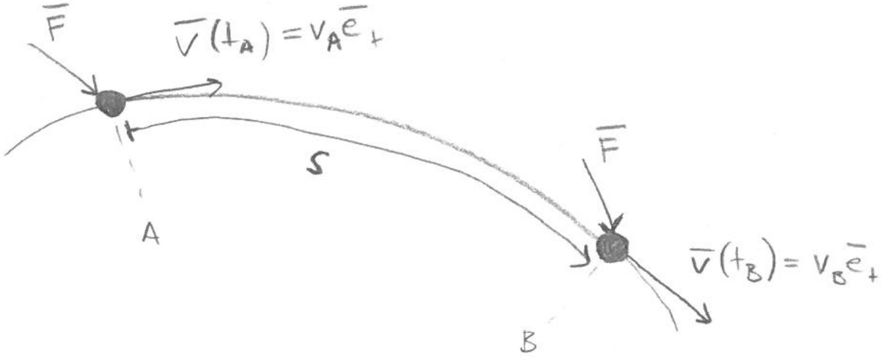
Repetition



Kraftekvationen

$$\bar{F} = m\frac{d\bar{v}}{dt} = m\bar{v}\cdot\frac{d\bar{v}}{d\bar{r}}$$

$$\Rightarrow \bar{F}\cdot d\bar{r} = m\bar{v}\cdot d\bar{v} \qquad \left(\text{Nat. koord. } d\bar{r} = ds\bar{e}_t,\ \bar{v} = \dot{s}\bar{e}_t\right)$$

$$F_t\, ds = m\dot{s}\, d\dot{s}$$

Vi integrerar

$$\underbrace{\int_A^B F_t\, ds}_{U_{A-B}} = \underbrace{\frac{mv_B^2}{2}}_{T_B} - \underbrace{\frac{mv_A^2}{2}}_{T_A}$$

"En kraft kan utföra ett arbete som ändrar ett föremåls rörelseenergi"

Arbete

$$U_{A-B} = \int_A^B \bar{F}\cdot d\bar{r}$$

Om konservativa krafter

$$U_{A-B} = V_A - V_B \qquad \left(V(\bar{r}) = -\int_{\bar{r}_0}^{\bar{r}} \bar{F}\cdot d\bar{r}\right)$$

godtycklig punkt

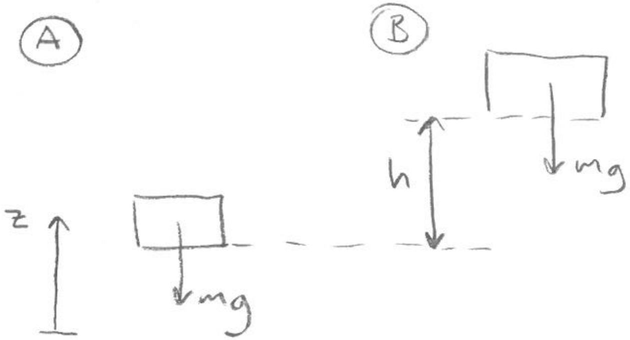
$\bar{r}_0$ ← referenspunkt!

Potentiell energi

→ Potentiell energi är relativt!
(Vi kan bara säga vad den är i förhållande till något annat!)

→ Vi kan själva välja referensnivå ("nollnivå")

Ex:



Potentialfunktion

$$V = mgz + C$$

Vi kan välja $C = -mgz_A \Rightarrow \boxed{V_A = 0 \quad V_B = mgh}$

"Lådan har potentiell energi $V = mgh$ i läge B relativt läge A"

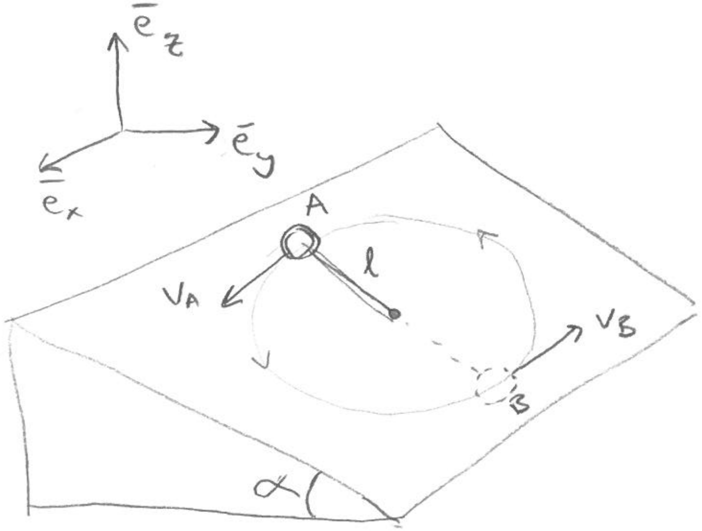
Samma sak med fjäder

Pot. fkn.

$$V = \frac{k}{2}(l - l_0)^2 + C$$

Med $C = -\frac{k}{2}(l_0 - l_0)^2 = 0 \Rightarrow \boxed{V_A = 0 \quad V_B = \frac{k}{2}(l - l_0)^2}$

Uppgift 9.18



Givet

- $m, \ell, \alpha, v_A, \mu$

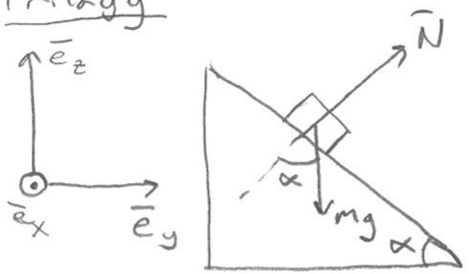
Sökt

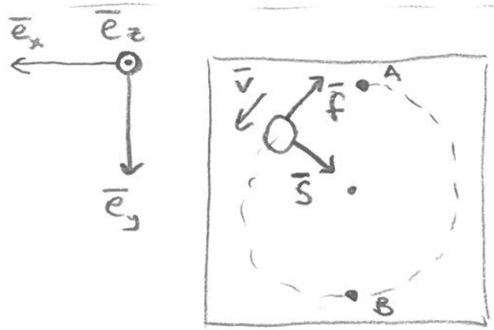
- v_B

Energiekvationen

$$U_{A-B} = T_B - T_A = \frac{mv_B^2}{2} - \frac{mv_A^2}{2} \quad (1)$$

Frilägg

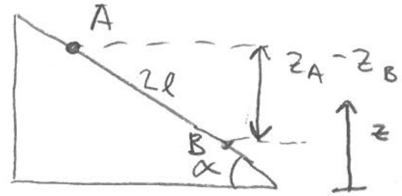




$$\left.\begin{aligned}\bar{S}\cdot d\bar{r} &= 0\\ \bar{N}\cdot d\bar{r} &= 0\end{aligned}\right\}$$ Uträttar inget arbete! (vinkelräta mot rörelsen)

Arbetet $\Rightarrow U_{A-B} = U_{A-B,mg} + U_{A-B,f} \quad (2)$

Tyngdkraftens arbete (konservativ)



$$U_{A-B,mg} = V_A - V_B = [V = mgz + C] =$$
$$= mg(z_A - z_B) =$$
$$= mg \cdot 2\ell \sin\alpha \quad (3)$$

Friktionskraftens arbete

$$U_{A-B,f} = \int_A^B \bar{f}\cdot d\bar{r} = [\bar{f}\cdot d\bar{r} = -\mu N dr] = -\mu N \int_A^B dr =$$
$$= [\text{kraft j.v. i } \bar{N}\text{-riktn} \Rightarrow N = mg\cos\alpha] =$$
$$= -\mu mg\cos\alpha \underbrace{\int_A^B dr}_{=\pi\ell} = -\mu mg\pi\ell\cos\alpha \quad (4)$$

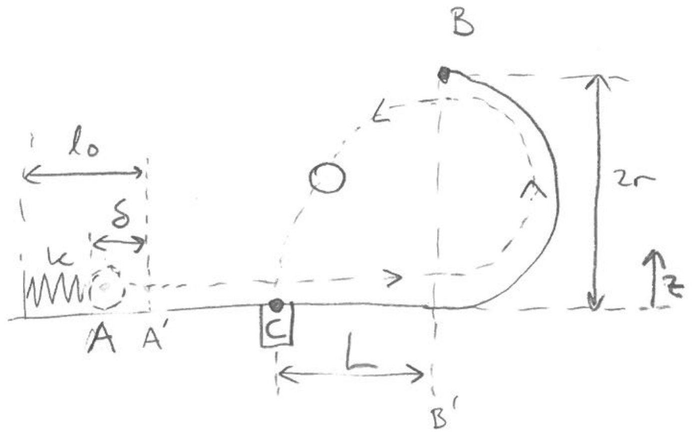
(1) - (4) $\Rightarrow$

$$\frac{mv_B^2}{2} - \frac{mv_A^2}{2} = 2mg\ell\sin\alpha - \mu\pi mg\ell\cos\alpha$$

$$v_B^2 = v_A^2 + 4g\ell\sin\alpha - 2\mu\pi g\ell\cos\alpha =$$
$$= v_A^2 + 2g\ell\cos\alpha(2\tan\alpha - \mu\pi)$$

$$\Rightarrow \quad \underline{\underline{v_B = \sqrt{v_A^2 + 2g\ell\cos\alpha(2\tan\alpha - \mu\pi)}}}$$

Uppgift 9.24



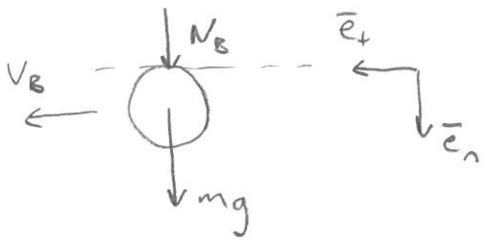
Givet

- V_B givet av $N_B = 0$
- m, g, r, k

Sökt

- δ
- L

Vad är v_B? Frilägg partikel i B



kraftekv.

$\bar{e}_n: \frac{m v_B^2}{r} = mg + \cancel{N_B}^{\,0}$

$\Rightarrow v_B = \sqrt{gr}$ (1)

Vad är δ? LKE

$U_{A-B} = \frac{m v_B^2}{2} - \cancel{\frac{m v_A^2}{2}}^{\,0} = [(1)] = \frac{mgr}{2}$ (2)

$U_{A-B} = U_{A-B,f_j} + U_{A-B,mg} = U_{A-A',f_j} + U_{B'-B,mg}$ (3)

Fjäderns arbete (konservativ)

$U_{A-A',f_j} = V_A - V_{A'} = \left[V = \frac{k}{2}(l-l_0)^2 + C_1\right]$

$\Rightarrow U_{A-A',f_j} = \frac{k}{2}\delta^2$ (4)

Tyngdkraftens arbete (konservativ) kulan antas liten

$U_{B'-B,mg} = V_{B'} - V_B = \left[V = mgz + C_1\right] =$

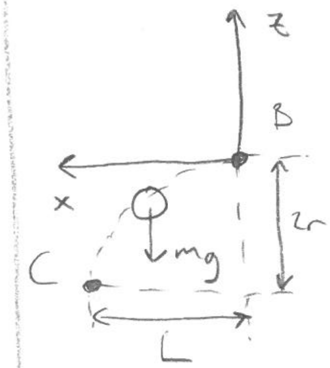
$= mg(z_A - z_B) = -mg \cdot 2r$ (5)

(2) - (5) $\Rightarrow$

$\frac{mgr}{2} = \frac{k}{2}\delta^2 - 2mgr$

$\delta^2 = (4mgr + mgr)\frac{1}{k} \quad \Rightarrow \quad \underline{\underline{\delta = \sqrt{\frac{5mgr}{k}}}}$

B → C är en kastbana!



$x = V_B t \quad \Rightarrow \quad t = \frac{x}{V_B}$ (6)

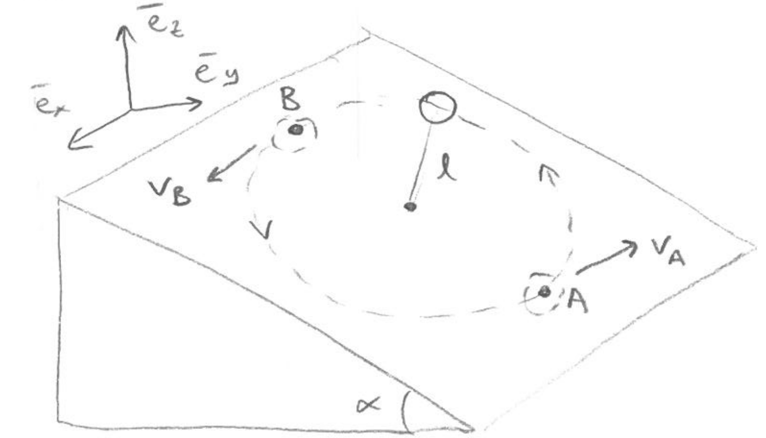
$z = -\frac{gt^2}{2} = [(6)] = -\frac{g}{2}\frac{x^2}{V_B^2}$ (7)

$\underline{z_C = -2r \text{ då } x_C = L}$

(7) $\Rightarrow -2r = -\frac{g}{2}\frac{l^2}{V_B^2} \quad \Rightarrow L^2 = \frac{4 v_B^2 r}{g} = \left[v_B^2 = gr\right] = \frac{4gr \cdot r}{g}$

$\Rightarrow \underline{\underline{L = 2r}}$

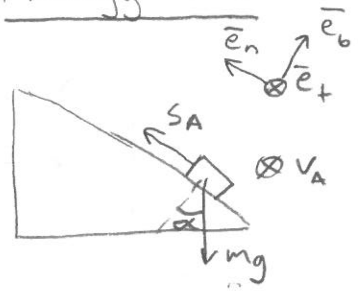
Uppgift 9.35



Givet
- ingen friktion
- m, g, ℓ, α
- $S_A = 2S_B$

Sökt
- S_A, S_B
- v_A, v_B

Frilägg: A

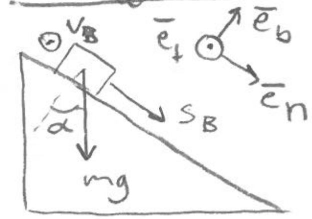


Kraftekv.

$$\bar{e}_n: \frac{mv_A^2}{\ell} = S_A - mg\sin\alpha$$

$$\Rightarrow mv_A^2 = \ell(S_A - mg\sin\alpha) \quad (1)$$

Frilägg: B



Kraftekv.

$$\bar{e}_n: \frac{mv_B^2}{\ell} = S_B + mg\sin\alpha$$

$$\Rightarrow mv_B^2 = \ell(S_B + mg\sin\alpha) \quad (2)$$

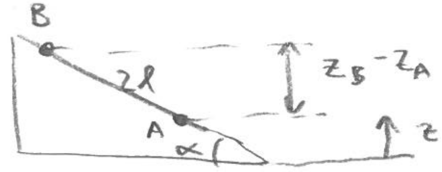
Energiekv. LKE

$$U_{A-B} = \frac{mv_B^2}{2} - \frac{mv_A^2}{2} \quad (3)$$

Arbetet

$$U_{A-B} = U_{A-B,mg} \quad (4)$$

Tyngdkraftens arbete



$$U_{A-B,mg} = V_A - V_B = \left[V = mgz + C_1\right] =$$

$$= mg(z_A - z_B) = -mg \cdot 2\ell\sin\alpha \quad (5)$$

(1) - (5) $\Rightarrow$

$$-2mg\ell\sin\alpha = \frac{\ell}{2}(S_B + mg\sin\alpha) - \frac{\ell}{2}(S_A - mg\sin\alpha) =$$

$$= \left[S_A = 2S_B\right] = \frac{S_B}{2} - S_B + mg\sin\alpha$$

$$\Rightarrow S_B = 4mg\sin\alpha + 2mg\sin\alpha = \underline{\underline{6mg\sin\alpha}} \quad (6)$$

$$S_A = 2S_B = \underline{\underline{12mg\sin\alpha}} \quad (7)$$

(1) & (7) $\Rightarrow$

$$v_A^2 = \frac{\ell}{m}(12mg\sin\alpha - mg\sin\alpha)$$

$$\Rightarrow \underline{\underline{v_A = \sqrt{11g\ell\sin\alpha}}}$$

(2) & (6) $\Rightarrow v_B^2 = \frac{\ell}{m}(6mg\sin\alpha + mg\sin\alpha)$

$$\Rightarrow \underline{\underline{v_B = \sqrt{7g\ell\sin\alpha}}}$$

Momentekvationen

($\bar{p} = m\bar{v}$ = rörelsemängd)

Kraftekv.

$$\bar{F} = m\bar{a} = \frac{d}{dt}(m\bar{v}) = \frac{d\bar{p}}{dt}$$

Momentet runt O

$$\bar{M}_O = \bar{r} \times \bar{F} = \bar{r} \times \frac{d\bar{p}}{dt} = \left[\frac{d\bar{r}}{dt} \times \bar{p} = \bar{0}\right] =$$

$$= \frac{d}{dt}(\bar{r} \times \bar{p})$$

Vi definierar rörelsemängdsmoment runt O

$$\bar{H}_O = \bar{r} \times m\bar{v} = \bar{r} \times \bar{p}$$

$$\Rightarrow \boxed{\bar{M}_O = \dot{\bar{H}}_O}$$

Translation

Yttre kraft ⇒ acceleration ⇒ $\bar{p}$ förändras

Ingen yttre kraft ⇒ $\bar{p}_{före} = \bar{p}_{efter}$

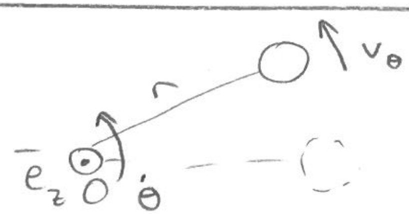
Rotation

Yttre moment ⇒ vinkelacceleration ⇒ $\bar{H}_O$ förändras

Inget yttre moment ⇒ $\bar{H}_O^{före} = \bar{H}_O^{efter}$

Oftast sker rotation runt en axel (z)

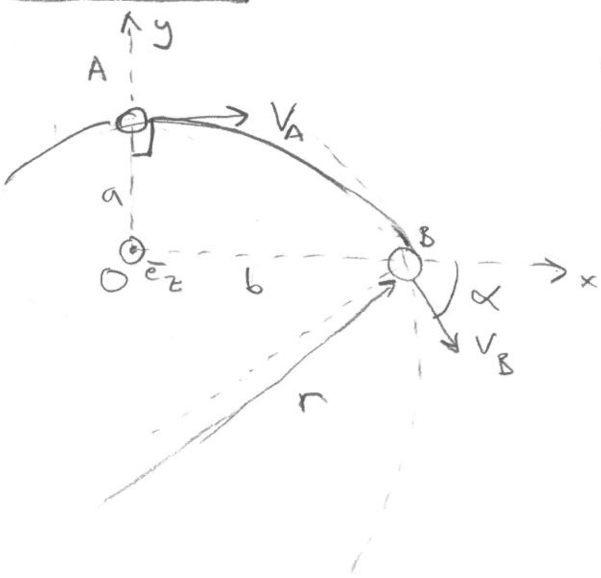
$$M_z = \dot{H}_z$$



$$\bar{H}_O = r\bar{e}_r \times mv_0\bar{e}_\theta = mv_0 r\bar{e}_z$$

$$\Rightarrow M_z = \frac{d}{dt}(mv_0 r)$$

Uppgift 10.7



__Givet__

- $V_A = V$
- $\rho_A = a$, $\rho_B = r$
- a, b, α, m, r

__Sökt__

- $\dot{H}_{oz}^{(A)}$ & $\dot{H}_{oz}^{(B)}$

__Rörelsemängdsmoment__ runt O

$$\bar{H}_o^{(A)} = \bar{r}_A \times m\bar{v}_A$$

$$\bar{H}_o^{(B)} = \bar{r}_B \times m\bar{v}_B$$

__Tidsderivera__

$$\dot{\bar{H}}_o^{(A)} = \cancel{\bar{v}_A \times m\bar{v}_A}^{\,0} + \bar{r}_A \times m\bar{a}_A = \bar{M}_o^{(A)} \quad (1)$$

$$\dot{\bar{H}}_o^{(B)} = \bar{r}_B \times m\bar{a}_B = \bar{M}_o^{(B)} \quad (2)$$

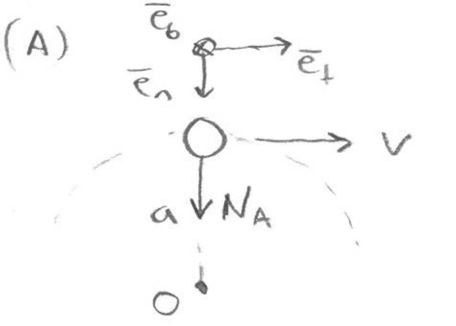
__Endast normalkraft på partikeln__

$$\Rightarrow U_{A-B} = \frac{mv_B^2}{2} - \frac{mv_A^2}{2} = 0$$

$$\Rightarrow v_B = v_A = v \quad \text{farten är konstant}$$

__Uttryck i naturliga koordinater__

(A)

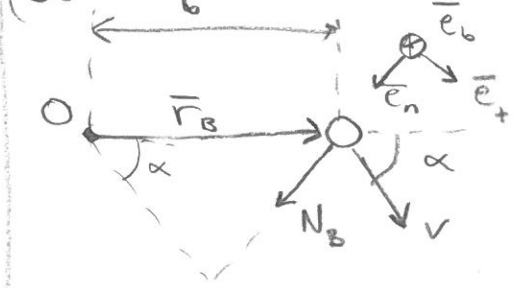


$$\left(\begin{aligned} \bar{r}_A &= -a\bar{e}_n \\ \bar{a}_A &= \cancel{\dot{v}_A}^{\,0}\bar{e}_t + \frac{v^2}{a}\bar{e}_n \end{aligned} \right)$$

$$\bar{M}_o^{(A)} = \bar{r}_A \times N_A\bar{e}_n = -a\bar{e}_n \times N_A\bar{e}_n = \bar{0}$$

$$(1) \Rightarrow \dot{H}_{oz}^{(A)} = 0$$

(B)



(Obs: $\bar{e}_b = -\bar{e}_z$)

$$\bar{r}_B = b\cos\alpha\,\bar{e}_t - b\sin\alpha\,\bar{e}_n$$

$$\bar{a}_A = \cancel{\dot{v}}^{\,0}\bar{e}_t + \frac{v^2}{r}\bar{e}_n$$

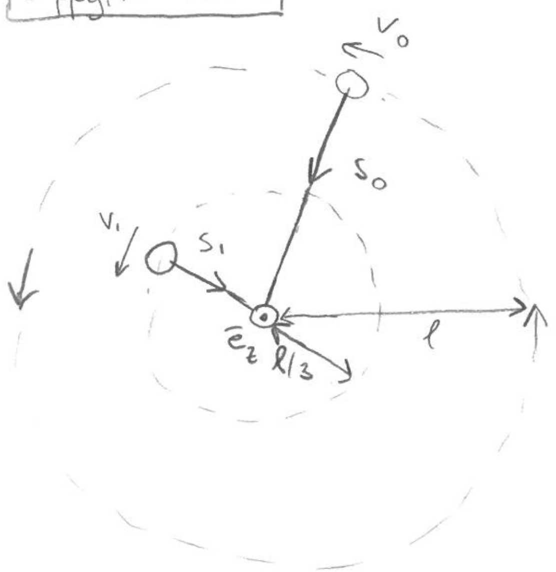
$$\bar{M}_o^{(B)} = \bar{r}_B \times N_B\bar{e}_n = b\cos\alpha\,\bar{e}_t \times N_B\bar{e}_n = bN_B\cos\alpha\,\bar{e}_b =$$

$$= [\bar{e}_b = -\bar{e}_z] = -bN_B\cos\alpha\,\bar{e}_z =$$

$$= \left[\text{kraftekv. } \bar{e}_n:\ N_B = \frac{mv^2}{r}\right] = -b\frac{mv^2}{r}\cos\alpha\,\bar{e}_z$$

$$(2) \Rightarrow \dot{H}_{oz}^{(B)} = M_{oz}^{(B)} = -b\frac{mv^2}{r}\cos\alpha$$

Uppgift 10.9



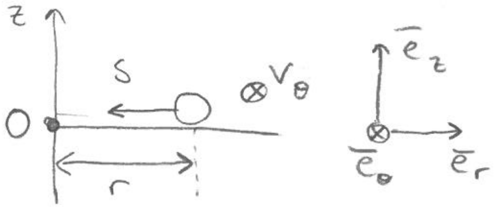
<u>Givet</u>

- v_0, m, ℓ

<u>Sökt</u>

- S_0, S_1
- U_{0-1}

<u>Rörelsemängdsmoment</u>



$\bar{H}_O = r\bar{e}_r \times mv_\theta \bar{e}_\theta$

$= mv_\theta r\, \bar{e}_z$

$H_z = mv_\theta r$

$\Rightarrow H_z^{(1)} = mv_0 \ell$

$H_z^{(2)} = mv_1 \cdot \frac{\ell}{3}$

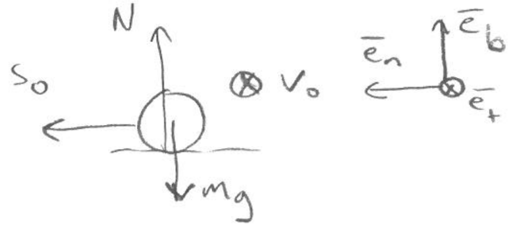
(3) $\Rightarrow mv_0 \ell = mv_1 \frac{\ell}{3} \Rightarrow v_1 = 3v_0$ (4)

<u>Momentekvationen</u> (vi har endast rotation runt z-axeln)

$\bar{M}_O = \dot{\bar{H}}_O$

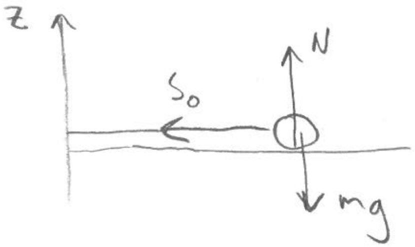
$\Rightarrow M_z = \dot{H}_z$ (1)

<u>Kraftekvationen</u> (Läge 0)



$\bar{e}_n: m\frac{v_0^2}{\ell} = S_0$

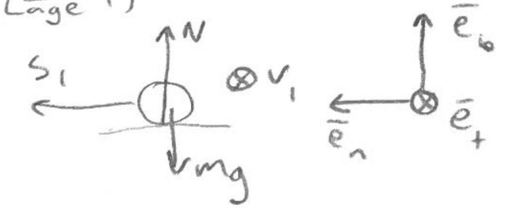
<u>Friläggn. av partikel</u> (Läge 0)



$M_z = 0$ (2)

Inga krafter ger moment runt z-axeln!

(Läge 1)



$\bar{e}_n: m\frac{v_1^2}{(\ell/3)} = S_1$ (5)

(1) & (2) $\Rightarrow$

$\dot{H}_z = 0 \Rightarrow H_z^{(0)} = H_z^{(1)}$ (3)

(4) & (5) $\Rightarrow$

$S_1 = \frac{m(3v_0)^2}{\ell/3} = 3 \cdot 9 \frac{mv_0^2}{\ell} = 27\frac{mv_0^2}{\ell}$

$\boxed{\text{Uppgift 10.9}}$ forts.

Arbetet $0 \to 1$

$$U_{0-1} = \frac{mv_1^2}{2} - \frac{mv_0^2}{2} = [v_1 = 3v_0] =$$

$$= 9\frac{mv_0^2}{2} - \frac{mv_0^2}{2} = \underline{\underline{4mv_0^2}}$$