Research on the interaction between streamwise streaks and Tollmien–Schlichting waves at KTH

Shervin Bagheri, Jens H. M. Fransson and Philipp Schlatter

Linné Flow Centre, KTH Mechanics, SE–100 44 Stockholm, Sweden

Abstract

This paper summarises the experimental and numerical investigations on how two different types of disturbances may, in a *positive* way, interact in a flat plate boundary-layer flow. The project, which mainly has been centered at KTH[1], has been performed in collaboration with colleagues from University of Bologna[2] and LadHyX CNRS Ecole Polytechnique[3], during the last years. The main phenomena — the stabilising effect of streamwise boundary-layer streaks on Tollmien-Schlichting waves (and other exponential disturbances) — have been captured both in experiments [1, 2] and with different numerical approaches such as direct numerical simulations [3], parabolic stability equation calculations [5] and large-eddy simulations [6]. We will here briefly review the methods and the main results of these studies, and discuss how they correlate with each other. For related references outside KTH the interested reader is referred to the journal publications in the reference list.

1 Introduction

Different types of drag forces are affecting both natural and artificial fliers and swimmers. For bluff bodies the largest drag component is linked to the pressure distribution around the body and is called form or pressure drag. Its value is strongly correlated with the dimension of the wake that is generated due to the separation of the boundary layers formed on the body surface. However, around streamlined bodies the boundary layer may remain attached on almost the entire surface and separate only in a very small region at the trailing edge. In this case, if the body is moving at subsonic speed, the drag will be mainly produced by the tangential viscous stresses inside the boundary layer and it will be called friction drag. Initially, very close to the leading edge, the boundary layer is laminar. The velocity profiles are steady and stable to small perturbations introduced by imperfections of the skin, turbulence in the incoming flow and other noise sources. Moving downstream the flow becomes unstable and small perturbations can be amplified, leading to transition to turbulence when they exceed a critical finite amplitude. Several methods have been studied and developed to reduce drag in a turbulent boundary layer. Some of those have been inspired by looking at the evolution in nature.

However, skin friction drag for a turbulent boundary layer, at Reynolds numbers typical of applications in aeronautical engineering, is one order of magnitude larger than the one of a laminar boundary layer. Therefore, it is possible to obtain a significant decrease of drag by extending as much as possible the laminar region, i.e. delaying transition. Numerous strategies have been tested to delay transition, most of these are *active* in the sense that they need some kind of external energy input. Here we are looking for some passive means, which can be effectively used as simple geometry modification or even as simple *add-on* to apply to already existing surfaces. We are motivated by the idea to find an approach that technology can easily pursue and, later on, optimise for a specific application.

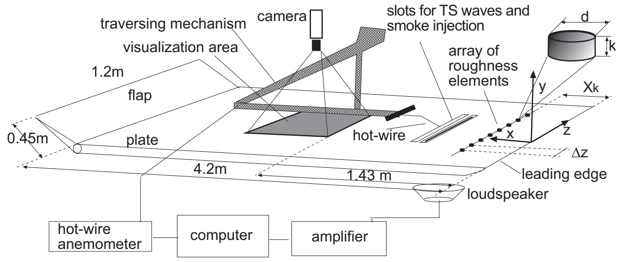

Figure 1: Schematic of the experimental setup.

One of the most generic configurations to test transition prediction and control tools is the boundary layer on a flat plate. In this case, the laminar boundary layer is given by the family of Falkner-Skan self-similar solutions and the unstable 2D waves are the so called Tollmien-Schlichting (TS) waves.

Almost any three-dimensional perturbation introduced into a boundary layer will eventually develop into streamwise elongated structures of high and low velocity alternating in the spanwise direction, so-called streaks. The mechanism responsible for the generation of these streaks is the so-called *lift-up effect*, which is due to the fact that a vertically displaced fluid particle will initially retain its streamwise momentum. In particular, any roughness on a flat plate will introduce three-dimensional disturbances which will eventually evolve into streamwise streaks. For a long time the common understanding has been that any kind of flow disturbance within the boundary layer has a promoting effect on transition. However, a number of recent studies has indicated that certain types of perturbations, in particular streamwise streaks, can delay the laminar-turbulent transition.

In this report we will present some recent results obtained experimentally and numerically regarding the effect of streamwise streaks on the growth and eventual breakdown of TS-waves in a zero-pressure-gradient boundary layer. In particular it will be shown that for boundary layers dominated by two-dimensional disturbances a substantial transition delay can be achieved by moderate-amplitude streaks.

[1]from where Luca Brandt is one of the main contributors.
[2]Alessandro Talamelli, active in the experimental work.
[3]Carlo Cossu, initiator of the project together with LB.

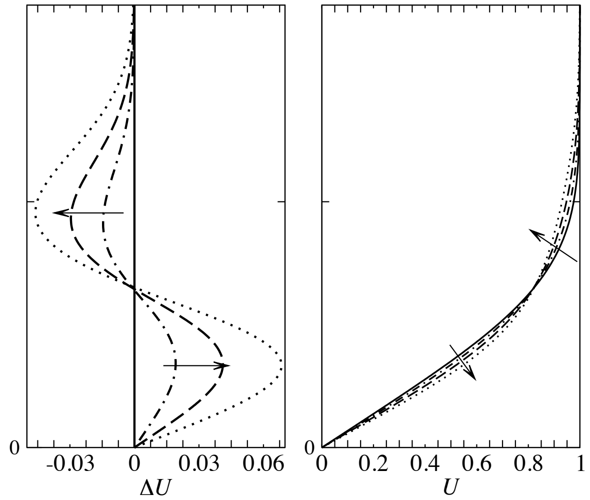

Figure 2: (a) The mean-flow distortion (ΔU) of the streaks with increasing amplitude in the direction of the arrow (published in Ref. [5]) and the corresponding boundary layer profiles (b) compared to the Blasius profile (solid line) obtained by PSE. Similar Figs. have previously been published in Ref. [3].

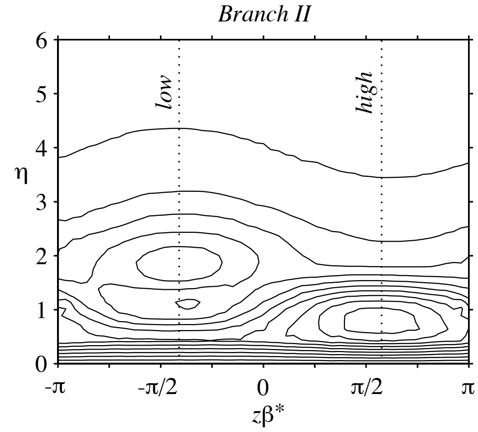

Figure 3: Experimental TS-amplitude distribution in the cross sectional plane of a streaky boundary layer for $F = 110$ close to branch II ($Re = 651$). Dotted lines indicate the location of low and high speed streaks. The contour levels are $(0.5 : 0.5 : 4.5) \times 10^{-3}$, and $\beta^* = 0.46$ mm^{-1}. Published in [2].

2 Experimental setup and numerical approaches

The experiments were carried out in the MTL (Minimum-Turbulence-Level) wind tunnel at KTH Mechanics in Stockholm on a 4.2 m long flat plate. For details of the wind tunnel and its flow quality see Ref. [1]. Figure 1 shows a schematic of the experimental setup. A single hot-wire probe is used to measure the streamwise velocity component. TS-waves are generated by time periodic suction and blowing at the wall through a slot in a plug mounted in the plate. The disturbance signal is generated by the computer through a D/A-board to an audio amplifier driving two loudspeakers. To visualise the flow state smoke is injected through a second slot and a CCD camera is used to record images downstream of the leading edge.

The enormous computational costs of direct numerical simulations (DNS) have constrained the study of the interaction of streaks and TS-wave only to a few runs, reported in Ref. [3]. For convection-dominated flows, such as the flat-plate boundary layer, all disturbances are convected downstream and grow only in a confined region between branch I and II. The influence of the disturbances upstream are insignificant and one can therefore neglect terms in Navier-Stokes equations that capture these effects, to obtain the nonlinear parabolised stability equations (PSE). The computational costs of PSE are only a fraction of that of the DNS. Therefore, PSE is well-suited for parametric studies. The method provides spatial evolution of single Fourier modes and their interactions and as it was shown in Ref. [5] they capture the interaction of streaks and TS-waves. Nonlinear PSE reproduces established experimental and numerical results on transition up to the breakdown stage.

Another method to avoid the prohibitive computational effort connected with DNS is to resort to large-eddy simulation (LES). This method solves the time-dependent, three-dimensional *filtered* Navier-Stokes equations and thus allows studies of the entire transition process from the initial laminar and slightly disturbed stages into the fully developed turbulent region. The basic idea of LES is to restrict the resolved scales to the large, energy carrying eddies (i.e. vortices) in the flow via a spatial low-pass filter. The influence of the small-scale fluctuations is incorporated via a SGS (subgrid-scale) model. The idea behind this scale separation is that the smaller eddies are more homogeneous and isotroptic than the large ones and depend little on the specific flow situation, whereas the large-scale vortices are strongly affected by the particular flow conditions (i.e. geometry, inflow perturbations etc.). LES has been applied successfully to a variety of turbulent and transitional flows yielding comparable accuracy as fully resolved DNS. The LES runs presented here are obtained with a spectral method based on about 20 million grid points. In that respect LES still requires large computational resources compared to PSE, but significantly less than DNS.

3 The effect of streaks on linear growth of TS-waves

The presence of the streaks close to the wall causes a spanwise thickening and thinning of the otherwise uniform boundary layer. Consequently, and in the same way as a pressure gradient, the influence of the streak on the *linear* stability of exponential disturbances manifests itself through the shape of the modified velocity profile. The spanwise average of the distorted mean flow in the presence of four different streaks is shown in Fig. 2. This distortion modifies the averaged velocity profile into a “fuller” shape close to the wall, and it causes the velocity to decrease in the outer part of boundary layer. The streak amplitude is here defined as

$$A_s = \frac{1}{2U_\infty}\left[\max_{y,z}(U - U_B) - \min_{y,z}(U - U_B)\right] ,$$

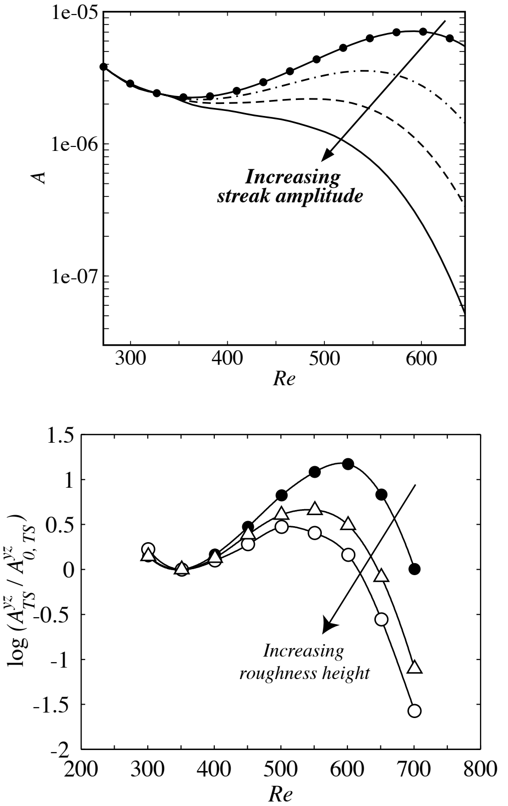

Figure 4: (a) The downstream development of the TS-amplitudes for initial streak amplitudes of $A_i = 14, 20$ and 26% of U_∞, respectively, obtained from PSE. Published in [5]. A similar Fig. has previously been published in Ref. [3] based on DNS. (b) Experimentally measured downstream amplitude growth of a TS-wave ($F = 130$) in the presence of 11 and 12% amplitude streaks (△ and ◯ respectively) are compared to the growth of TS-waves in the absence of streaks (•). Solid lines are fitted spline curves to the experimental data. Published in [2].

in both the numerics and the experiments. U_B denotes the Blasius boundary layer profile and (y, z) the coordinates in the cross plane.

As soon as the two-dimensional (2D) TS-wave enters the streaky boundary layer it is redistributed into a 3D disturbance. In Fig. 3 an example of the TS mode ($F = (\omega/Re) \times 10^6 = 110$) distribution close to branch II is shown. The Reynolds number $Re = \sqrt{Re_x} = U_\infty \delta/\nu$ is based on the free-stream velocity (U_∞) and the boundary layer thickness ($\delta = \sqrt{x\nu/U_\infty}$). The TS-wave in the presence of streaks is modulated and has two near-wall peaks in the low-speed region of the streak (usually described as an "M" shaped profile), whereas the profile in the high-speed streak resembles more the classical 2D TS-wave. This really illustrates how important it is to define the TS amplitude as an average both in the spanwise and the wall-normal direction in order to perform a fair comparison with the Blasius case.

The increase of the mean velocity close to the boundary layer is partly responsible for the stabilising effect of streaks on TS-waves as shown by PSE calculations in Fig. 4(a) and by experiments in Fig. 4(b). The other stabilising effect is caused by an extra en-

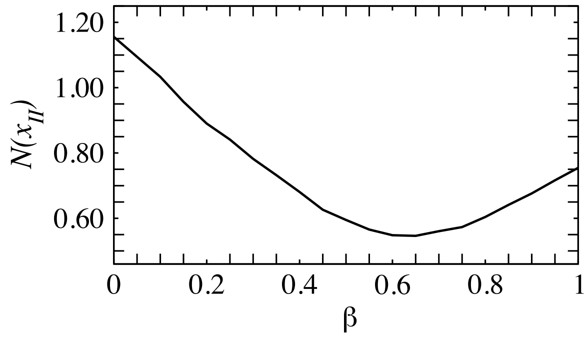

Figure 5: The N-factor at branch II of the TS-wave ($F = 132$) as a function of the spanwise wavenumber of streaks. The maximum amplitudes of the streaks have been fixed at $A_s = 10\%$. Published in [5].

ergy production term, namely $-\langle uw \rangle \partial U/\partial z$, compared to the Blasius case. This production term turns out to be of negative nature and hence together with the viscous dissipation they can overcome the wall-normal production term $(-\langle uv \rangle \partial U/\partial y)$ [4].

For details and definition of the disturbance amplitude measure see Refs. [5] and [1]. A damping effect on the TS-wave ($F =\approx 130$) is observed for all streaks and increases with the streak amplitude. For streaks with a sufficiently large amplitude the TS-wave is even completely stabilised. The streak amplitude in the experiments is controlled via the roughness element height and for the numerical computations via the initial condition, all other parameters are kept constant.

The results obtained by LES are detailed in another article in this issue and essentially show the same behaviour for the linear evolution of the interaction between streaks and TS-waves.

How the stabilising effect of streaks on TS-waves depends on the spanwise wavenumber — i.e. the streak spacing — is investigated by means of a parametric study using PSE. It was found that by distributing the streaks "optimally" in the spanwise direction, it is possible to completely stabilise a TS-wave, with considerably lower streak amplitudes than for $\beta = \beta^* \delta = 0.45$, which is optimal for maximum streak growth. For the TS-waves with high frequencies a reduction of the maximum streak amplitude of almost a factor of two can be achieved. The streaks which most efficiently reduce the growth rate of a given disturbance attain their maximum amplitudes close to branch I of that disturbance. These streaks generate a "fuller" velocity profile in the unstable domain of the TS-waves. In Fig. 5, the N-factor (ratio of the energy and branch II and I) of a TS-wave is plotted as a function of the spanwise wavenumber of the streaks, β. Here, the maximum streak amplitudes are kept constant at $A_s = 10\%$ of U_∞. We see that the N-factor attains a minimum value, just above $\beta = 0.6$, which indicates that there exists an optimal streak spacing to minimise the amplification of the TS-wave.

4 Nonlinear interaction and transition delay

If a streak is unable to completely stabilise a TS-wave in the linear regime, the TS-wave will eventually grow

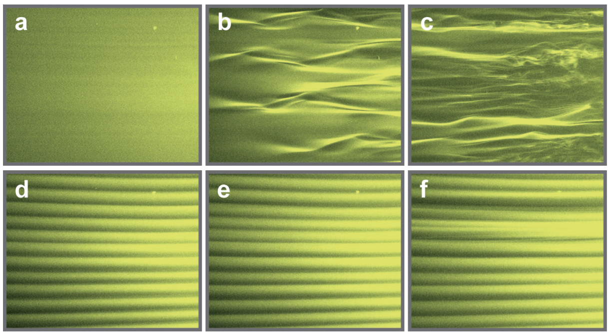

Figure 6: Smoke flow visualisations from above with flow from left to right. (a, b, c) show the case without streaks with increasing excitation (0, 150, 201) mV, respectively. The flow in (c) is turbulent. (d, e, f) show the case with the streaks present and for increasing excitation (0, 330, 450) mV, respectively. Even in (f), where the excitation is more than a factor of two compared to (c), the flow remains laminar. Some of these Figs. were published in [2].

and reach a finite amplitude and may cause transition to turbulence further downstream. It is therefore important to investigate how the presence of streaks influences exponential disturbances at higher amplitudes. Noteworthy is that it is not trivial that transition delay has been accomplished just because linear TS-waves have been damped. When exponential disturbances reach amplitudes larger than a few percent, they may saturate nonlinearly and transform the flow into a new, more complicated state. The stability of this new modified mean flow is called secondary instability, and has for TS-dominated transition been roughly associated with two scenarios. A subharmonic or H-type transition and the fundamental or K-type transition transition, where both theory and numerical simulations indicate that H-type scenario is more likely to occur than the K-type scenario.

4.1 Experimental findings

To visualise the flow, smoke is injected through a slot (see Fig. 1) and the flow state is recorded through a CCD camera. Without forcing, the 2D laminar flow is observed (Fig. 6a), whereas forcing with the amplifier set at 150 mV triggers a transitional flow (Fig. 6b), and a turbulent flow at 201 mV (Fig. 6c). The same experiment is then repeated in the spanwise modulated boundary-layer flow. The modulation is obtained by pasting on the flat plate cylindrical roughness elements (diameter = 4.2 mm and height = 1.4 mm) periodically spaced in the spanwise direction by 14.7 mm and at a distance of 80 mm from the leading edge. Here it should be emphasised that the net drag increase due to the presence of the streaks is less than 3.5% compared to the 2D base flow. The streaky base flow induced by the roughness is shown in Fig. 6(d). When subject to the same disturbance forcing as before this streaky flow remains laminar in the visualisation region. Even when the excitation amplitude is more than doubled, to 330 mV and 450 mV, as shown in Figs. 6(e) and (f) the flow is kept laminar.

Quantitative data have been obtained by hot-wire

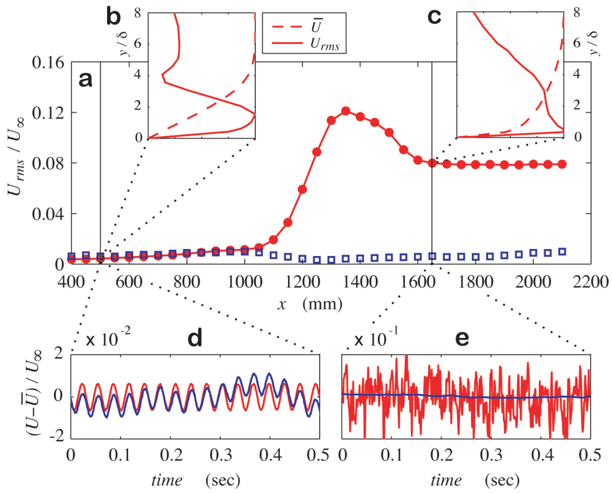

Figure 7: (a) Evolution of the dimensionless temporal root mean square streamwise velocity fluctuations U_{rms}/U_∞ in the streamwise direction x when TS-waves are excited at $f = 32$Hz and 1000 mV. (b) and (c) show the wall-normal disturbance profile at two selected positions ($x = 400$ and $x = 1650$ mm) in the absence of streaks. (d) and (e) show the corresponding time signals with and without streaks. Light lines and filled symbols correspond to the two dimensional boundary layer, without streaks, whereas dark lines and unfilled symbols to the boundary layer with the controlling streaks present. In the presence of streaks the flow remains laminar. A similar Fig. was published in [2].

measurements of the streamwise velocity component U along the plate at constant dimensionless height, $y/\delta = 2.2$. To force three-dimensional disturbances on top of the dominant two-dimensional TS-wave, a classical procedure was used by pasting thin 50×50 mm^2 tape-spacers on the flat plate just downstream of the blowing/suction slot. The streamwise distribution of the temporal root mean square (rms) of the velocity fluctuations and two typical hot-wire time signals, induced by a forcing of 1000 mV, are reported in Fig. 7. In the absence of streaks the classical K-type transition scenario is observed. Linear TS-waves (Fig. 7b, d) of small amplitude evolve into finite amplitude non-linear waves, characterised by the appearance of spikes in the hot-wire traces. This leads to an increase of the rms-velocity to a peak level where the flow is transitional. Transition is completed when the perturbation rms-velocity reaches a plateau ($x = 1650$ mm) where a fully turbulent signal is observed (Fig. 7e). In the presence of streaks the flow is laminar: the forced perturbations remain of small amplitude and sinusoidal all along the measurement region as it can be seen comparing the signals with and without streaks at $x = 1650$ mm in Fig. 7(e). Thus, we show that it is possible, using the streaks, to maintain TS waves at low amplitudes in a boundary layer flow which otherwise would be transitional in the absence of the streaks [2].

Increasing the excitation amplitudes of the TS-waves, transition is observed also in the streaky boundary layer, even though further downstream than in the 2D boundary layer and no delay is ob-

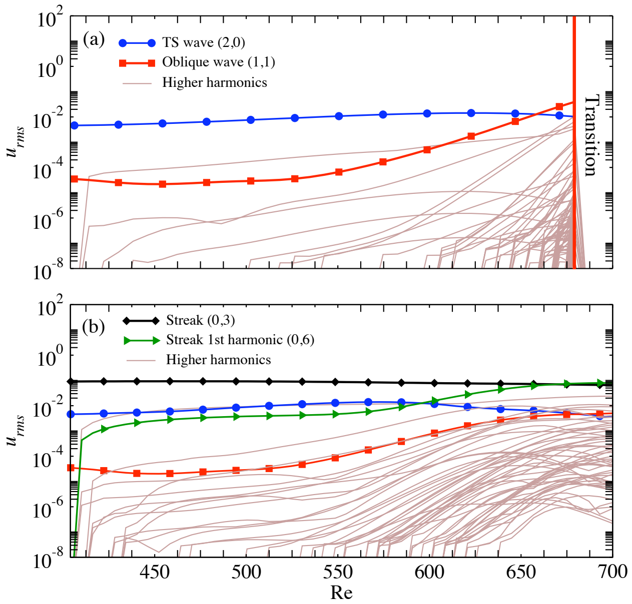

Figure 8: The downstream development of 50 modes of the H-type scenario in (a). In (b) a streak of initial amplitude 9% is added. Only 50 out of 250 excited modes are shown. The TS-wave and the subharmonic wave are marked with filled circles and squares, respectively. In (b) the streaks is marked with diamonds and the (0,6)-mode is marked with left triangle.

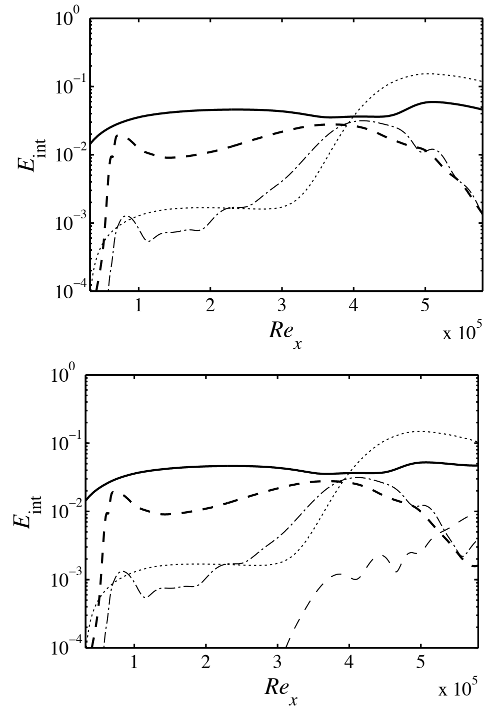

Figure 9: Energy integrated in the wall-normal direction for selected modes for streak (amplitude 2.6% at the inlet) and TS-waves. *Top:* TS-wave and streaks without noise, *bottom:* TS-wave and streaks with small-amplitude noise triggering transition close to $Re_x \approx 550000$. ——— Steady streak $(0, \beta_0)$, - - - - TS-wave $(\omega_0, 0)$, ········ first harmonic of streak $(0, 2\beta_0)$, —·— first harmonic of streaky TS-wave (ω_0, β_0), - - - - oblique mode $(\omega_0, \beta_0/3)$. Published in Ref. [6].

served for forcing amplitudes larger than 2000 mV. Transition is however never observed to occur earlier in the streaky flow than in its 2D counterpart. These results are in accordance with the observation in Ref. [7] that forcing TS-waves with high enough amplitudes into a streaky boundary layer transition is promoted. These findings also imply that in the present configuration there is no significant destabilising action due to the non-linear interactions between streaks and TS-waves and therefore the low-amplitude stabilising action of the streaks on the TS-waves is able to delay transition. The results are essentially unchanged when random noise is added to the forcing. Transition delay is also observed when white noise with amplitude of 10% of that of the periodic signal is added in the excitation signal, showing that the delay mechanism is robust for strongly colored noise. Experiments were also performed by forcing large amplitude, 2000 mV, two-dimensional white noise without the sinusoidal signal. TS-waves of all frequencies are thus excited with random phase shifts. Even in this case, the streaks delay transition by 385 mm (cf. [2]).

4.2 Numerical results

The great advantage of numerical simulations it that the initial and inflow conditions can be specified to a high degree of accuracy; in the present case the laminar Blasius boundary layer, and the amplitudes of both the streak and TS-wave. Moreover, during the postprocessing step always the whole flow field is available to collect complex statistics (e.g. the evolution of individual Fourier modes) and to produce 3D visualisations of the flow structures present during the nonlinear transitional stage and turbulent breakdown. On the other hand, as detailed in section 2 the high cost of direct simulation requires the use of simplified models, in this case PSE and LES, which have to be validated carefully against DNS and experiments.

A set of nonlinear PSE calculations was performed with similar conditions as in traditional studies of the H-type scenario. The computational domain begins at $\mathrm{Re} = 404$, with a fundamental frequency $F = 62$ of the TS-wave and a spanwise wavenumber $\beta_0 = 0.14$. Initial conditions are given for a TS-wave in mode $(2, 0)$ and a subharmonic wave $(1, 1)$ with *rms* amplitudes of 0.46% and 0.0035% of U_∞, respectively. In the classical H-type scenario, the secondary instability is caused by the interaction of these two modes, in which a large number of higher modes are excited (see Fig. 8a). It is the subharmonic wave $(1, 1)$ that grows the most. The transition to turbulence takes place close to $\mathrm{Re} = 650$ when u_{rms} of the subharmonic wave is about 3.9%. Transition to turbulence sets in when suddenly a large number higher modes are forced and show a very rapid growth. In the presence of a streak with amplitude 9%, the flow is much more complicated as a few hundred of modes are excited much earlier than in the classical H-type scenario without streak, see Fig. 8(b). However, due to the presence of streaks, all modes are decaying after $\mathrm{Re} \approx 670$, indicating a delay or even inhibitation of transition. We observe that the second harmonic of the streak mode,

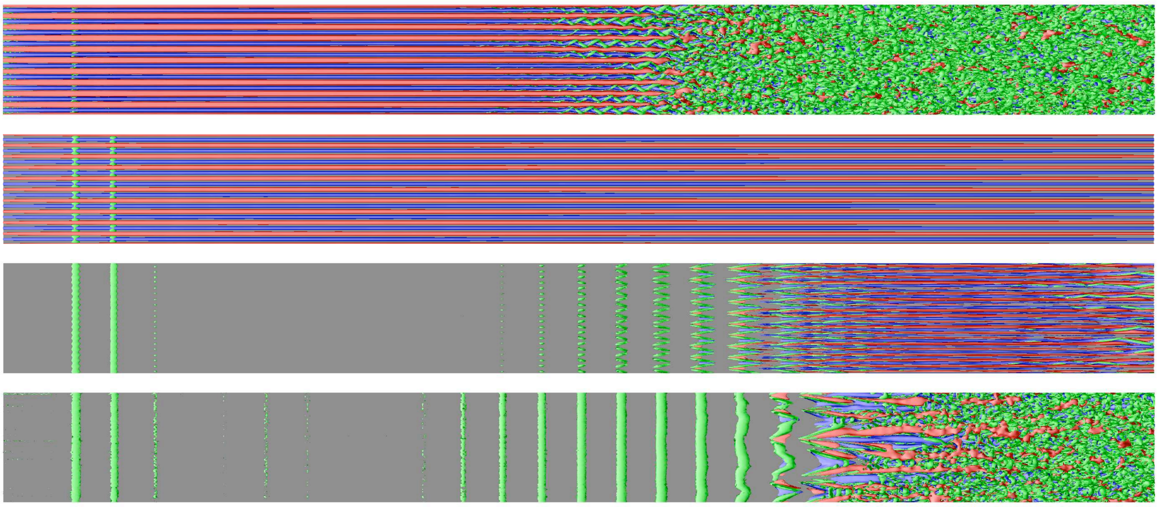

Figure 10: Top view of the three-dimensional flow structures for (from top to bottom) streaks with amplitudes 29%, 10%, 2.6% and 0% (uncontrolled) at the inlet ($Re_x \approx 32000$). Green isocontours represent the $\lambda_2 = -0.00008$ vortex-identification criterion, red and blue isocontours are positive and negative disturbance velocity $u' = \pm 0.07$, respectively. Flow from left to right. Published in Ref. [6].

i.e. the $(0, 6)$ mode, overtakes the streak mode $(0, 3)$ at around $\mathrm{Re} = 600$. This indicates the development of a streak with double spanwise wavenumber than the initiated streak.

The large-eddy simulations of the nonlinear evolution of the interaction between streaks and TS-waves were performed on a numerical domain extending from $Re_x = 30000$ to $Re_x = 600000$. The streaks are introduced at the inlet plane as optimally growing linear perturbations with spanwise wavenumber β_0; the TS-waves are forced further downstream at $Re_x = 60000$ with a harmonic volume force ($F = 120$) acting in the wall-normal direction. The TS amplitude at branch I ($Re_x \approx 150000$) is approximately 0.76%. Additionally, in order to trigger transition in the uncontrolled case small-amplitude three-dimensional noise was superimposed at that position. Note that the spanwise width is chosen such that approximately four periods of the secondary instability of the primary TS-waves fit in the domain (see Fig. 10).

The influence of varying streak amplitude while maintaining a constant amplitude of the 2D TS-wave is shown in Fig. 10 with a top view of the flow structures arising in the various cases. For the streak with the highest amplitude (29% at the inlet) the TS-wave is strongly damped. However, this high streak amplitude leads to secondary instability of the streaky base flow itself characterised by a spanwise oscillation of the low speed streak. Reducing the streak amplitude below the critical value of 26%, giving stable streaks, Fig. 10(b), it is clear that the TS-waves are completely damped and transition is inhibited; also evident is the slow downstream decay of the streaks. Further decreasing the streak amplitude, complex interaction between the TS-waves and the streaks is observed. A development similar to the fundamental K-type scenario is initiated with Λ-shaped vortices on top of the low-speed streaks. However, this interaction does not lead to turbulent breakdown, rather a new quasi-steady base flow characterised by the $(0, 2\beta_0)$ is established, corresponding to a doubling of the streaks. This new baseflow may be susceptible to an instability caused by oblique waves (see Fig. 9) and undergo transition to turbulence. Finally, the lowermost visualisation in Fig. 10 shows the uncontrolled case with exponentially growing two-dimensional TS-waves and aligned Λ-structures typical of the K-type scenario are clearly visible. In summary, Fig. 10 depicts the various possible transition mechanisms in a boundary layer with streamwise streaks: From the classical K-type scenario (at low streak amplitudes) to bypass transition of high-amplitude streaks. In between these limiting cases, stabilisation and transition delay is achieved by means of spanwise modulation of the base flow.

The case of richest nonlinear interaction between streaks and TS-waves of the same order of magnitude, leading to the streak doubling, is further examined in Fig. 9. The top figure shows the clean case, i.e. only 2D TS-waves and streaks are excited without any additional noise. On the other hand, the bottom figure displays the flow when three-dimensional noise is also added by a streamwise localised forcing. The streak doubling characterised by the mode $(0, 2\beta_0)$ discussed above is clearly observed in the clean case as well. This feature is thus neither affected nor triggered by the presence of noise. However, the first mode to actually grow is the first harmonic of the streaky TS-wave, (ω_0, β_0). Noise obviously needs to be added to trigger transition, as can be induced by Fig. 9(b): It is interesting to see that the subharmonic mode $(\omega_0, \beta_0/3)$ is the first mode to grow after the streak doubling. Note that the spanwise wavenumber $\beta_0/3$ roughly corresponds to the wavenumber of the most unstable secondary instability for the given TS-wave. The noise forced upstream obviously survives during the streak doubling process and subsequently determines the transition scenario observed further downstream.

5 Conclusions

In conclusion, we have shown that transition to turbulence in a boundary layer can be delayed by a passive method, i.e. by modifying the base flow with streamwise high and low-speed streaks. The observed delay can be ascribed to the reduction of the exponential growth of TS-waves evolving in boundary-layer flows in the presence of streaks. By controlling the relative amplitude of both the streaks and the TS-waves a destabilising non-linear interaction between these two types of perturbation can be avoided. If the streak amplitude is too high (i.e. above approximately 26%) the streak itself will undergo wavy secondary instability; and for low streak amplitudes the streak and TS-waves can interact leading to a new base flow with strong streaks of twice the wavenumber of the original streaks. In turn this new base flow can be unstable to oblique modes which are not sufficiently damped by the streaks. However, it is important to note that – as long as the critical amplitude of the streaks is not exceeded and the flow disturbances are dominated by two-dimensional waves – transition to turbulence is never promoted due to the presence of the streaks.

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