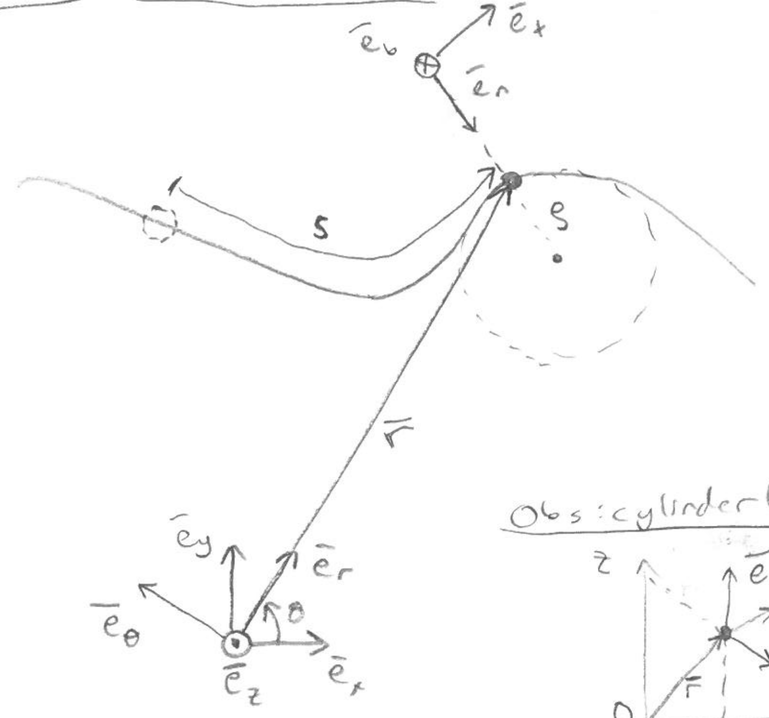
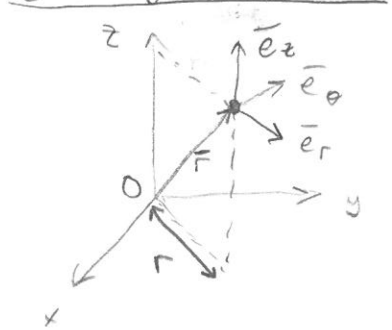
Repetition: Kraftekvationen



Obs: cylinderkoord.

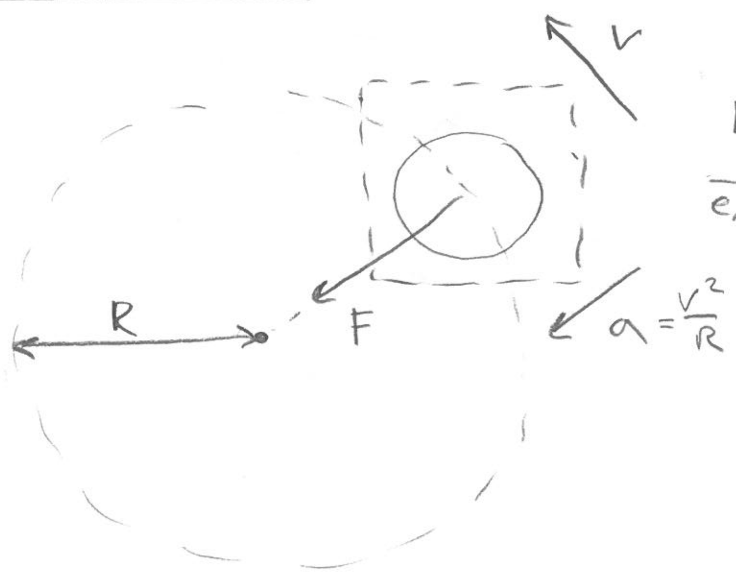


r används som avstånd från z-axeln, dvs. $|\bar{r}| \neq r$!

$$\boxed{\bar{F} = m\bar{a}}$$

Kartesiska	Naturliga	Cylinder
$m\ddot{x} = F_x$	$m\ddot{s} = F_t$	$m(\ddot{r} - r\dot{\theta}^2) = F_r$
$m\ddot{y} = F_y$	$m\dfrac{\dot{s}^2}{\varrho} = F_n$	$m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = F_\theta$
$m\ddot{z} = F_z$	$0 = F_b$	$m\ddot{z} = F_z$

Inertialsystem: Kinematik

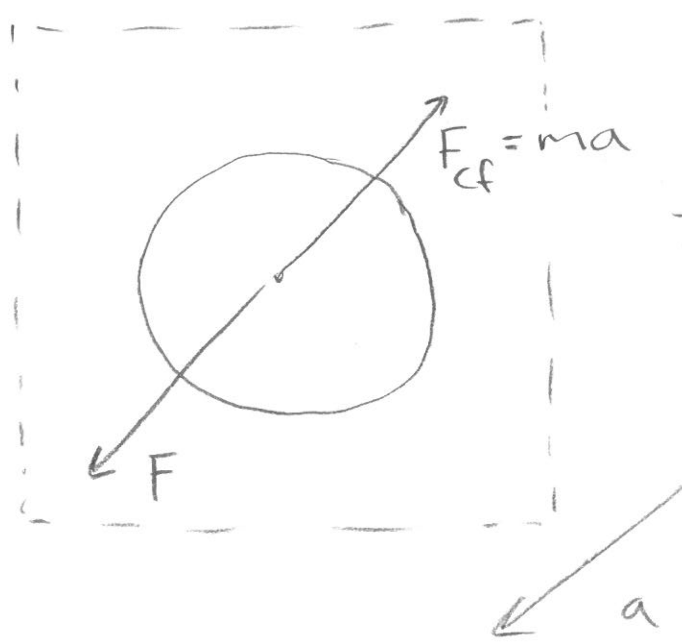


$a = \dfrac{v^2}{R}$

Kraftekv.

$\bar{e}_n: m\dfrac{v^2}{R} = F$

Icke-inertialsystem: Jämvikt



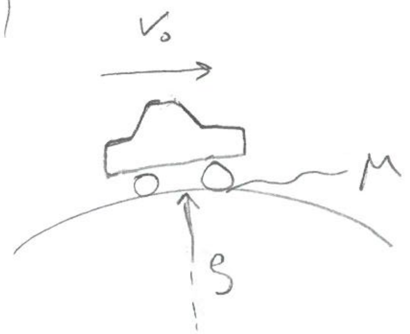
$F_{cf} = ma$

Jämvikt

$\bar{e}_n: F - F_{cf} = 0$

Uppgift 8.33

(I)

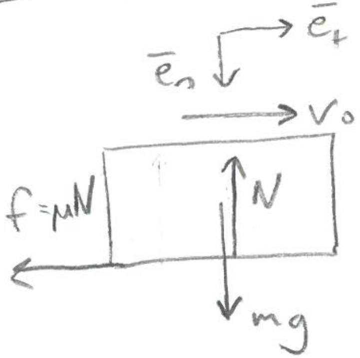


(II)

Givet
- $\dot{s}_I = \dot{s}_{II} = v_0$
- μ, ρ
- Tvärbromsning med låsta hjul

Sökt
- $\dot{v}_I = \ddot{s}_I$
- $\dot{v}_{II} = \ddot{s}_{II}$

Friläggning (I)



Kraftekvationen

$\bar{e}_t : m\ddot{s}_I = m\dot{v}_I = -\mu N$ (1)

$\bar{e}_n : m\frac{\dot{s}_{II}^2}{\rho} = m\frac{v_0^2}{\rho} = mg - N$

$\Rightarrow N = m\left(g - \frac{v_0^2}{\rho}\right)$ (2)

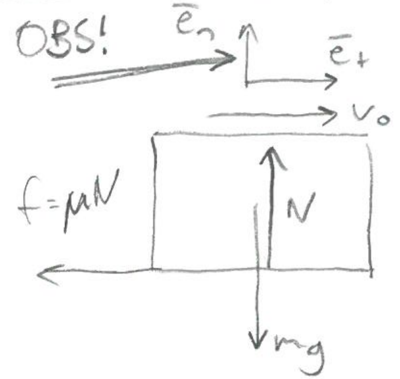
(obs $N=0$ om vi kör för fort!
$\Rightarrow$ Vi hoppar över krönet!)

(1) & (2) $\Rightarrow$

$m\dot{v}_I = -\mu m\left(g - \frac{v_0^2}{\rho}\right)$

$\Rightarrow \dot{v}_I = -\mu\left(g - \frac{v_0^2}{\rho}\right)$

Friläggning (II)



Kraftekvationen

$\bar{e}_t : m\dot{v}_{II} = -\mu N$ (3)

$\bar{e}_n : m\frac{v_0^2}{\rho} = N - mg$ OBS!

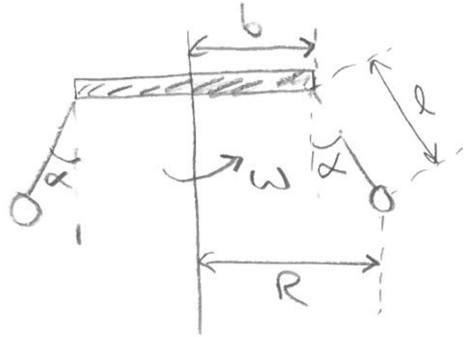
$N = m\left(g + \frac{v_0^2}{\rho}\right)$ (4)

(3) & (4) $\Rightarrow$

$\dot{v}_{II} = -\mu\left(g + \frac{v_0^2}{\rho}\right)$

Notera: $|\dot{v}_{II}| > |\dot{v}_I|$, dvs vi bromsar kraftigare i fall II då den ökade normalkraften ger högre friktionskraft!

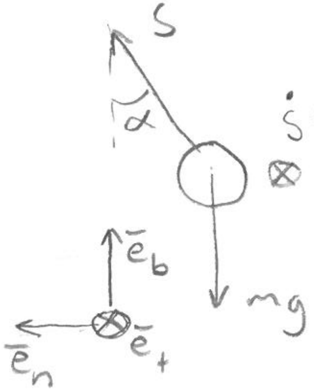
Uppgift 8.4c



Givet
- b, l, α

Sökt
- ω

Frilägging



$\dot{s} = \omega R$ ⊗

Kraftekv.

$\bar{e}_n: m\frac{(\omega R)^2}{R} = S\sin\alpha$ (1)

$\bar{e}_b: 0 = S\cos\alpha - mg$

$\Rightarrow S = \frac{mg}{\cos\alpha}$ (2)

---

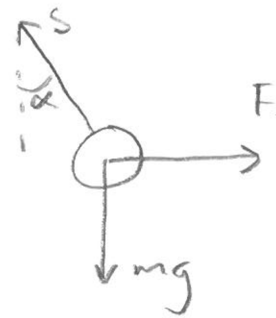
(1) & (2) $\Rightarrow$

$\cancel{m}\omega^2 R = \cancel{m}g\tan\alpha$

$$\Rightarrow \omega = \sqrt{\frac{g\tan\alpha}{R}} = [R = b + l\sin\alpha] =$$

$$= \underline{\sqrt{\frac{g\tan\alpha}{b + l\sin\alpha}}}$$

Som jämvikt i icke-inertialsystem



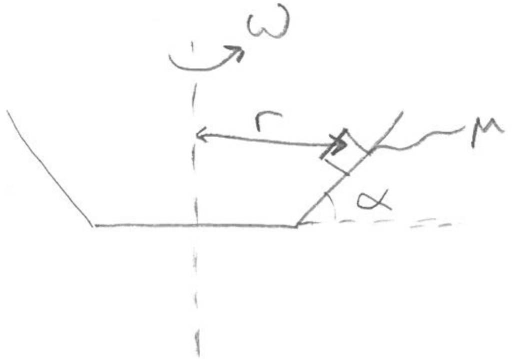
$F_{trög} = m\underbrace{\omega^2 R}_{\text{systemets acceleration}}$

Jämviktsekvationer

$$\left.\begin{aligned} \rightarrow&: m\omega^2 R - S\sin\alpha = 0 \\ \uparrow&: S\cos\alpha - mg = 0 \end{aligned}\right\}$$

Samma ekvationer som (1) & (2)

Uppgift 8.46



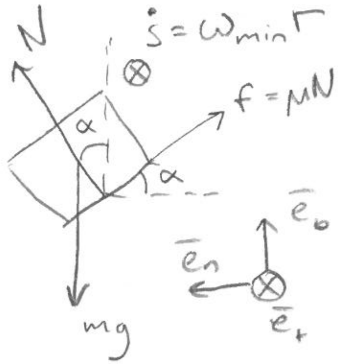
Givet

- α, r, μ

Sökt

- ω_{min} & ω_{max} för att förhindra glidning

Friläggning för ω_{min}



Kraftekv.

$\bar{e}_t : \; m\underbrace{\ddot{s}}_{=0} = f_t$

$\Rightarrow f_t = 0$ (friktionskraft har ej tangentiell komp.)

$\bar{e}_n : m\omega_{min}^2 r = N\sin\alpha - \mu N\cos\alpha$ ①

$\bar{e}_b : 0 = N\cos\alpha - mg + \mu N\sin\alpha$

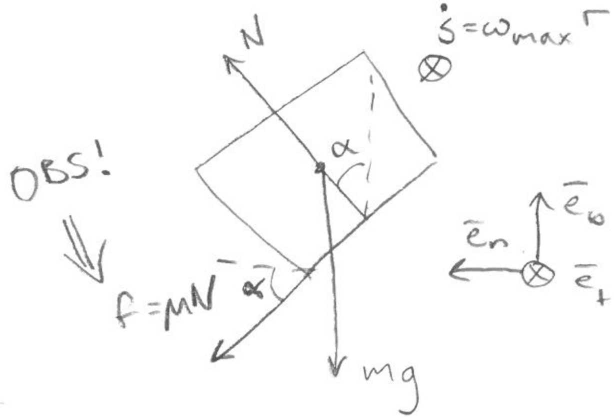
$\Rightarrow N = \dfrac{mg}{\cos\alpha + \mu\sin\alpha}$ ②

① & ② $\Rightarrow$

$$\cancel{m}\,\omega_{min}^2 r = \frac{\cancel{m}g}{\cos\alpha + \mu\sin\alpha}(\sin\alpha - \mu\cos\alpha)$$

$$\Rightarrow \omega_{min} = \sqrt{\frac{g}{r}\,\frac{\sin\alpha - \mu\cos\alpha}{\cos\alpha + \mu\sin\alpha}}$$

Friläggning för ω_{max}



Kraftekv

$\bar{e}_n : \; m\omega_{max}^2 r = N\sin\alpha + \mu N\cos\alpha$ ③ OBS!

$\bar{e}_b : \; 0 = N\cos\alpha - mg - \mu N\sin\alpha$ OBS!

$\Rightarrow N = \dfrac{mg}{\cos\alpha - \mu\sin\alpha}$ ④

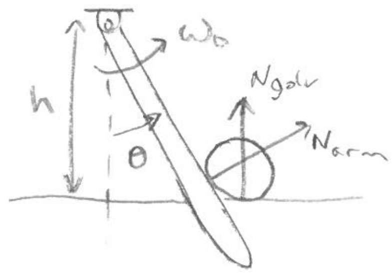
③ & ④ $\Rightarrow$

$$\cancel{m}\,\omega_{max}^2 r = \frac{\cancel{m}g}{\cos\alpha - \mu\sin\alpha}(\sin\alpha + \mu\cos\alpha)$$

$$\Rightarrow \omega_{max} = \sqrt{\frac{g}{r}\,\frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha - \mu\sin\alpha}}$$

För träning: Hur skulle det se ut om vi räknade i det icke-inertiella systemet?

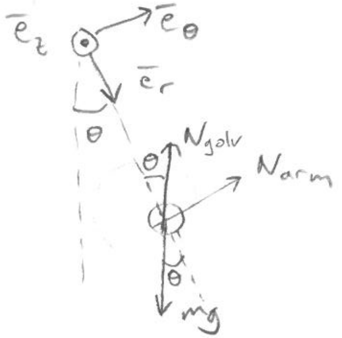
$\boxed{\text{Uppgift } 8.64}$



Givet
- m, ω_0

Sökt
- $N_{golv}(\theta)$

Frilägg. cylinder-koord.



Kraftekv.

$\bar{e}_r: m(\ddot{r} - r\dot{\theta}^2) = mg\cos\theta - N_{golv}\cos\theta$ (1)

$\left(\bar{e}_\theta: m(2\dot{r}\dot{\theta} + r\ddot{\theta}) = N_{arm} - mg\sin\theta + N_{golv}\sin\theta\right)$ (2)

Vi identifierar

$r = \frac{h}{\cos\theta} = h(\cos\theta)^{-1}$

$\dot{r} = -h(\cos\theta)^{-2}\cdot(-\sin\theta)\cdot\dot{\theta} = h\sin\theta(\cos\theta)^{-2}\dot{\theta}$ (3)

$\ddot{r} = h\cos\theta(\cos\theta)^{-2}\dot{\theta}^2 - 2h\sin\theta(\cos\theta)^{-3}\cdot(-\sin\theta)\cdot\dot{\theta}^2$ (4)
$+ h\sin\theta(\cos\theta)^{-2}\ddot{\theta}$

$\dot{\theta} = \omega_0$ (5)

$\ddot{\theta} = 0$ (6)

(1) & (3)→(6) ⇒

$$m\left(\cancel{\frac{h\omega_0^2}{\cos\theta}} + \frac{2h\omega_0^2\sin^2\theta}{\cos^3\theta} - \cancel{\frac{h\omega_0^2}{\cos\theta}}\right) = mg\cos\theta - N_{golv}\cos\theta$$

$$2mh\omega_0^2\frac{\sin^2\theta}{\cos^4\theta} = mg - N_{golv}$$

$$\Rightarrow \underline{\underline{N_{golv} = mg - 2mh\omega_0^2\frac{\tan^2\theta}{\cos^2\theta}}}$$

Checka in tangentialkomp. av kraftekv.

$$m\ddot{s} = F_t \xrightarrow{\text{tricket}} m\dot{s}\frac{d\dot{s}}{ds} = F_t$$

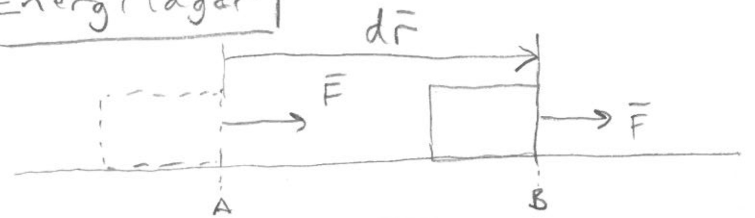
$$\Rightarrow m\dot{s}\,d\dot{s} = F_t\,ds$$

Integrering från $A \to B$ ger $(F_t = \text{konstant})$

$$m\int_A^B \dot{s}\,d\dot{s} = F_t \int_A^B ds$$

$$\underbrace{\frac{mv_B^2}{2} - \frac{mv_A^2}{2}}_{\text{Vi kallar detta kinetisk energi}} = \underbrace{F_t \cdot \overbrace{s_{A\to B}}^{\text{sträckan från } A\to B}}_{\text{Vi kallar detta för kraftens arbete}}$$

Energilager



$$\boxed{U_{A-B} = T_B - T_A}$$ Lag om kinetisk energi (LKE)

kinetisk energi

$$T = \frac{mv^2}{2}$$

Arbete

$$U_{A-B} = \int_A^B \bar{F} \cdot d\bar{r}$$

Obs! Om $\bar{F} \perp d\bar{r} \Rightarrow U_{A-B} = 0$

Om $\bar{F} \parallel d\bar{r}$ & $\bar{F} = \text{konstant}$

$$\Rightarrow U_{A-B} = F\int_A^B dr = F \cdot s_{A\to B}$$

"kraften gånger sträckan"

Spec.fall. konservativa krafter (ej friktion t.ex.)

Potentialfunktion

$$V = -\int^{\bar{r}} \bar{F} \cdot d\bar{r}$$

Arbetet blir då

$$U_{A-B} = V_A - V_B$$ (oberoende av väg från A till B!)

kombinerat med LKE

$$\Rightarrow T_A + V_A = T_B + V_B$$

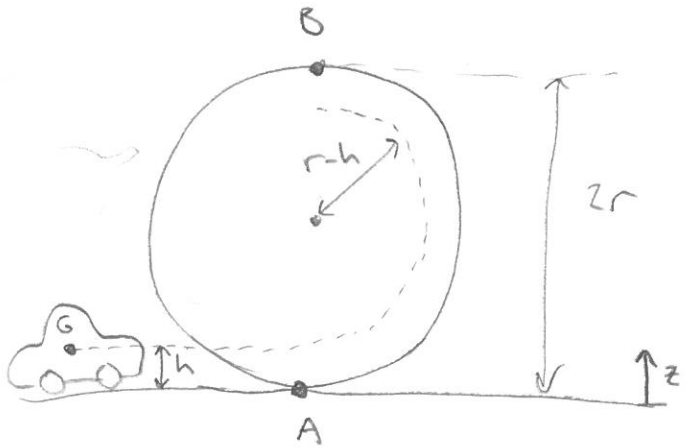
$$\Rightarrow \boxed{T + V = \text{konstant}}$$

Potentialfunktioner att komma ihåg

Tyngdkraft: $V_{mg} = mg\underbrace{z}_{\text{Höjdkoordinat}} + \underbrace{C}_{\text{Godt. konstant}}$

Fjäderkraft: $V_{fj} = \frac{\overbrace{k}^{\text{Fjäderkonstant}}}{2}(\underbrace{l}_{\text{Fjäderns längd}} - \underbrace{l_0}_{\text{Fjäderns nat. längd}})^2 + C$

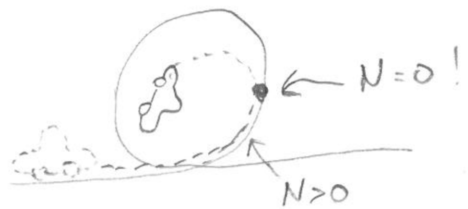
Uppgift 4.2



Givet
- r, h
- fri rullning

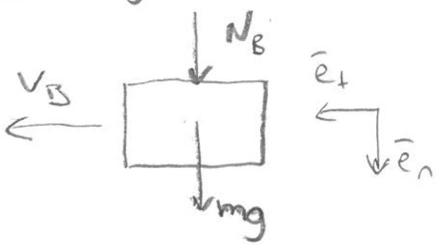
Sökt
- $v_{A,min}$ för att kunna fullborda ett varv

Vad händer om farten är för låg?
Normalkraften från banan blir noll och bilen tappar kontakt



$\Rightarrow$ Vid kritiska läget : $\boxed{\begin{array}{l} v_B = v_{B,min} \\ N_B = 0 \end{array}}$

Frilägging punkt B



kraftekv.

$$\bar{e}_n : m\frac{v_{B,min}^2}{(r-h)} = mg + \cancel{N_B}^{0}$$

$$\Rightarrow v_{B,min}^2 = g(r-h) \quad ①$$

Vad är $v_{A,min}$?

$$U_{A-B} = T_{B,min} - T_{A,min} = \frac{m v_{B,min}^2}{2} - \frac{m v_{A,min}^2}{2} \quad ②$$

Endast tyngdkraften som utför arbete

$$U_{A-B} = \int_A^B \bar{F}_{mg} \cdot d\bar{r} = \int_h^{2(r-h)+h} -mg\bar{e}_z \cdot dz\bar{e}_z =$$

$$= \int_h^{2(r-h)+h} -mg\,dz = \left[-mgz\right]_h^{2(r-h)+h}$$

$$= -2mg(r-h) \cancel{-mgh + mgh} = -2mg(r-h) \quad ③$$

①, ② & ③ $\Rightarrow$

$$\frac{\cancel{m}g(r-h)}{\cancel{2}} - \frac{\cancel{m}v_{A,min}^2}{\cancel{2}} = -\cancel{2}^{4}\cancel{m}g(r-h)$$

$$v_{A,min}^2 = g(r-h) + 4g(r-h) = 5g(r-h)$$

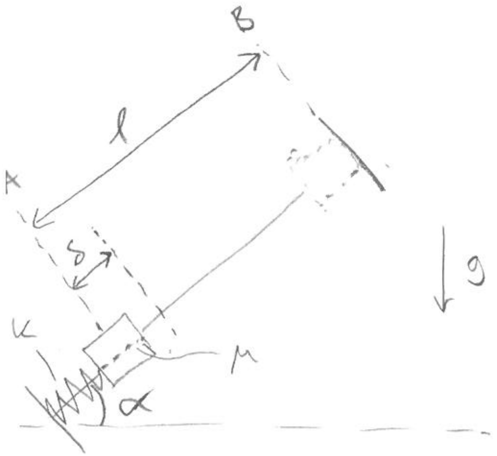
$$v_{A,min} = \sqrt{5g(r-h)}$$

Notera att vi kunde använda pot. funktion

$$U_{A-B} = V_A - V_B = \left[V = mgz + C_1\right] = mg(z_A - z_B) =$$

$$= -mg \cdot 2(r-h)$$

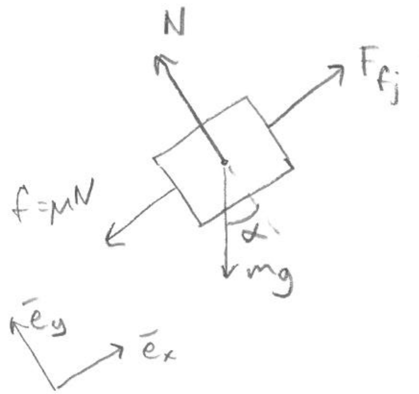
Uppgift 7.7



Givet
- m, α, μ, l, k
- $v_B = 0$, $v_A = 0$

Sökt
- $\delta = \Delta l$

Frilägggning i början (under kontakt m. fjäder)



Krafteker.

$x: \; m\ddot{x} = F_{fj} - \mu N - mg\sin\alpha$

$y: \; 0 = N - mg\cos\alpha$

$\Rightarrow N = mg\cos\alpha$ (1)

Lag om arbete

$U_{A-B} = \cancel{T_B}^{0} - \cancel{T_A}^{0} = 0$ (2)

$U_{A-B} = U_{A-B,mg} + U_{A-B,f} + U_{A-B,fj}$ (3)

Tyngdkraftens arbete

$$U_{A-B,mg} = \int_A^B m\bar{g} \cdot d\bar{r} = \int_0^l -mg\sin\alpha \, dx = $$

$$= -mgl\sin\alpha \quad (4)$$

Friktionens arbete

$$U_{A-B,f} = \int_A^B -\mu N \bar{e}_x \cdot d\bar{r} = \int_0^l -\mu mg\cos\alpha \, dx =$$

$$= -\mu mgl\cos\alpha \quad (5)$$

Fjäderns arbete

$$U_{A-B,fj} = \int_A^B \bar{F}_{fj} \, d\bar{r} = V_A - V_B = \begin{bmatrix} V_B = 0 \\ V_A = \frac{1}{2}k\delta^2 \end{bmatrix} =$$

$$= \frac{1}{2}k\delta^2 \quad (6)$$

(2)-(6) $\Rightarrow$

$$-mgl\sin\alpha - \mu mgl\cos\alpha + \frac{1}{2}k\delta^2 = 0$$

$$\delta = \sqrt{\frac{2mgl}{k}(\sin\alpha + \mu\cos\alpha)}$$