in LES of wall-bounded flows is well known and the commonly used assumption of an isotropic linear relationship between the SGS stress and the resolved strain-rate tensor is not truly valid for such flows (Horiuti, 2003; Marstorp *et al.*, 2009; Wang & Bergstrom, 2005; Kosovic, 1997; Rasam *et al.*, 2011). The value of nonlinear SGS stress models in LES of more complex flows is still an open issue and needs to be more investigated.

Although application of nonlinear SGS stress models is important in LES of turbulent flows in complex geometries, it can also have an impact on LES results in more simple geometries like channel flows with and without system rotation. In (Rasam *et al.*, 2011), we showed that application of isotropic SGS stress models, e.g. the eddy viscosity model, leads to LES predictions that are strongly resolution dependent. It was also shown that LES results using such models at coarse resolutions, where the SGS anisotropy is appreciable, are very inaccurate due to the lack of the nonlinear SGS stress contribution. Therefore, resolution requirements to produce reliable LES predictions using eddy viscosity models, approaches that of the DNS (Baggett *et al.*, 1997; Choi & Moin, 2012). We showed that using the explicit algebraic subgrid stress model (EASSM), which is a nonlinear mixed model (Marstorp *et al.*, 2009), gives LES predictions that are more accurate and less resolution dependent. The EASSM is derived from the modeled transport equations of the SGS stress anisotropy. The model has been shown to improve LES predictions of rotating and non-rotating turbulent channel flow at various Reynolds numbers and different axes of rotation, especially at coarse resolutions, over the commonly used isotropic dynamic Smagorinsky model (Rasam *et al.*, 2011, 2012, 2013). In this study we will extend our investigation of the performance of the EASSM to the LES of more complex flows. For this purpose, LES of turbulent flow over a periodic hill (see e.g. Frohlich *et al.*, 2005; Breuer *et al.*, 2009; Manhart *et al.*, 2011) will be carried out using the EASSM and the results will be compared to those of the highly resolved LES of Frohlich *et al.* (2005).

In LES of flows in complex geometries low-order numerical methods are often used and the numerical discretization schemes that are usually employed have inherent numerical dissipation, due to the truncation errors, which is often of the same order as the SGS force (see e.g. Chow & Moin, 2003). Therefore, contrary to our previous studies (Marstorp *et al.*, 2009; Rasam *et al.*, 2011, 2012, 2013), where an accurate pseudo-spectral Navier–Stokes solver was employed, in this study we use a finite volume solver which has a second-order spatial accuracy and inherent numerical dissipation. We test the performance of the EASSM in a simple case, which is an essential step before we apply it to more complex geometries. To quantify the effect of the numerical dissipation on the performance of the SGS models, LES results with no SGS model are also presented for comparison. As the first step, LES of channel flow at $Re_\tau = 590$ using the EASSM and the *code_Saturne* is presented in this paper, where the bulk Reynolds number is similar to that of the periodic-hill case (Frohlich *et al.*, 2005). The LES results for the periodic-hill case will be presented at the conference.

Explicit algebraic subgrid stress model (EASSM)

The EASSM has the following formulation for the SGS stresses, τ_{ij},

$$\tau_{ij} = \frac{2}{3} K^{\text{SGS}} \delta_{ij} + \beta_1 K^{\text{SGS}} \widetilde{S}^*_{ij} + \beta_4 K^{\text{SGS}} (\widetilde{S}^*_{ik} \widetilde{\Omega}^*_{kj} - \widetilde{\Omega}^*_{ik} \widetilde{S}^*_{kj}),$$

which consists of an eddy viscosity (second term on the right-hand side) and a nonlinear term (third term). In this formulation, $\widetilde{\cdot}$ denotes a grid-filtered quantity, $\widetilde{S}^*_{ij}$ and $\widetilde{\Omega}^*_{ij}$ are the resolved normalized strain- and rotation-rate tensors

$$\widetilde{S}^*_{ij} = \frac{\tau^*}{2} \left(\frac{\partial \widetilde{u}_i}{\partial x_j} + \frac{\partial \widetilde{u}_j}{\partial x_i} \right), \quad \widetilde{\Omega}^*_{ij} = \frac{\tau^*}{2} \left(\frac{\partial \widetilde{u}_i}{\partial x_j} - \frac{\partial \widetilde{u}_j}{\partial x_i} \right),$$

where τ^* is the time scale of the SGS motions. β_1 and β_4 are coefficients that determine the relative contribution of the eddy viscosity and the nonlinear terms and are given by

$$\beta_4 = -\frac{33}{20} \frac{1}{\left[(9c_1/4)^2 + |\widetilde{\Omega}^*_{ij}|^2 \right]}, \quad \beta_1 = \frac{9}{4} c_1 \beta_4,$$

The present EASSM uses Yoshizawa's model (Yoshizawa, 1986) for K^{SGS}

$$K^{\text{SGS}} = c\,\Delta^2 |\widetilde{S}_{ij}|^2,$$

where $\Delta = \sqrt[3]{\Omega}$ (Ω is the volume of a computational cell) and c is dynamically determined using the Germano identity (Moin *et al.*, 1991) with local averaging over the neighboring cells and is given as

$$c = \frac{1}{2} \frac{\widehat{\widetilde{u}_k \widetilde{u}_k} - \widehat{\widetilde{u}}_k \widehat{\widetilde{u}}_k}{\widehat{\Delta}^2 |\widehat{\widetilde{S}}_{ij}|^2 - \Delta^2 \widehat{|\widetilde{S}_{ij}|^2}}$$

where $\widehat{\cdot}$ represents the test filter which is a top-hat filter (local averaging over the cells sharing a common face) with a length scale, $\widehat{\Delta} = 3\Delta$. The model parameter c_1 is determined from the dynamic coefficient c and the SGS time scale, τ^*, is modeled using the inverse shear

$$c_1 = c'_1 \sqrt{c'_3} \frac{c^{1.25}}{(2C_s)^{2.5}}, \quad \tau^* = c'_3 \frac{1.5 C_k^{1.5} \sqrt{c}}{2C_s} |\widetilde{S}_{ij}|^{-1}$$

where $c'_1 = 4.2$, $c'_3 = 2.4$, $C_k = 1.6$ is the Kolmogorov constant and $C_s = 0.1$, see Marstorp *et al.* (2009) for details. In the LES using the pseudo-spectral method the grid filter is $\Delta = \sqrt[3]{\Omega}$, where Ω is the volume of a computational cell. Test filtering is performed using a sharp cutoff filter in the homogeneous directions in the Fourier space at a filter width $\widehat{\Delta} = 2\Delta$.

Numerical method

Two numerical methods are used for the simulations. *Code_Saturne* (`www.code-saturne.org`) has been used for most of the LESs performed in this paper. It is an unstructured collocated finite volume solver for incompressible flows (Archambeau *et al.*, 2004), developed by Électricité de France (EDF), and has been employed extensively for simulations of industrial and academic flows (Revell *et al.*, 2006; Aounallah *et al.*, 2007; Monfort *et al.*, 2010; Afgan *et al.*, 2011; Dehoux *et al.*, 2012). It uses a SIMPLEC algorithm for pressure-velocity coupling, which requires Rhie and Chow (Rhie & Chow, 1983) interpolation to avoid odd-even oscillations. The code uses a second-order central differencing in space and a second-order Crank–Nicholson scheme in time.

LES results for the EASSM are obtained both using the *Code_Saturne* (`www.code-saturne.org`) and a pseudo-spectral method. The pseudo-spectral method which is used here (Chevalier *et al.*, 2007) employs Fourier representation in wall-parallel directions (x and z), using periodic boundary conditions, and Chebyshev representation in the wall-normal direction (y), using the Chebyshev–tau method. Aliasing errors are removed using the $3/2$-rule (Hussaini & Zang, 1987). The time integration is carried out using a four-step third-order Runge–Kutta scheme for the nonlinear terms and a second-order Crank–Nicholson scheme for the linear terms.

Table 1. Summary of the channel flow simulations. EASSM/PS and EASSM/SAT: Explicit algebraic SGS stress model with pseudo-spectral method and *code_Saturne*, respectively; DSM: Dynamic Smagorinsky model; NM: No SGS model.

Case	SGS model	Δ_x^+	Δ_z^+	Δ_y^+ $(min \sim max)$	Re_τ
1	EASSM/SAT	55	28	$0.69 \sim 15.9$	566
	EASSM/PS	58	29	$0.71 \sim 28.9$	592
	DSM	50	25	$0.60 \sim 13.8$	506
	NM	56	28	$0.68 \sim 15.7$	571
2	EASSM/SAT	38	19	$0.70 \sim 16.2$	582
	EASSM/PS	38	19	$0.31 \sim 19.2$	586
	DSM	35	17	$0.61 \sim 15.0$	531
	NM	39	19	$0.71 \sim 16.4$	591

Channel flow simulations at $Re_\tau = 590$

Simulations are carried out at two resolutions, see table 1. The table also shows the acronyms that are used here. For the finite volume simulations an algebraic grid using a tangent-hyperbolic distribution in the wall-normal direction is used. For the simulations using the pseudo-spectral method roots of the Chebyshev polynomials represent the grid-point distribution in the wall-normal direction. The flow domain is a rectangular box with $2\pi\delta$ in the streamwise and $\pi\delta$ in the spanwise directions, where δ is the channel half height. LES results using the EASSM are compared to the DNS results of Rasam *et al.* (2013), which are similar to those of Moser *et al.* (1999), and the LES results using the dynamic Smagorinsky model (DSM) (Germano *et al.*, 1991; Lilly, 1992).

Figure 1 shows the mean velocity profiles in wall units.

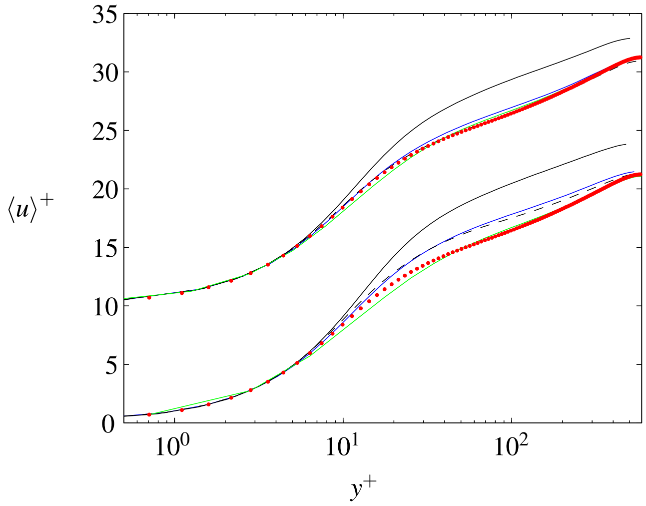

Figure 1. Mean velocity profiles in wall units. Different resolutions are shifted in the ordinate direction for the clarity of the plot and the upper curves are for a higher resolution. Red dots: DNS; Blue solid line: EASSM/SAT; Green solid line: EASSM/PS; Black solid line: DSM; Black dashed lines: NM. For the list of acronyms, see the caption of table 1.

Note that different resolutions are separated by a shift in the ordinate direction. The figure shows that the DSM largely under-predicts the wall shear stress, therefore the mean velocity profile is over-predicted by this model at both resolutions. At the coarser resolution (case 1), LES without a SGS model (NM) also over-predicts the mean velocity profile showing that the discretization errors are larger than the true SGS dissipation. This is due to the low-order discretization scheme used in the code. The over-prediction of NM reduces with increasing resolution, which is due to the decrease in the discretization error with increasing resolution. The EASSM predictions are close to the NM results, which shows that the model contribution to the SGS dissipation is low. The EASSM/SAT prediction approaches that of the DNS with increasing resolution. The EASSM/PS results are accurate at both resolutions and is close to the EASSM/SAT at the finer resolution.

The streamwise Reynolds stresses, R_{uu}, at the two resolutions are shown in figures 2(a-b). The EASSM predictions represent the resolved plus modeled Reynolds stresses, while for the DSM the modeled part is not available. At the coarser resolution, all models (except the EASSM/PS) and the NM using the *code_Saturne* largely over-predict the near-wall peak of R_{uu}, see figure 2(a-b). The EASSM/SAT prediction is slightly better than those of the DSM and the NM. All model predictions approach that of the DNS with increasing resolution and the EASSM/SAT prediction is more accurate than the DSM and the NM. The EASSM/PS predictions agree well with the DNS and are better than those of the EASSM/SAT. It is worth mentioning that the results from the pseudo-spectral method and the *code_Saturne* cannot be compared quantitatively at the same grid resolutions, since they have different numerical accuracy, which affects the overal performance of the EASSM and are presented here for more information.

The wall-normal Reynolds stresses, R_{vv}, are shown in figures 3(a-b). At the coarser resolution, see figure 3 (a), all the models and the NM using the *code_Saturne* under-predict R_{vv}. The DSM prediction at this resolution shows a much larger under-prediction than the EASSM/SAT and the NM and the EASSM/SAT and the NM predictions are similar. Model predictions approach that of the DNS with increasing resolution. The EASSM/SAT prediction is slightly better than the other model predictions, see figure 3(b). The EASSM/PS prediction is close to that of the DNS at both resolutions. The spanwise Reynolds stresses, R_{ww}, at the two resolutions are shown in figures 4(a-b). The performances of the models are similar to that for the R_{vv} predictions.

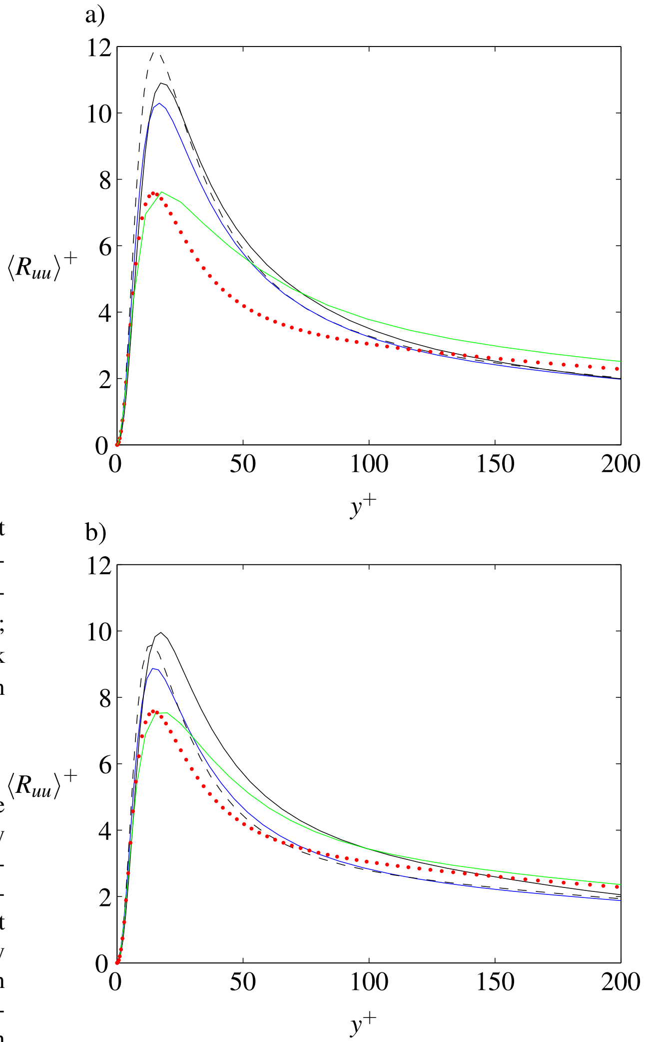

Figure 2. Streamwise Reynolds stresses in wall units; a) coarse resolution (Case 1) b) Fine resolution (case 2). Refer to the caption of figure 1 for the legends.

Conclusions and outlook

We performed LES of channel flow at $Re_\tau = 590$ using the explicit algebraic subgrid scale model (EASSM) (Marstorp *et al.*, 2009). *Code_saturne* which is a finite volume unstructured Navier–Stokes solver suitable for LES of complex geometries, was employed in the simulations. LES results showed that the EASSM predictions agree reasonably well with the DNS data at fine resolutions. Comparison of the results with the dynamic Smagorinsky model (DSM) for the prediction of the wall shear stress showed that the DSM largely under-predicts the wall shear even at the fine resolution, while the EASSM prediction was close to the DNS results. It was found that when the

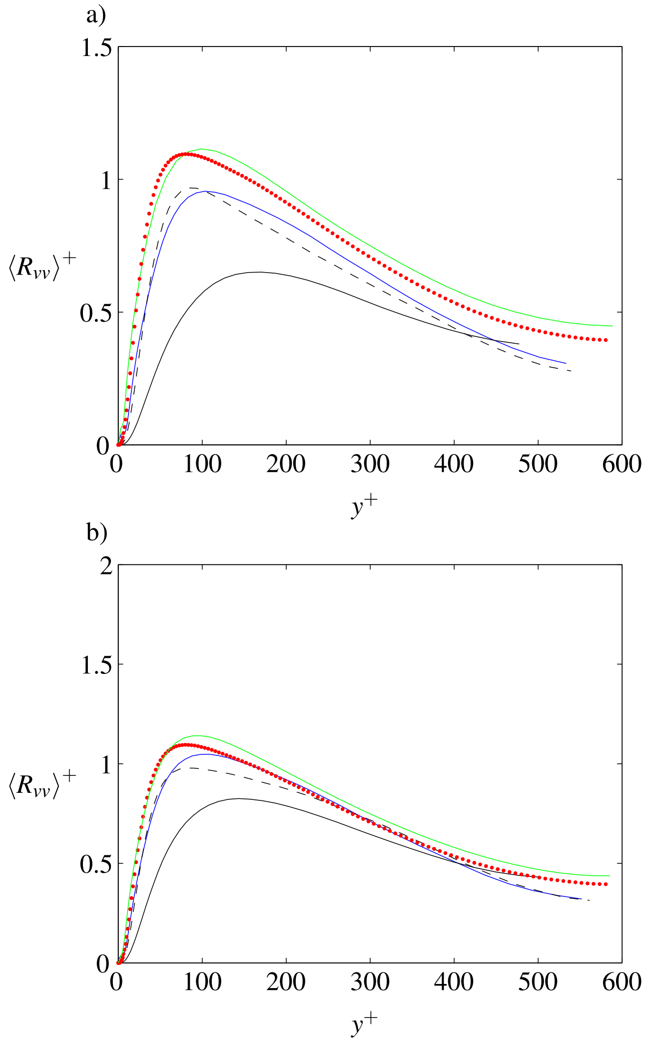

Figure 3. Cross stream Reynolds stresses in wall units; a) coarse resolution (Case 1) b) Fine resolution (case 2). Refer to the caption of figure 1 for the legends.

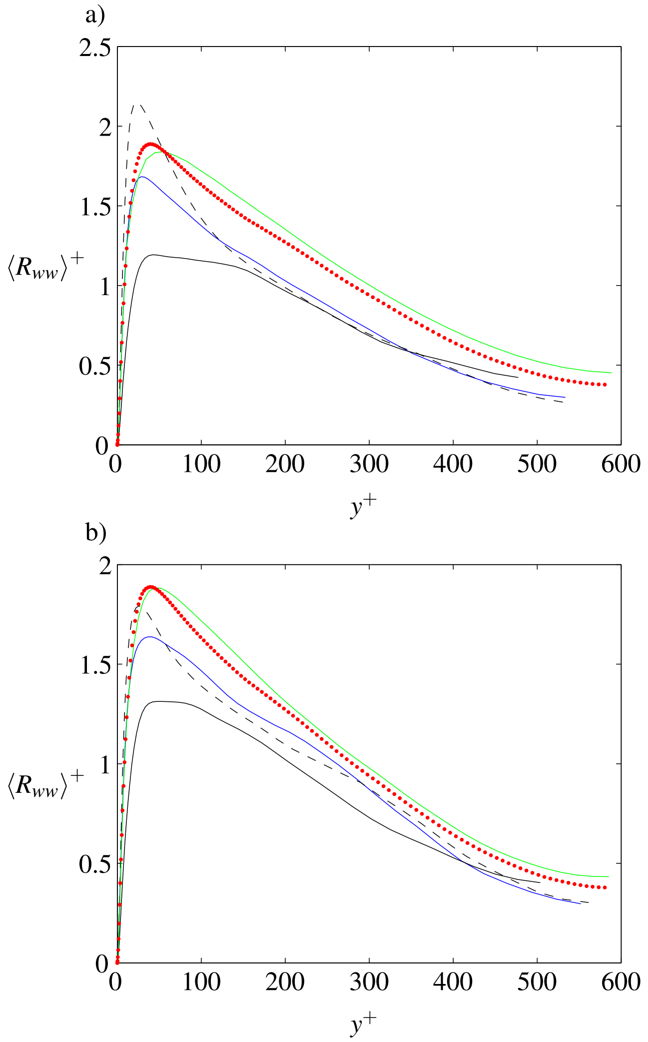

Figure 4. Spanwise Reynolds stresses in wall units; a) coarse resolution (Case 1) b) Fine resolution (case 2). Refer to the caption of figure 1 for the legends.

EASSM was used with the *code_Saturne*, which has inherent numerical dissipation, its prediction of the SGS dissipation was smaller than when it was used with a pseudo-spectral solver without numerical dissipation, which is a useful characteristic of the model.

In the next step towards the application of the EASSM to more complex flows, we are performing LES of periodic hill (Frohlich *et al.*, 2005) using the EASSM and the *Code_saturne*. The bulk Reynolds number in this flow based on the hill height is similar to the bulk Reynolds number considered in the channel flow simulations. The results will be presented at the conference.

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