Nonlinear interaction of streaks and Tollmien-Schlichting waves in boundary layer flows

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Introduction

The linear growth of Tollmien-Schlichting Waves is reduced in the presence of Streaks in the Blasius boundary layer.

Passive Control: Use streaks/optimal perturbations to stabilize the boundary layer. (J. Fransson et al. (2006))

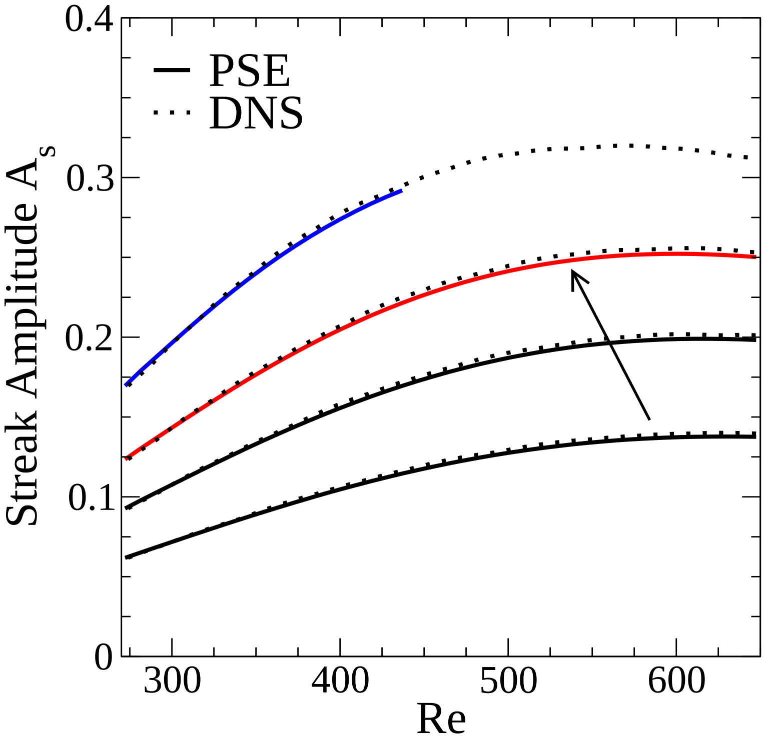
Objective of this work: Investigate in the presence of optimal streaks,

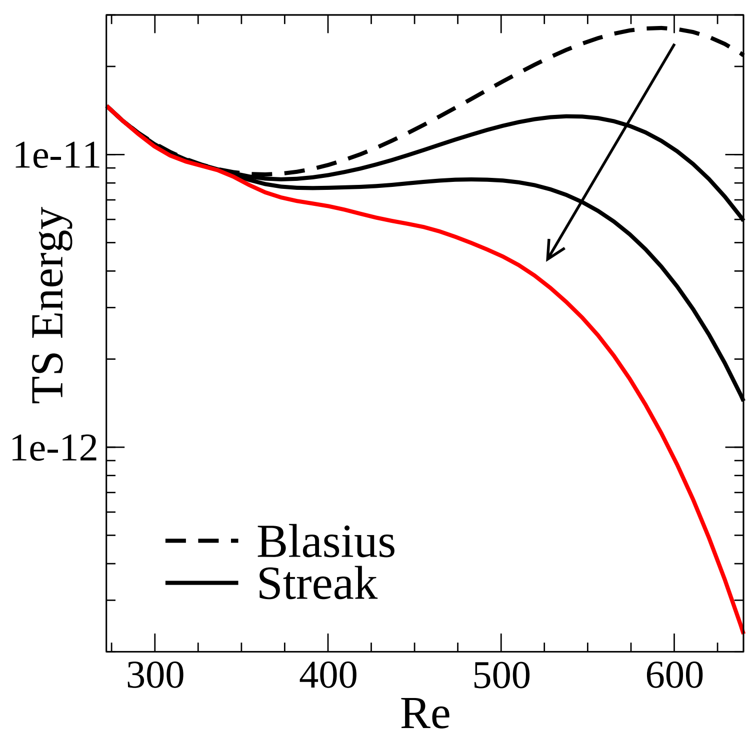
1. *Linear growth* of exponential disturbances.
   - What are the possibilities of optimizing the stabilization process, e.g. what is optimal spanwise wavenumber β of the streak?
2. *Nonlinear Interaction* of TS waves and Streaks.
   - Are exponential disturbances with amplitudes of the same order as the streak stabilized or destabilized?

I. Linear Growth

The Effect of Streaks on TS waves.

$$A_s = \frac{1}{2}\left[\max_{y,z}\{u_s\} - \min_{y,z}\{u_s\}\right]$$





The stabilizing effect increases with the streak amplitude.

Secondary inviscid instability on the streak if the amplitude is to high ($\sim 26\%$).

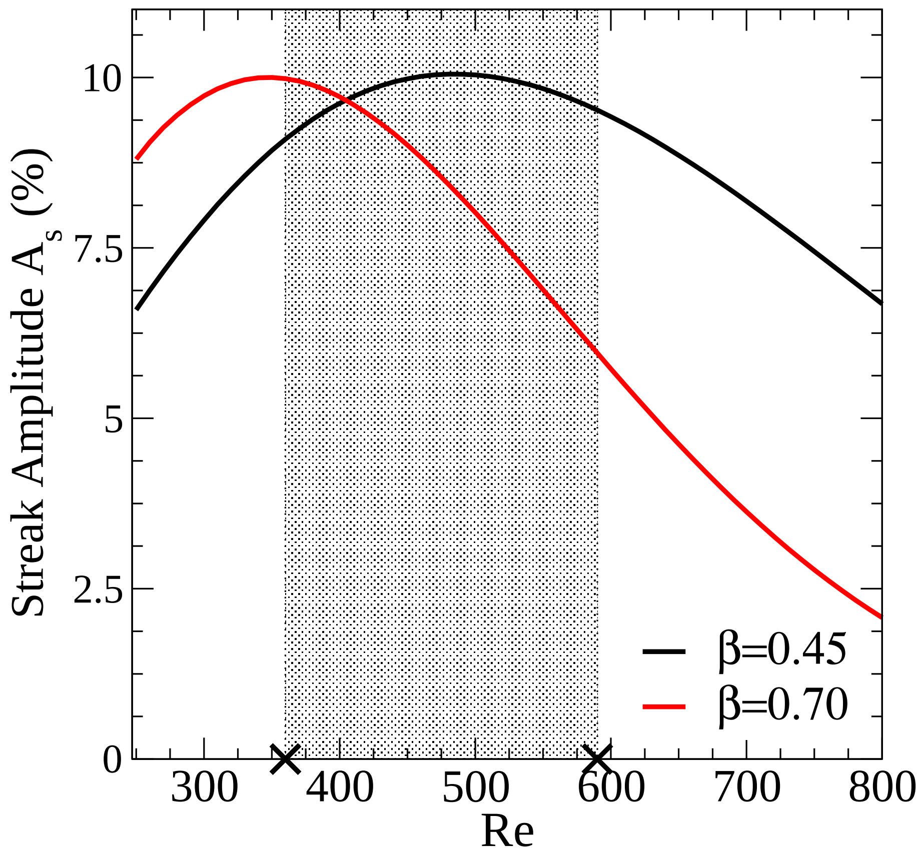
I. Linear Growth

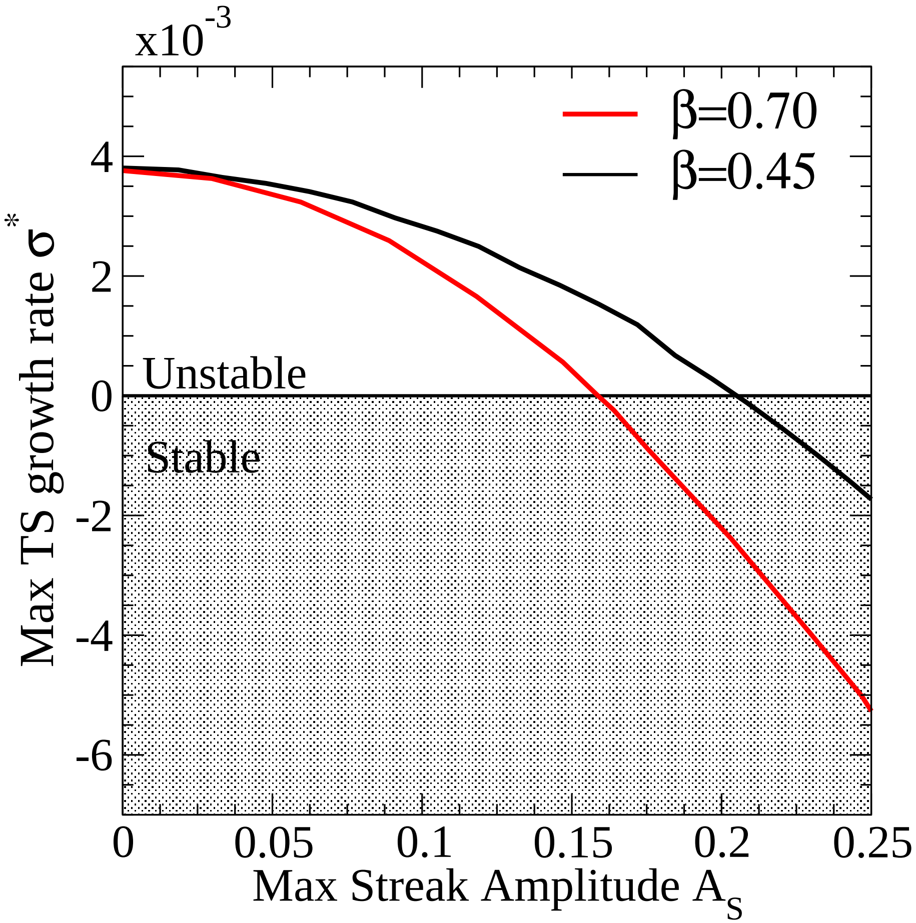
Parametric Studies of Streak

i) Vary Initial streak amplitude A_s: $0–25\%U_\infty$. Fix spanwise wavelength: $\beta = 0.45$..

ii) Vary Spanwise wavelength of streak β: $0.01–1$. Fix maximum amplitude: $A_s^* = 10\%$.

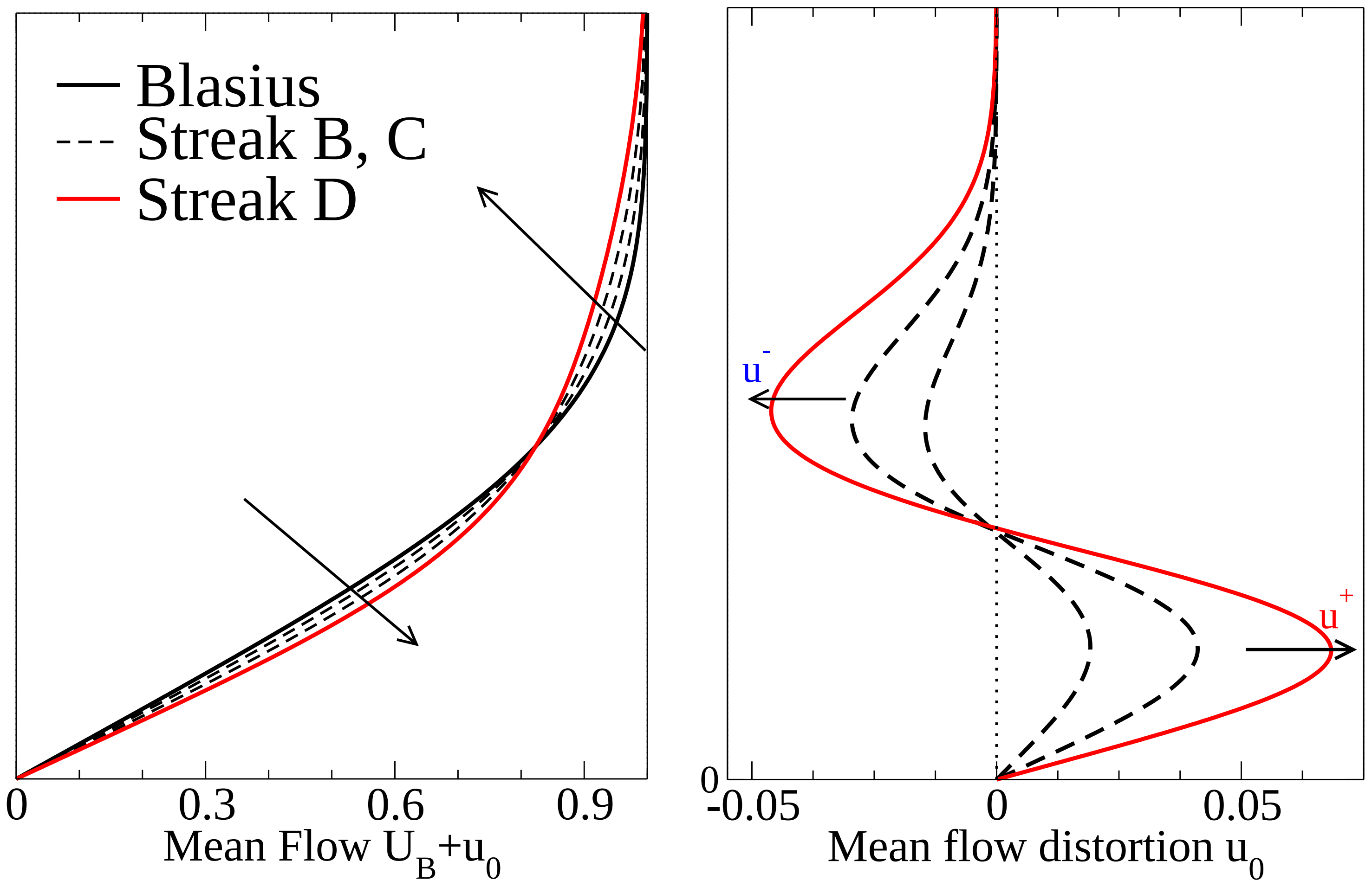
The Optimal Stabilizing Effect





The optimal stabilizing streak attains its maximum amplitude at the location of Branch I of the TS wave $(F = 131E-6)$.

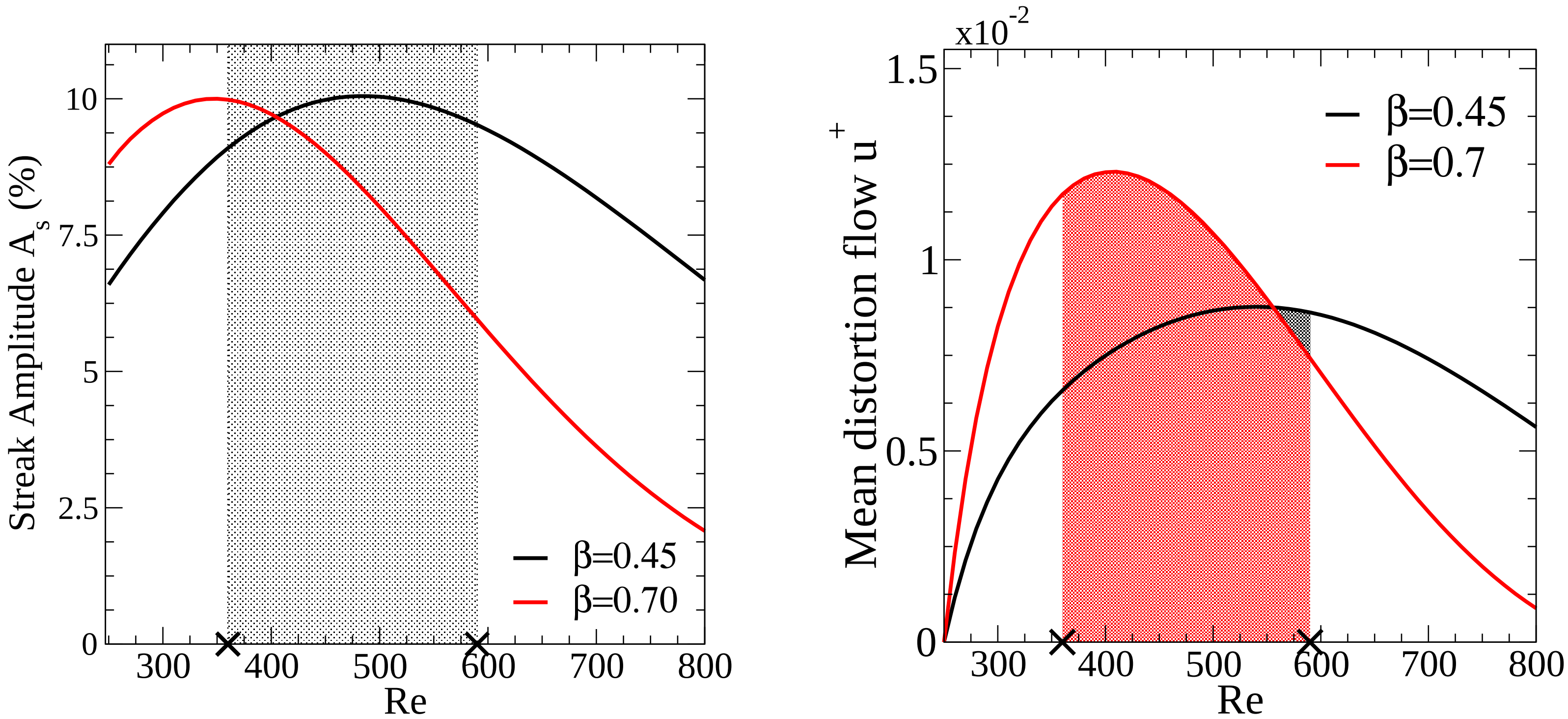
Streaky Boundary Layer



Define the mean flow excess and mean flow deficit

$$u^+ = \max_y\{u_0\} \quad u^- = \min_y\{u_0\}.$$

I. Linear Growth
The Optimal Stabilizing Effect



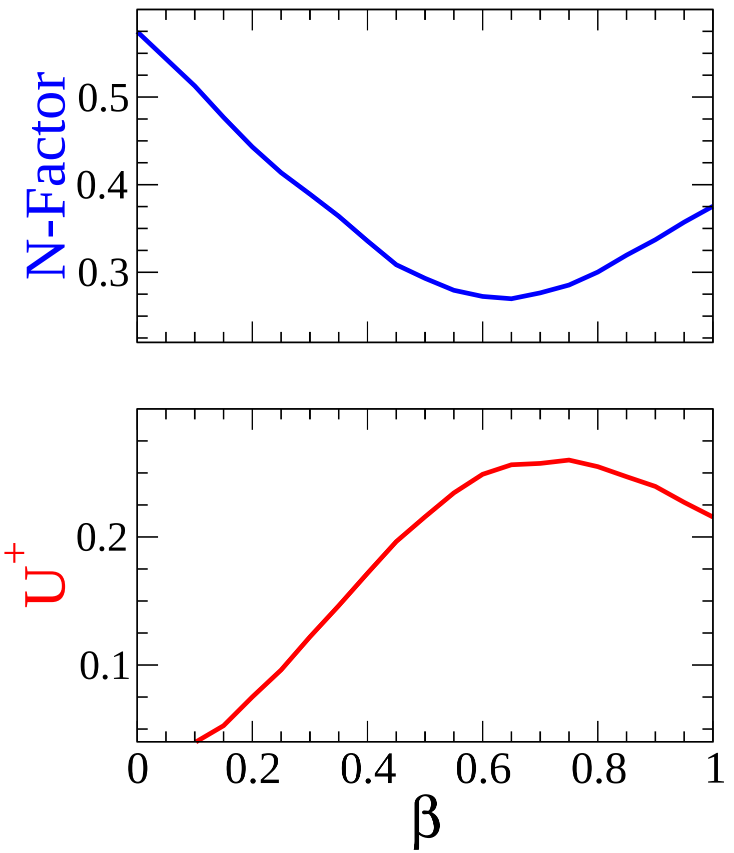
The mean flow excess of streaks is the largest, for streaks that attain their maximum amplitudes near Branch I of TS waves.

Compute the total sum of u^+ along the plate, i.e. the total mean flow excess,

$$U^+ = \int_{x_0}^{x_1} u^+ \, dx. \tag{1}$$

I. Linear Growth

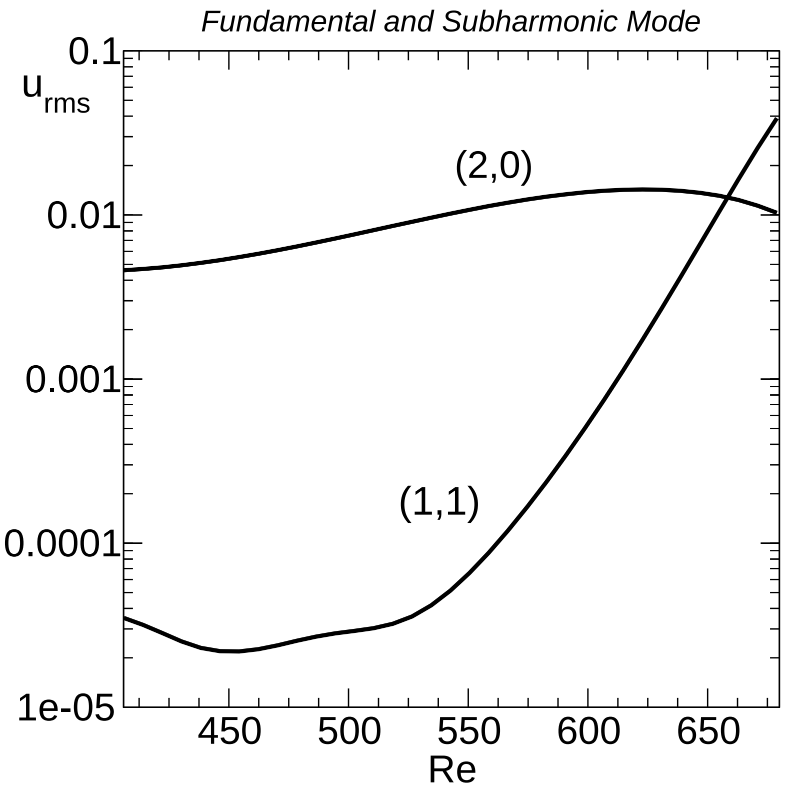
The Optimal Stabilizing Effect: Fixed Streak Amplitude, Vary β

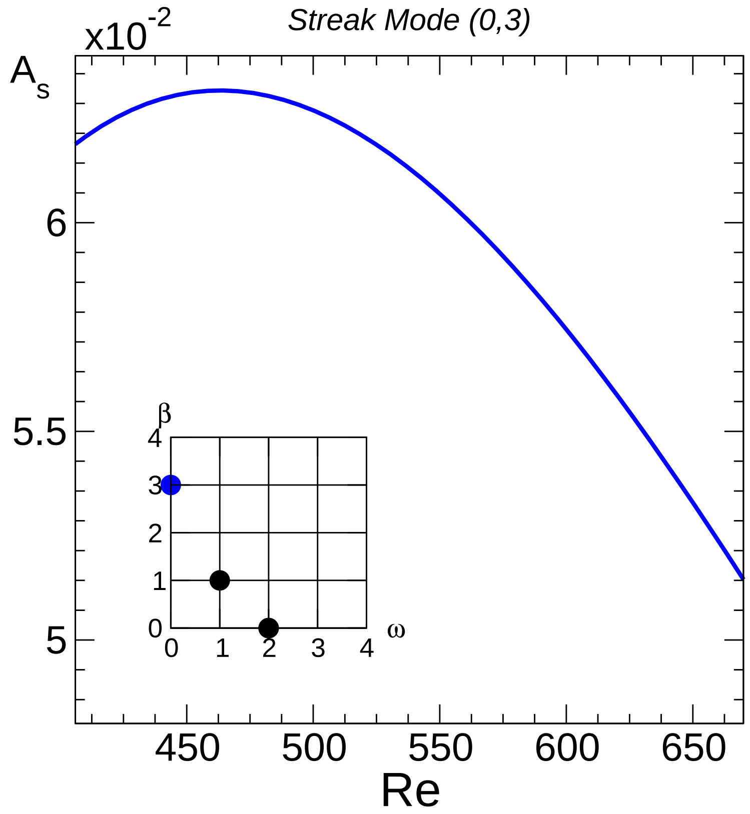


The streak that maximizes the total mean flow distortion U^+ gives the optimal stabilization effect.

II. Nonlinear Growth

Subharmonic secondary instability.

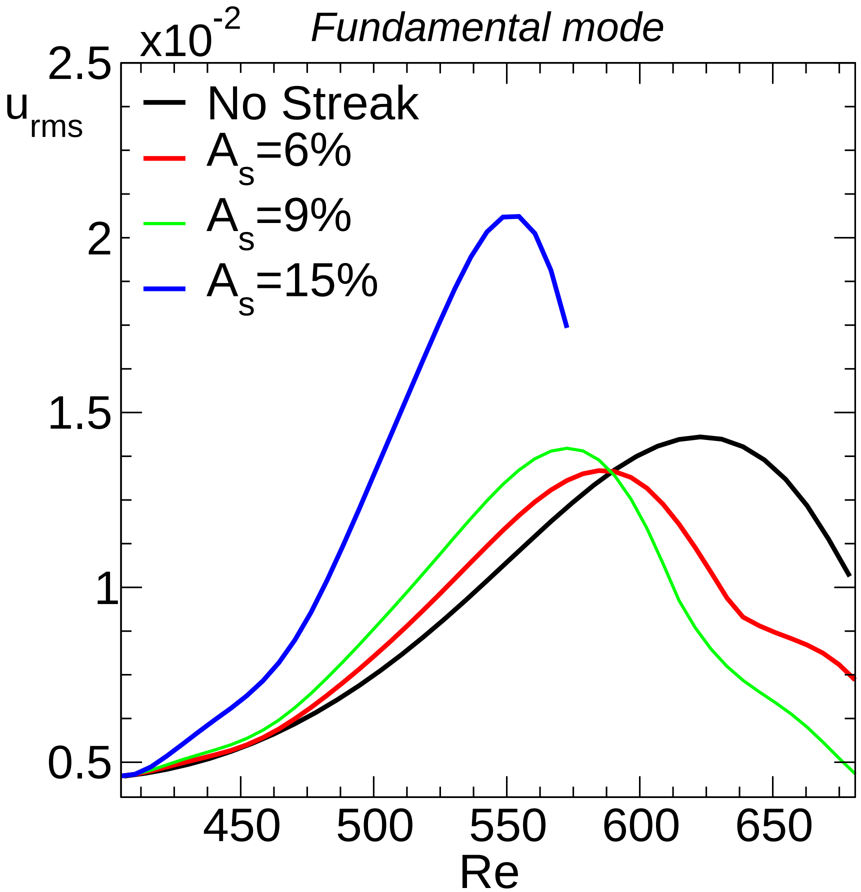


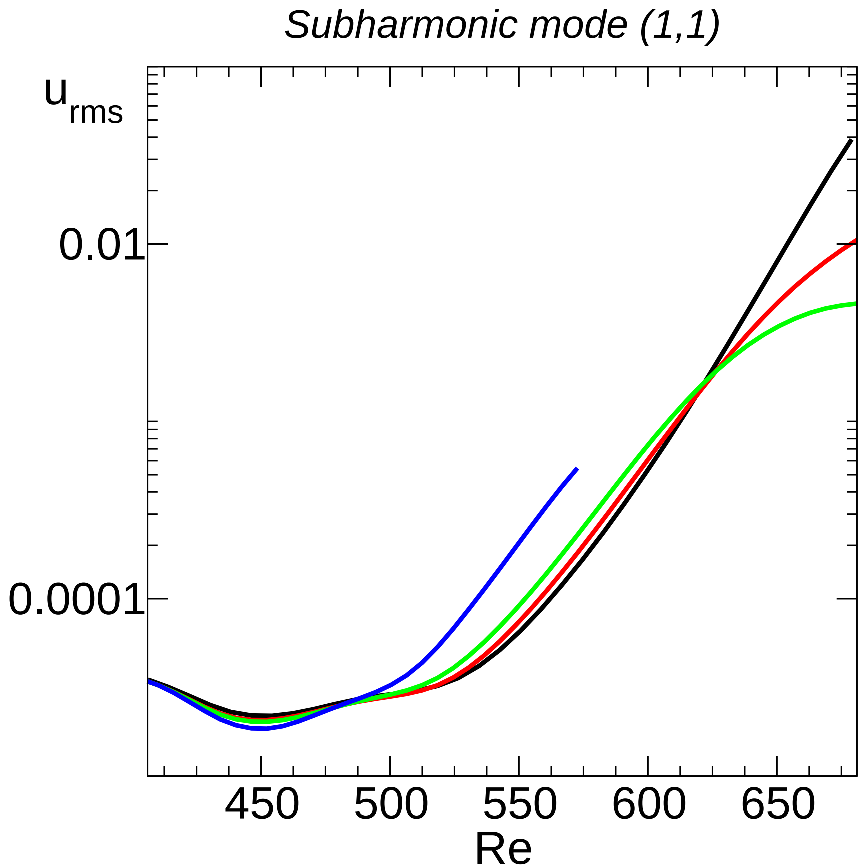


Oblique waves $(1,1)$ and $(1,-1)$ with initial amplitude $A_{ob} = 0.0035\%\ U_\infty$ and $\beta = 0.14$.
TS wave with initial amplitude $A_{TS} = 0.46\%\ U_\infty$ at $Re = 406$.

II. Nonlinear Growth

Secondary instability with Streak

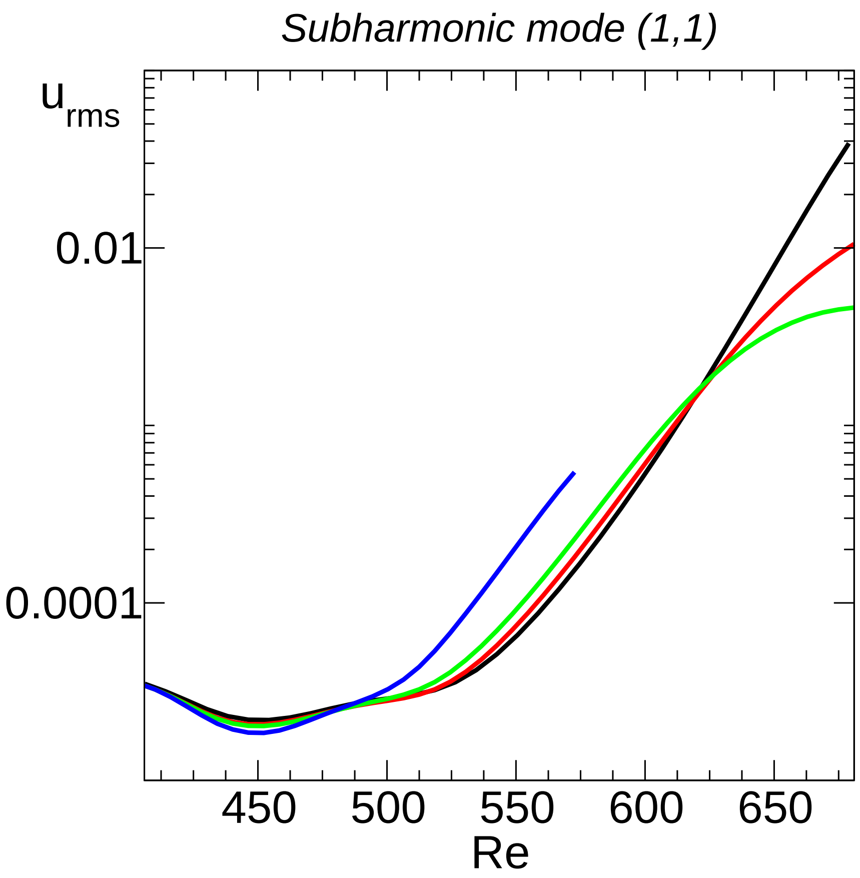


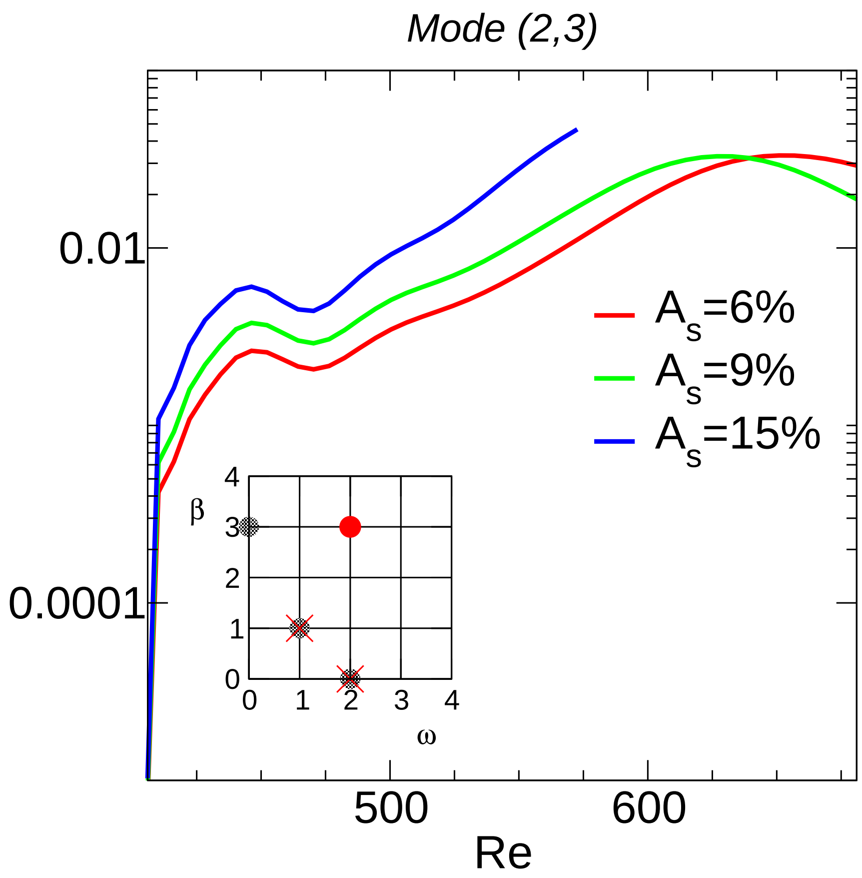


$A_s = 0$: $3.89\% U_\infty$ at $Re = 680$.

$A_s = 9\%$: $0.46\% U_\infty$ at $Re = 680$.

Destabilized Modes





If the streak amplitude is too large, nonlinear interaction of streak, subharmonic and fundamental mode can force other modes to have larger growth rates than the subharmonic wave!

Conclusions

I. Linear growth of exponential disturbances:

- The optimal stabilization effect is obtained for streaks, which cause the largest distortion of flow between Branch I & II of the TS wave.
- These streaks attain the maximum amplitude close to Branch I of the TS wave.
- The stabilization process is also observed for 3D exponential disturbances and for pressure gradients.

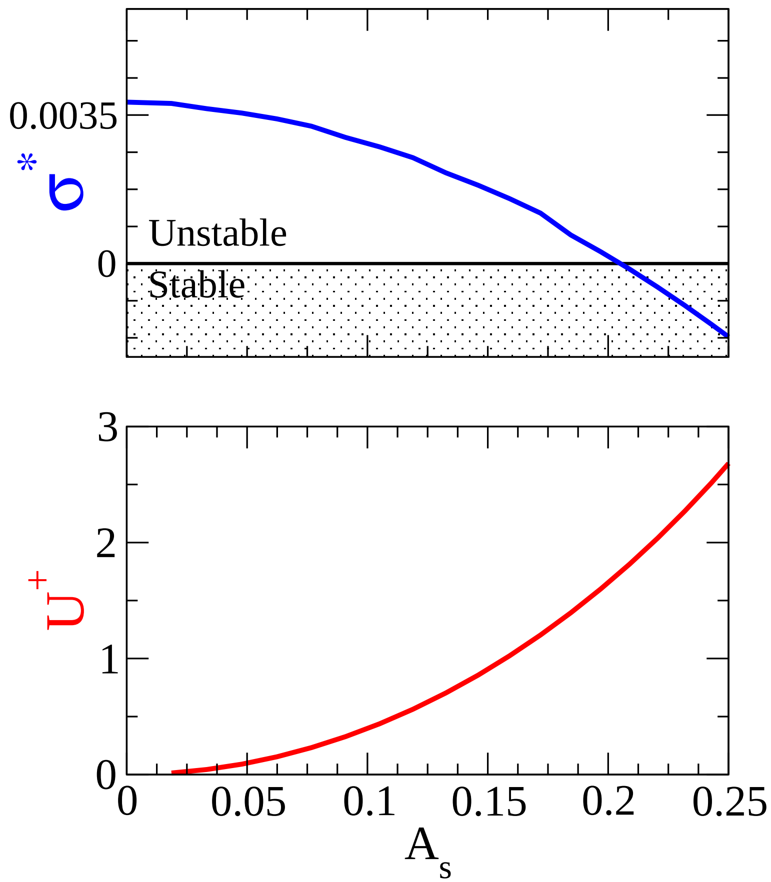
II. Nonlinear Interaction of TS waves and Streaks:

- Both stabilizing and destabilizing interaction has been observed. Finite amplitude subharmonic and TS waves are damped in the presence of streaks, but for high streak amplitudes transition due to other forced modes occurs. Other configurations of modes and transition scenarios are being investigated.

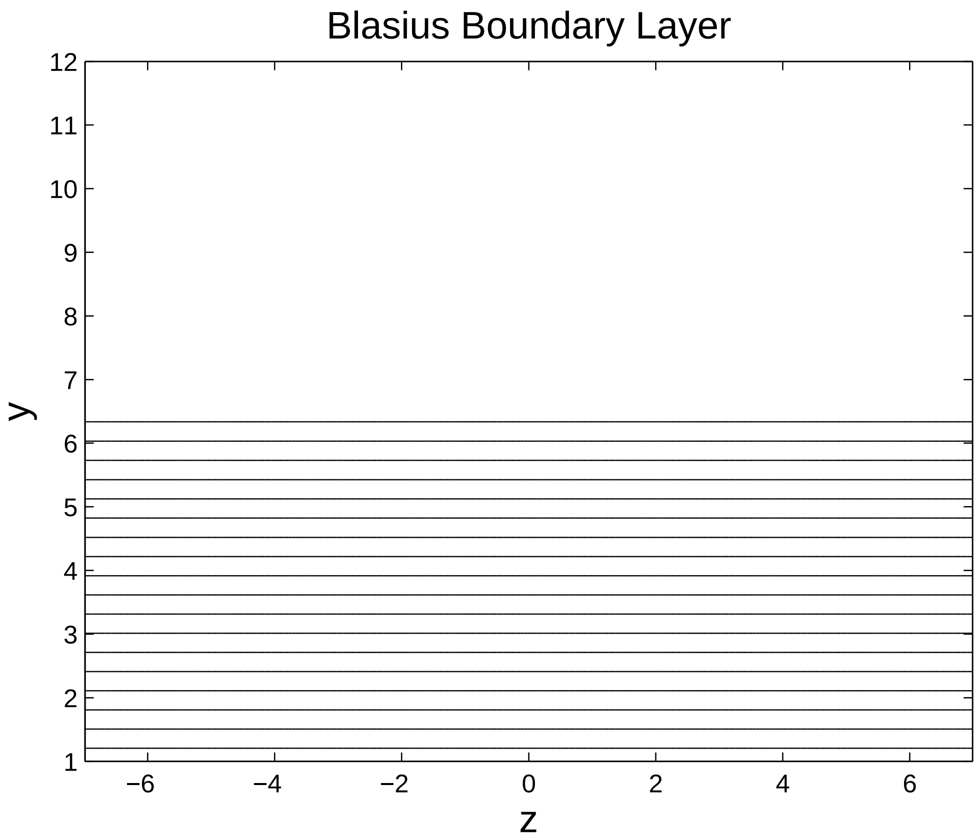
Extra Slides

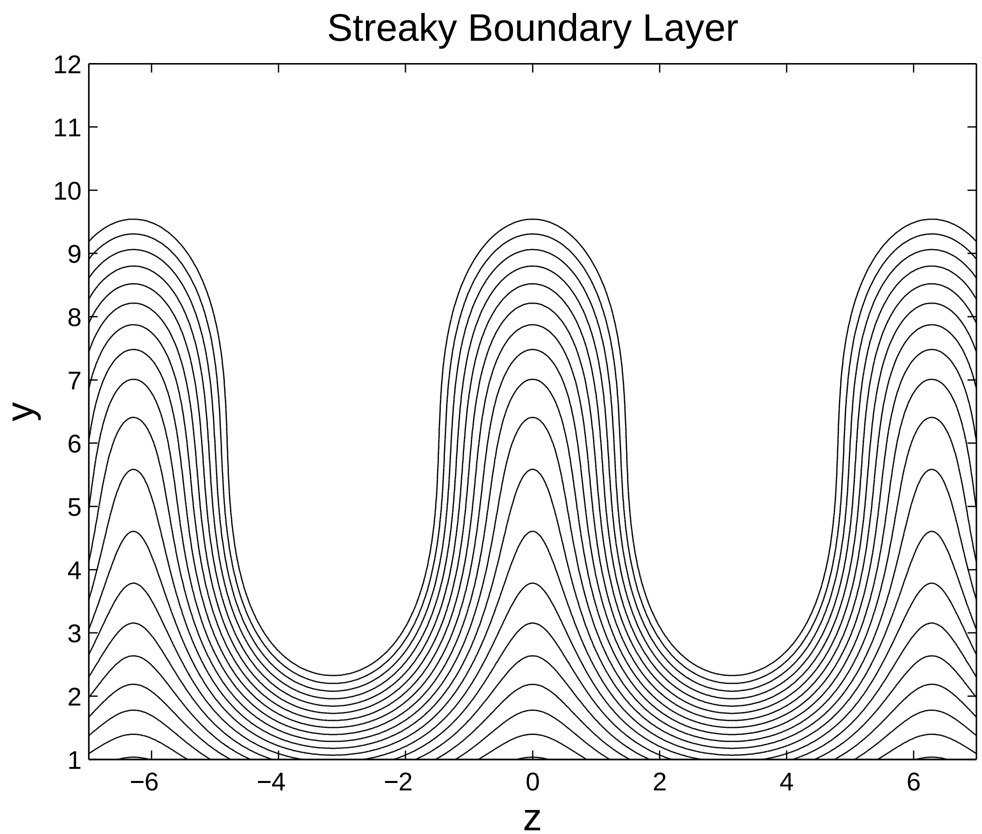
I. Linear Growth

The Optimal Stabilizing Effect: Fixed β, Vary Streak Amplitude



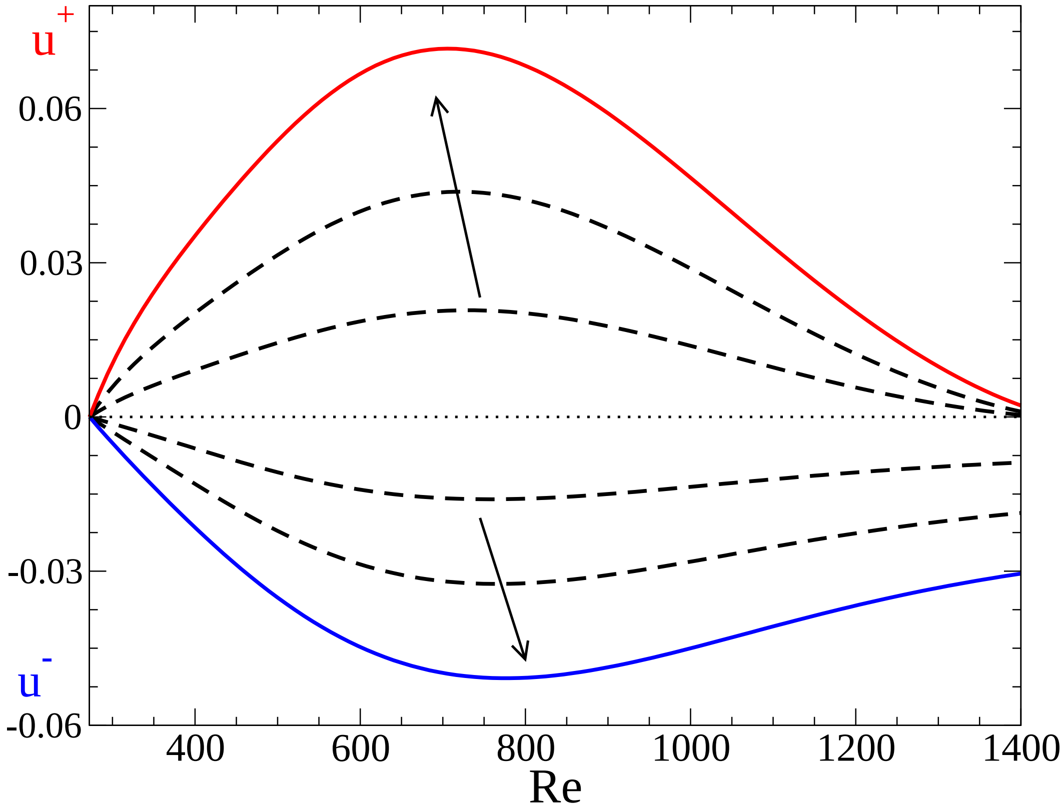
Streaky Boundary Layer





Spanwise modulation of the Boundary layer thickness.

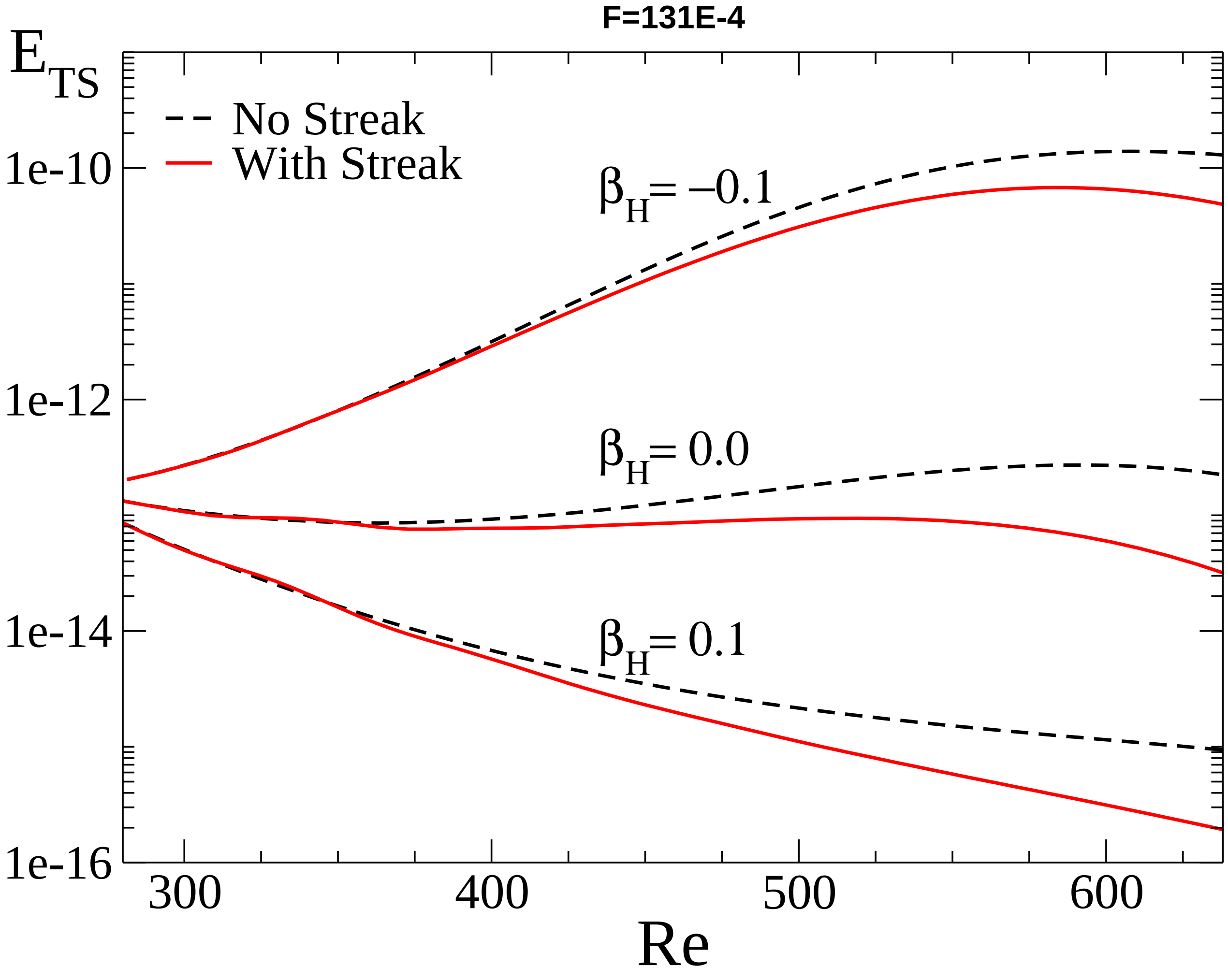
Measurement of the boundary layer



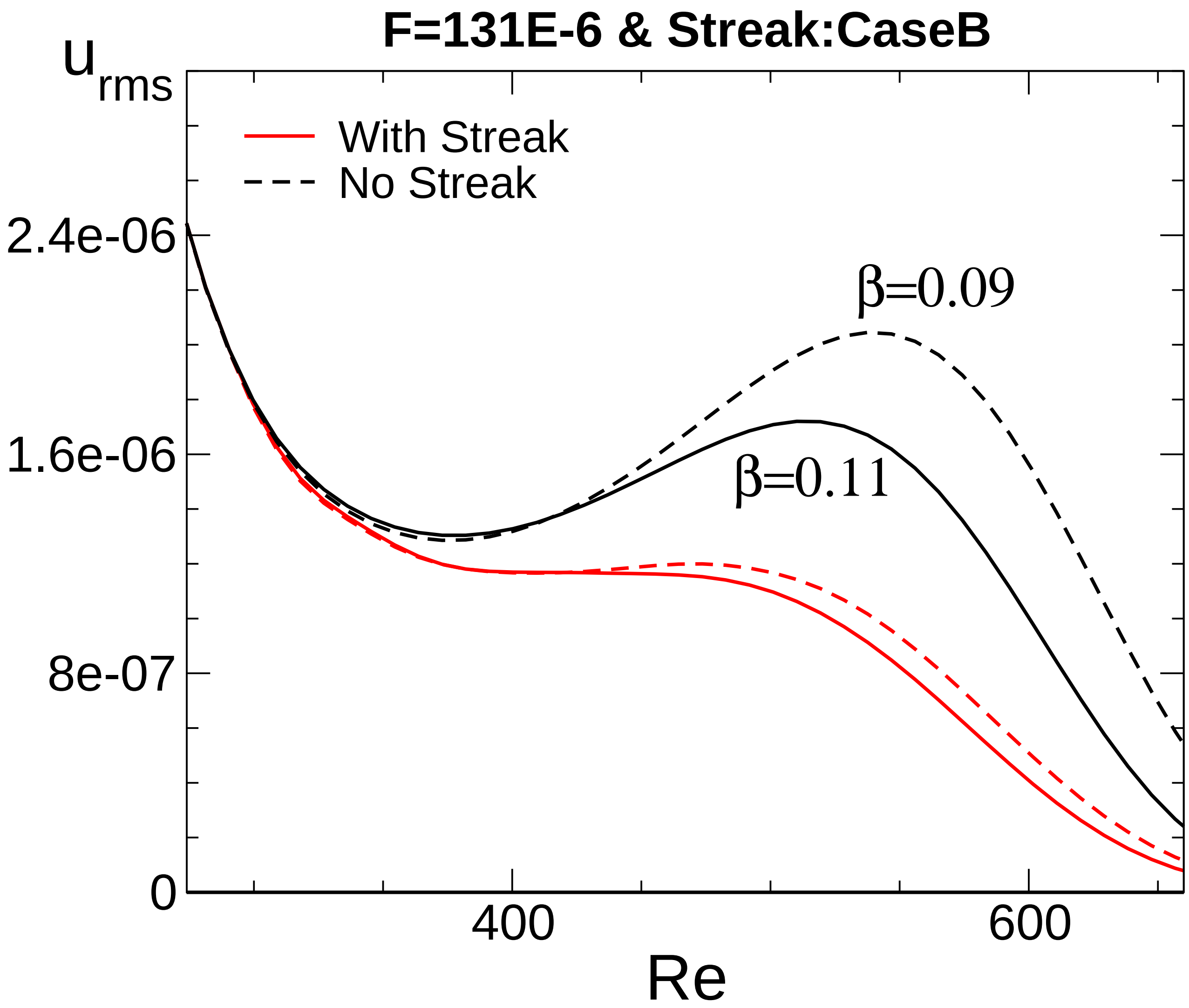
Compute the total sum of u^+ along the plate, i.e. the total mean flow excess,

$$U^+ = \int_{x_0}^{x_1} u^+ \, dx. \tag{2}$$

The Effect of Pressure Gradients on Streaks

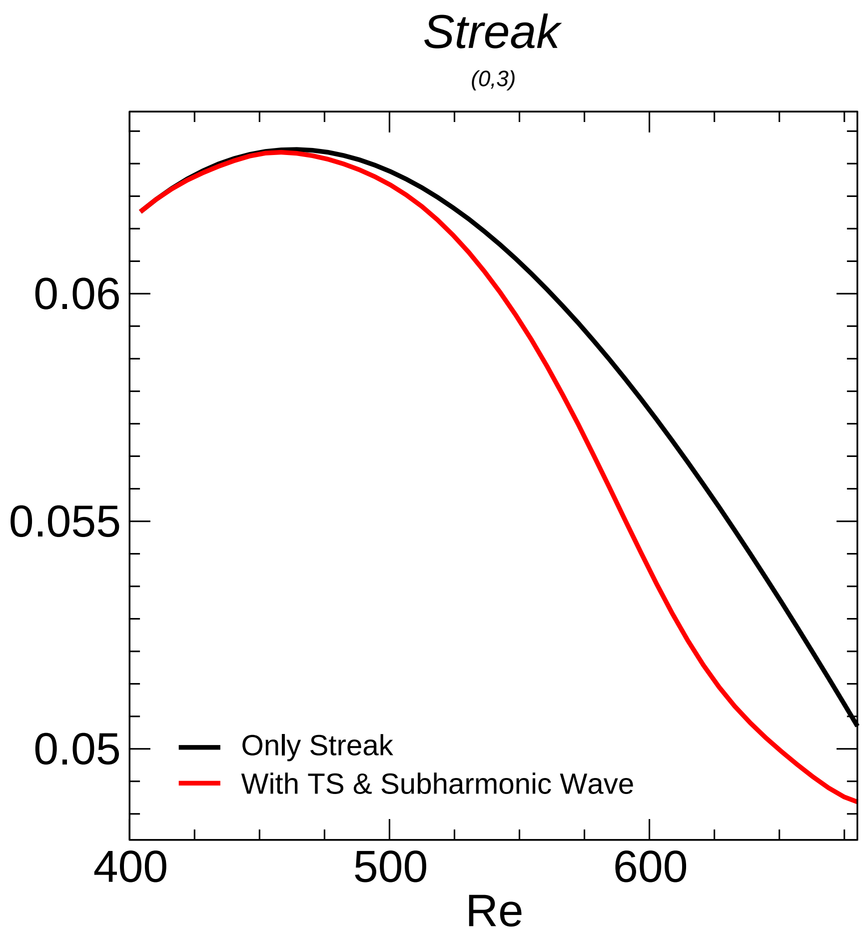


Oblique Waves



The stabilization effect is observed for 3D exponential disturbances.

The action on the Streak



Streak with maximum amplitude $A^* = 6\%\ U_\infty$ and spanwise wavelength $\beta = 0.42$.