The stabilizing effect of streaks on Tollmien-Schlichting and oblique waves: A parametric study

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(Received 5 February 2007; accepted 28 April 2007; published online 9 July 2007)

The stabilizing effect of finite amplitude streaks on the linear growth of unstable perturbations [Tollmien-Schlichting (TS) and oblique waves] is numerically investigated by means of the nonlinear parabolized stability equations. We have found that for stabilization of a TS-wave, there exists an "optimal" spanwise spacing of the streaks. These streaks reach their maximum amplitudes close to the first neutral point of the TS-wave and induce the largest distortion of the mean flow in the unstable region of the TS-wave. For such a distribution, the required streak amplitude for complete stabilization of a given TS-wave is considerably lower than for $\beta=0.45$, which is the optimal for streak growth and used in previous studies. We have also observed a damping effect of streaks on the growth rate of oblique waves in Blasius boundary layer and for TS-waves in Falkner-Skan boundary layers. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2746047]

In boundary-layer flows, the transition from a laminar state to a turbulent one is usually caused by growth and breakdown of small amplitude perturbations. For a long time, the common understanding has been that any kind of flow perturbation inside the boundary layer has a promoting effect on transition. However, a number of recent studies[1–4] have indicated that certain types of perturbations inside the boundary layer can postpone the laminar-turbulent transition. A general feature of these perturbations seems to be a modification of mean velocity profile to a more stable one. In two-dimensional mean flows, these are streaky structures that create regions of alternating negative and positive streamwise velocity perturbations. Streaks are usually found inside the boundary layers subjected to high free-stream turbulence. A damping effect of moderate amplitude free-stream turbulence on Tollmien-Schlichting (TS) waves has been observed in some experiments.[5] Numerical investigations of Cossu and Brandt[2] showed a clear stabilizing effect of streaks on growth of TS waves in Blasius flow. They reported an increasing damping effect with increasing streak amplitude. These results were later verified by experimental works of Fransson *et al.*,[3] who generated the streaks by means of small roughness elements. Recently, Fransson *et al.*[4] also showed that these streaks can truly delay the transition. Here, the transition was triggered by means of high-amplitude two-dimensional disturbances generated through random suction and blowing at the wall. These new results have received great attention; e.g., Ref. 6. However, in all these studies, both experimental and numerical, a single spanwise spacing ($\beta=0.45$) of streaks has been used, which corresponds to the most growing streaks. Therefore, we aim to investigate whether other distributions of streaks are more efficient for stabilizing TS-waves, so that a lower streak amplitude would be required for transition delay. This is important because the amplitude of the streaks should not exceed the threshold for secondary instability and instead promoting the transition to turbulence. The present work is based on a parametric study of the streak spacing. The feasibility of such a study, requires a relative fast computational method, such as the nonlinear parabolized stability equations (PSE).[7]

Numerical procedure. We consider flow disturbances that are periodic in time t and spanwise direction z. These disturbances are decomposed in Fourier modes as

$$\mathbf{q}(\mathbf{x},t)=\sum_{m=-M}^{M}\sum_{n=-N}^{N}\tilde{\mathbf{q}}_{mn}(x,y)\exp(in\beta_0 z-im\omega_0 t).$$

Here, $\tilde{\mathbf{q}}_{mn}=\hat{\mathbf{q}}_{mn}(x,y)\exp(i\int\alpha_{mn}\mathrm{d}x)$ is the amplitude function of the mode $(m\omega_0,n\beta_0)$ [referred to as (m,n)], where β_0 denotes the fundamental spanwise wavenumber, ω_0 the fundamental frequency, and α is the complex-valued streamwise wavenumber. Further, x and y are the streamwise and wall-normal coordinates, respectively. The evolution of each mode is described by the nonlinear PSE as given, e.g., by Bertolotti *et al.*[7] In addition, we use a scaling proposed by Levin and Henningson[8] to modify the PSE to correctly describe the evolution of streaks. These equations are then discretized using a fourth-order compact scheme for the wall-normal derivatives and first- or second-order backward Euler for the streamwise derivatives. It is well known that original PSE suffer from numerical instability for small streamwise step-size. Here, we use the technique proposed by Andersson *et al.*[9] to stabilize the numerical integration. As initial condition for the streak, we use optimal disturbances[10] at the leading edge, which in a linear framework lead to the maximum perturbation energy at a certain downstream position. These are computed with a spectral code used in Ref. 8 based on an

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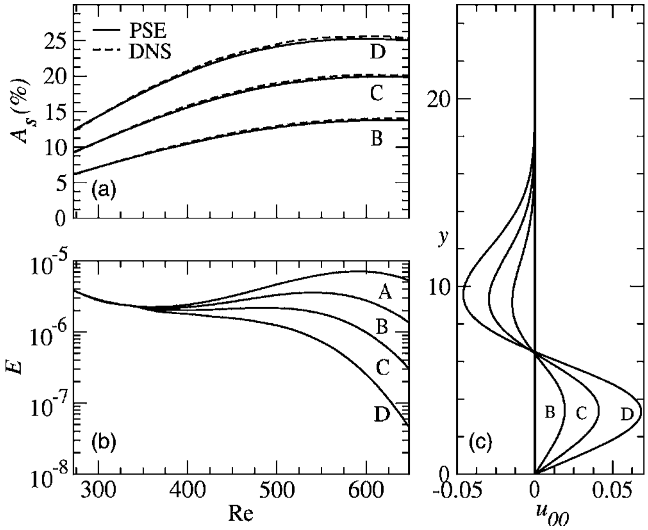

FIG. 1. (a) Comparison of DNS and PSE simulations of the nonlinear downstream development of three streaks with increasing amplitudes. (b) The evolution of the TS-wave in presence of streaks. (c) The mean-flow distortion at Re=640 caused by streaks.

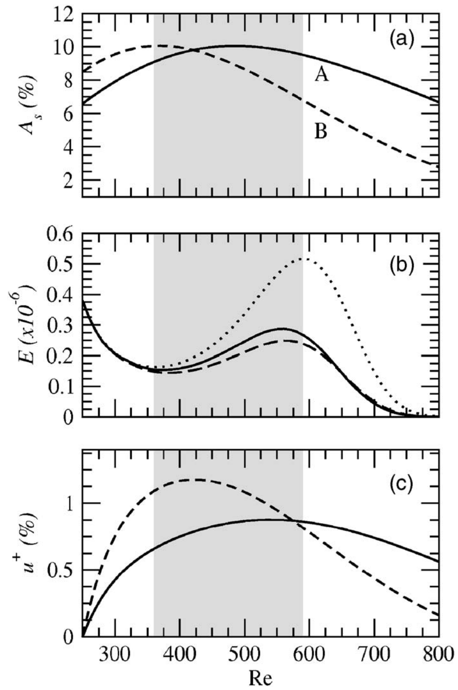

FIG. 2. (a) Evolution of amplitudes of two streaks with β=0.45 (solid) and β=0.65 (dashed). (b) TS-wave with F=131.6 in absence (dotted) and presence (dashed, solid) of streaks. (c) The maximum value of the mean flow distortion caused by the streaks.

adjoint optimization technique described in Ref. 10.

The procedure of the simulations is as follows. An optimal disturbance is initiated close to the leading edge. Its linear downstream development is followed up to a specified streamwise position, where the nonlinear calculations begin by the assignment of an initial amplitude, defined as

$$A_s = \tfrac{1}{2}[\max_{y,z}\{u_s\} - \min_{y,z}\{u_s\}].$$

Here, u_s is the sum of the streamwise velocity component of all $(0,n)$-modes. At this location, a single exponential disturbance is initialized, i.e., (m,n)-mode, with an amplitude sufficiently low to insure its linear behavior. Unless otherwise stated, this location is upstream of the first neutral point of the exponential disturbances at $\mathrm{Re}_0=\sqrt{x_0 U_e/\nu}=250$, where U_e is the streamwise velocity at the edge of the boundary layer and ν the kinematic viscosity. The length scale used here is $\sqrt{\nu x_0/U_e}$. Usually, 20–30 modes were sufficient to correctly describe the evolution of the disturbances.

Validation. The results obtained using the procedure described above, are verified against the direct numerical simulations of Cossu and Brandt.[2] As in Ref. 2, we consider the instability of a TS-wave of frequency $F=(\omega_0/\mathrm{Re})\times 10^6 = 131.6$ in the presence of a set of streaks $(\beta_0/\mathrm{Re}=6.36\times 10^{-4})$ with different amplitudes [Fig. 1(a)]. The initial profiles of the streaks are optimized for maximum growth at Re=707 and the nonlinear calculations begin at Re_0=272. As reported in Cossu and Brandt[2] and shown in Fig. 1(b), the stabilizing effect on the TS-wave is observed for all streak amplitudes. Here, the following norm of the disturbance $E=(\int_0^\infty \mathbf{u}\cdot\mathbf{u}^* dy)^{1/2}$, is used as a measure of the TS-wave size. In Fig. 1(b), case A corresponds to zero streak amplitude. For moderate streak amplitudes (B, C) a damping of the growth of the TS-wave is observed, whereas for a sufficiently large streak amplitude (D) the TS-wave is completely stabilized. In Fig. 1(c), the mean-flow distortion u_{00}, i.e., streamwise velocity component of the $(0,0)$-mode, is shown. This is induced by streaks and it modifies the velocity profile into a "fuller" shape close to the wall. This seems to be the main mechanism behind the stabilization effect of the streaks.[2]

Effects of the spanwise wavenumber of the streak. Previous studies[2–4] have focused solely on the effects of the streak amplitude. As the development of streaks also depends on its spanwise wavelength, it is of interest to investigate its effects on TS-wave instability. Therefore, we vary the spanwise wavenumber of streaks in the range [0.1, 1]. The initial profiles of these streaks are optimized for maximum growth at Re=400. Assigning the same initial amplitude for each of them results in streaks with different maximum amplitudes. Since the stabilizing effect depends strongly on the streak amplitude, it is difficult to draw a definite conclusion about the significance of different values of β. Therefore, it seems reasonable to compare streaks with different β but same maximum amplitude. Here, for each streak we choose an appropriate initial amplitude such that the *maximum* amplitudes, i.e., A_s^*, of each of them is 10% of the free-stream velocity. To illustrate the effect of streak parameter β, we begin by investigating the stability of a two-dimensional TS-wave with frequency F=131.6 in the presence of the two streaks shown in Fig. 2(a). The streaks A and B have the spanwise wavenumbers, i.e., β_A=0.45 and β_B=0.65, respectively, and fixed maximum amplitude A_s^*=10%. In Fig. 2(b), we show that in the absence of streaks the TS-wave (dotted line) grows exponentially (with a rate predicted by the linear theory) as it enters the unstable domain at branch I, the shaded domain, and decays as it is propagated downstream away from the domain. In the presence of streaks a damping effect is observed, which is larger for streak B (dashed line)

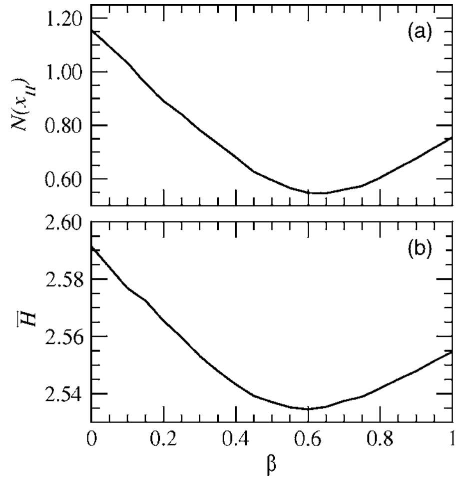

FIG. 3. (a) The N-factor at branch II of the TS-wave ($F=131.6$) as a function of the spanwise wavenumber of streaks. The maximum amplitudes of the streaks have been fixed at $A_s^*=10\%$. (b) The averaged shape factor $\bar{H}$ as a function of the same set of streaks as in (a).

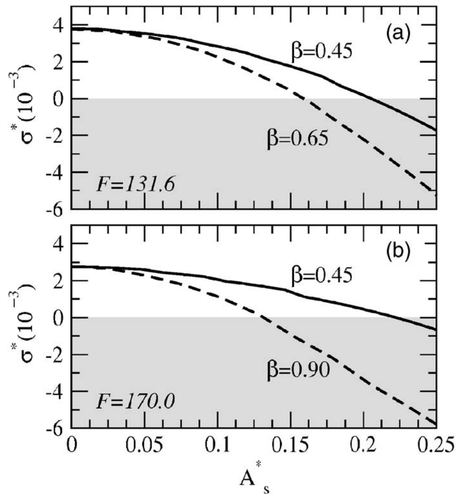

FIG. 4. The maximum growth rate σ^* of TS-waves, $F=131.6$ in (a) and $F=170$ in (b), in the presence of streaks.

than for streak A (solid line), despite the fact that streak A maintains a *larger* amplitude in the most part of the unstable domain of the TS-wave. Streak B, on the other hand, attains its maximum amplitude close to the location of branch I of the TS-wave, and then rapidly decays downstream. This can be explained if the distortions of the mean flow, i.e., $(0,0)$-mode, induced by these two streaks are compared. In Fig. 2(c), the development of the maximum mean-flow distortion, i.e., $u^+=\max_y\{u_{00}\}$, for streaks A and B is shown. It is apparent that streak B modifies the flow considerably more than streak A, between branches I and II, due to larger values of u^+. This is caused by the larger amplitude of streak B upstream of branch I.

As a measure of the amplification of the TS-waves, we compute the N-factor, defined as $N(x)=\ln[E(x)/E(x_I)]$. In Fig. 3(a), $N(x_{\mathrm{II}})$ for the TS-wave with $F=131.6$ is plotted as a function of the spanwise wavenumber β of the streaks. Here, the maximum streak amplitudes are kept constant; i.e., $A_s^*=10\%$. As shown in Fig. 3(a), $N(x_{\mathrm{II}})$ attains a minimum value for $\beta\approx 0.65$. This indicates that there exists an optimal streaky boundary layer, when the objective is to minimize the amplification of the TS-wave. It should be mentioned that, due to nonlinear effects, there is a slight upstream shift of the location of the A_s^* with increasing initial streak amplitude [see Fig. 1(a)]. Therefore, the "optimal" β depends weakly on the streak amplitude. In order to relate the total modification of the mean flow caused by streaks, to their stabilization effects we compare the N-factor with the averaged shape factor $\bar{H}$. Here, $\bar{H}$ is averaged in the streamwise direction between branches I and II of the TS-wave. In Fig. 3(b), $\bar{H}$ is plotted as a function of β. In the absence of streaks, the shape factor of a Blasuis profile is $\bar{H}=2.59$, whereas in the presence of streaks $\bar{H}$ is smaller, indicating a fuller velocity profile. Furthermore, $\bar{H}$ attains a minimum value in the presence of streaks with $\beta=0.6$, i.e., close to the β that minimizes the N-factor of the TS-wave [shown in Fig. 3(a)]. This indicates that the commonly used streak with $\beta=0.45$ is not the most efficient stabilizing streak. This value of β corresponds to the vortices generated at the leading edge which experience the largest linear growth.[10] We have performed the same parametric study of β for two other frequencies: $F=170$ and 90. For both frequencies, the N-factor, i.e., $N(x_{\mathrm{II}})$, attains a minimum at approximately the spanwise wavenumber (0.9 and 0.45, respectively) for which the streamwise averaged shape factor is the smallest. Again, the streak that is the most efficient for stabilizing a TS-wave attains its maximum amplitude close to branch I of that TS-wave.

Now we aim at finding the minimum streak amplitude necessary for the complete stabilization of a TS-wave. We consider two different streaks: the optimal growing streak ($\beta=0.45$) and a streak with $\beta=0.65$, chosen such that the maximum streak amplitudes are close to branch I of the given TS wave. The maximum amplitudes are varied between 0% and 25%, and the maximum growth rates $\sigma^*=\max_x\{\sigma\}$ of the TS-wave are computed for each streak. The physical growth rates are calculated from the relation[7] $\sigma=-\alpha_i+(\partial/\partial x)\ln(E)$. When $\sigma^*<0$ the TS-wave is completely stabilized. For complete stabilization of a TS-wave with $F=131.6$, the necessary amplitude of the streak with $\beta=0.65$ is $A_s=15\%$, whereas for $\beta=0.45$ the corresponding amplitude is $A_s=20\%$ [see Fig. 4(a)]. For $F=170$, the necessary amplitude is reduced from $A_s=22\%$ to $A_s=0.12\%$, when β is increased from 0.45 to 0.9 [see Fig. 4(b)]. As the TS-wave frequency is decreased, the location of branch I moves downstream and consequently streaks with smaller β are required to stabilize the flow.

Stabilization of oblique waves. The focus of previous investigations[2,3] has been on reducing the linear growth of two-dimensional TS-waves, as these disturbances are the first to become unstable in a Blasius boundary layer. However, certain transition scenarios,[7] require the existence of oblique waves. Here, we choose two unstable oblique waves with frequency $F=131.6$ and spanwise wavenumbers $\beta_0=0.09$ and 0.1123, respectively. For these values of β_0, a streak with a spanwise wavenumber $\beta=0.45$ is initiated at Re_0

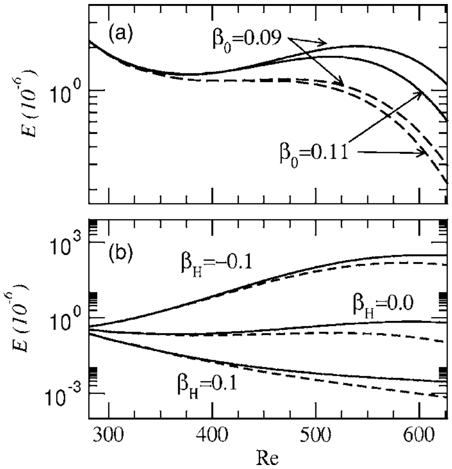

FIG. 5. (a) The downstream development of oblique waves ($F=131.6$) in the absence (solid) and the presence (dashed) of streaks. (b) The downstream development of a TS-wave ($F=131.6$) in the absence (solid) and the presence (dashed) of streaks in boundary layer with adverse, zero and favorable pressure gradient.

$=272$ as modes $(0,5\beta_0)$ and $(0,4\beta_0)$, respectively. The oblique disturbances are initiated as a pair of modes $(1,\pm 1)$ with sufficiently small amplitude to insure a linear behavior. The results are shown in Fig. 5(a), where we compare the norm E of the oblique waves in the presence (dashed line) and the absence (solid line) of a streak with the maximum amplitude $A_s^*=10\%$. Similar to TS-waves, the linear growth of the oblique waves is found to be damped when streaks are present.

Effects of pressure gradient. We have also investigated the effects of streaks on the linear growth of exponential disturbances in boundary-layer flows with pressure gradients. In particular, boundary layers with free-stream velocities given as $U_e=U_\infty x^m$, $m=\beta_H/(2-\beta_H)$, where β_H is the Hartree parameter. In Fig. 5(b), the evolution of a TS-wave with frequency $F=131.6$ in boundary layers with favorable ($\beta_H=0.1$), zero ($\beta_H=0$), and adverse pressure ($\beta_H=-0.1$) gradients are shown by the solid lines. By introducing a streak at $\mathrm{Re}_0=278$ with spanwise wavenumber $\beta=0.45$ and amplitudes $A_s\approx 13\%-16\%$, the growth of TS-waves is damped (shown by dashed lines).

Conclusions. We have found that the stabilization effect of streaks on the linear growth of TS-waves in Blasius boundary layer, observed in previous studies, to also apply to three dimensional disturbances and Falkner-Skan boundary-layer flows. We have also found that by distributing the streaks "optimally" in the spanwise direction, it is possible to completely stabilize a TS-wave, with considerably lower streak amplitudes. For the TS-waves with high frequencies, a reduction of the maximum streak amplitude by a factor of almost 2 can be achieved. The streaks which most efficiently reduce the growth rate of a given disturbance attain their maximum amplitudes close to the branch I of that disturbance. These streaks generate a "fuller" velocity profile in the unstable domain of the TS-waves. By computing the streamwise averaged shape factor of the modified boundary layer, one can estimate the stabilization effect of streaks without actually calculating the interaction with the targeted TS (or oblique) waves. It should also be mentioned that the optimal growing streak, often associated with the spanwise wavenumber $\beta=0.45$, is not the most efficient one to suppress TS-waves of all frequencies.

We wish to thank Luca Brandt, Carlo Cossu, Phillip Schlatter and Dan Henningson for their helpful comments.

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