Model reduction and feedback control of the Blasius boundary-layer

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APS/DFD07 Meeting
Salt-Lake city, Utah, USA
Nov 18-20, 2007

Motivation and aim of investigation

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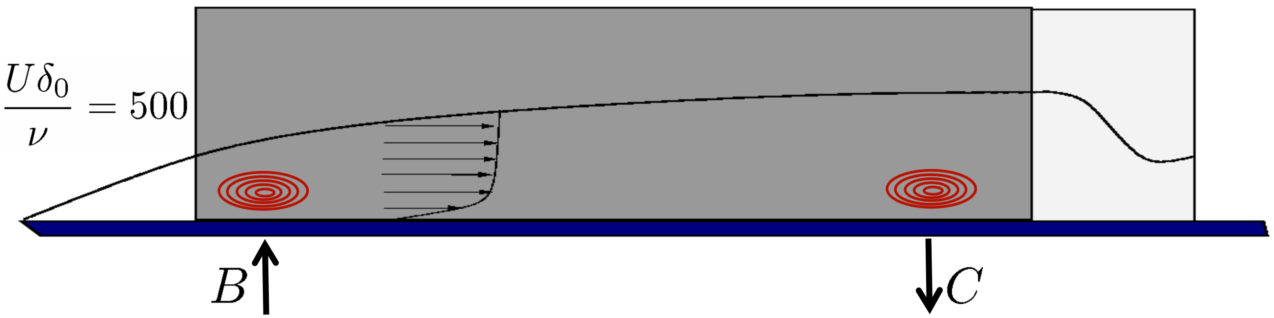
- Discretized Navier-Stokes equations too complex for modern control design (H_2/H_∞)
  - $n>10^5$ degrees of freedom
- Model reduction: approximate the high-dimensional system with a low-dimensional system
  - $m<100$ degrees of freedom
  - How can we find a low-dimensional model that describes the input-output behaviour?

  ➔ balanced truncation
- Objective: Design a reduced feedback controller (H_2/H_∞) to reduce the growth of linear perturbations and eventually delay transition to turbulence.

State-space formulation of Navier-Stokes

$$Re = \frac{U\delta_0}{\nu} = 500$$



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- Linearized Navier-Stokes in state-space formulation:

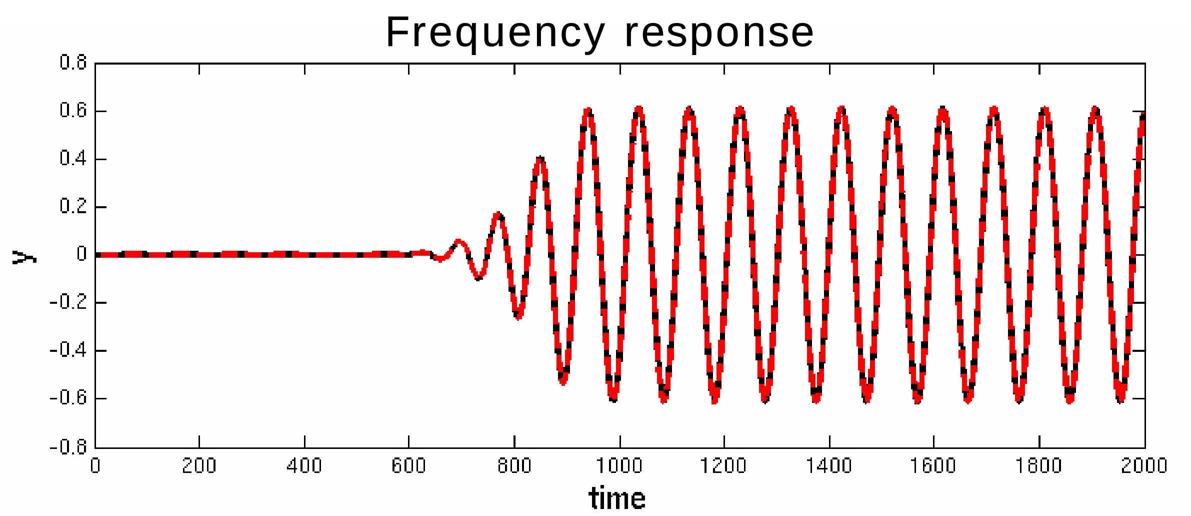
$$\begin{aligned} \dot{q} &= Aq + Bu \quad q(0) = 0 \\ y &= Cq \end{aligned}$$

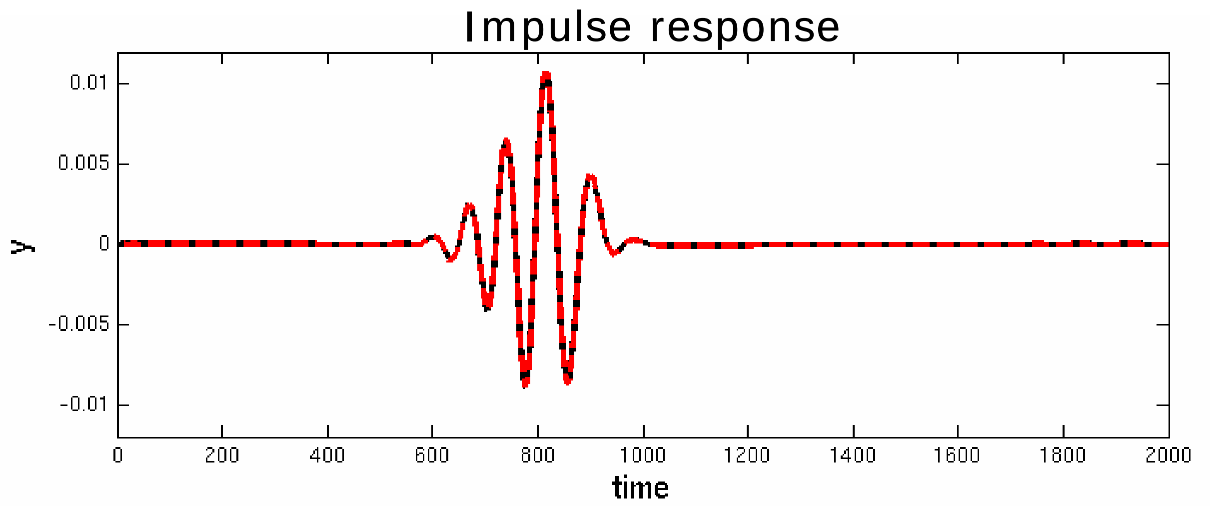
- For control design must analyze:
  - Perturbations → Cost function
  - Perturbations → Sensors
  - Actuators → Cost function
  - Actuators → Sensors

- Laplace transform to frequency domain:

$$y = \underbrace{C(sI - A)^{-1}B}_{G(s)}\, u$$

Input-output behavior of 2D Blasius flow





DNS: $n=10^5$

ROM: $m=50$

Snapshot-based balanced truncation

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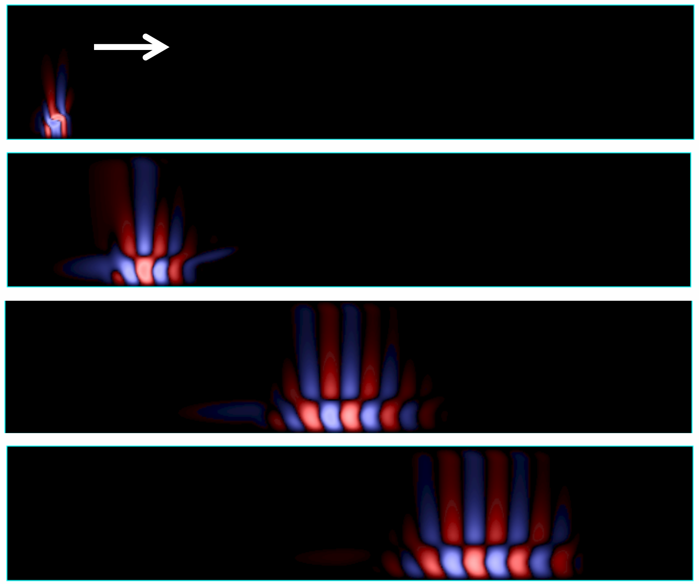
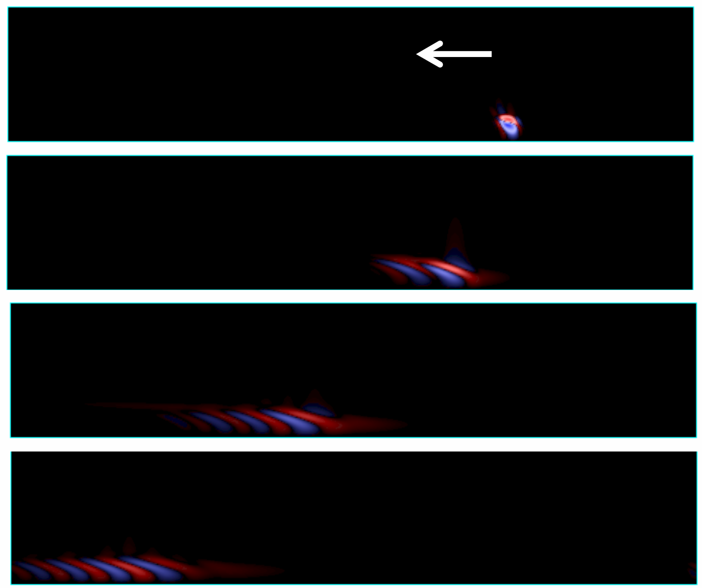
Snapshots of direct simulation:

$$X = [q(t_1) \dots q(t_m)]\Delta_t$$



Snapshots of adjoint simulation:

$$Y = [q^+(t_1) \dots q^+(t_m)]\Delta_t$$

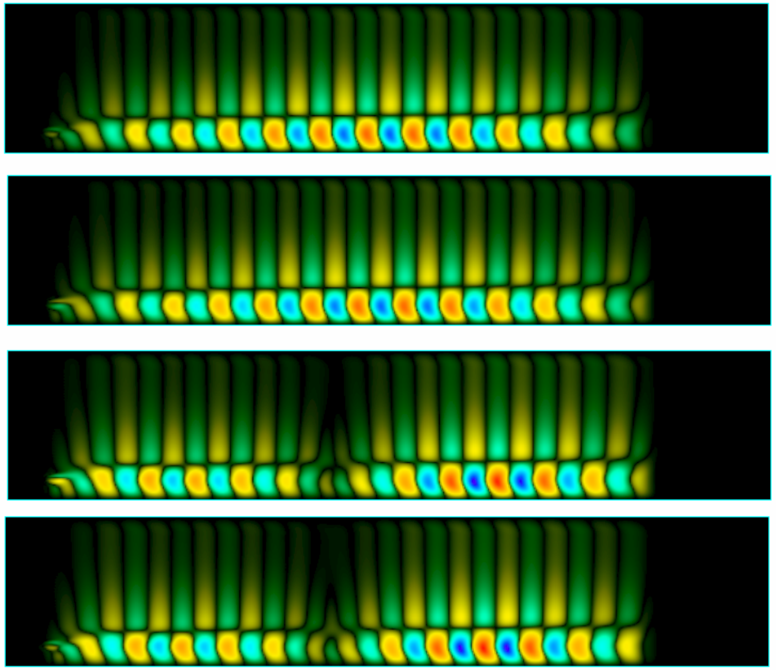


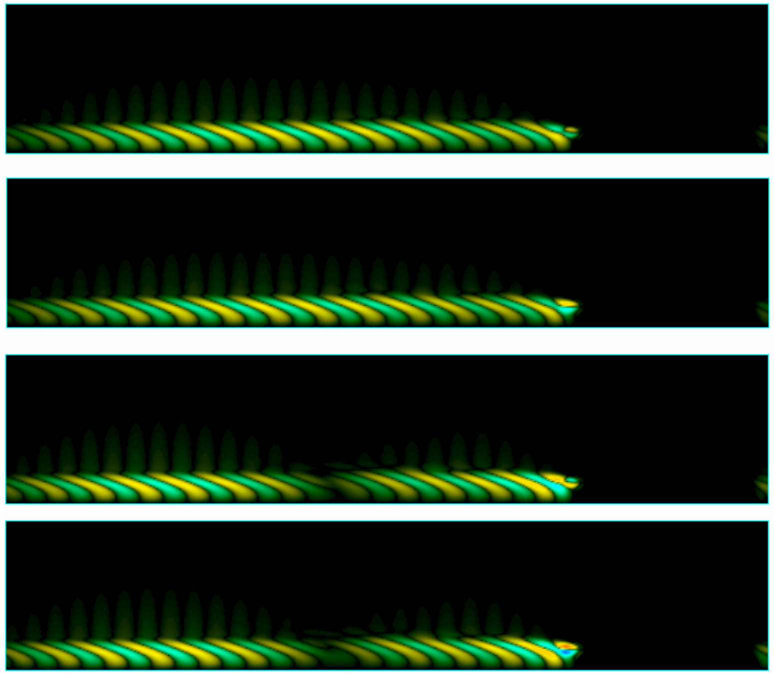
One SVD of small system $Y^T X = U \Sigma V^T$
(Rowley 2005)

Balanced modes as expansion basis

- One SVD of small system: $Y^T X = U \Sigma V^T$
- Balanced modes and adjoint balanced modes:

$T = [T_1 \dots T_m] = X V \Sigma^{-1/2}$ $\quad$ $S = [S_1 \dots S_m] = Y U \Sigma^{-1/2}$





- Projection on balanced modes to obtain reduced system:

$$\begin{aligned} \dot{\hat{q}} &= S^T A T \hat{q} + S^T B u \\ y &= C T \hat{q} \end{aligned}$$

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Model reduction error

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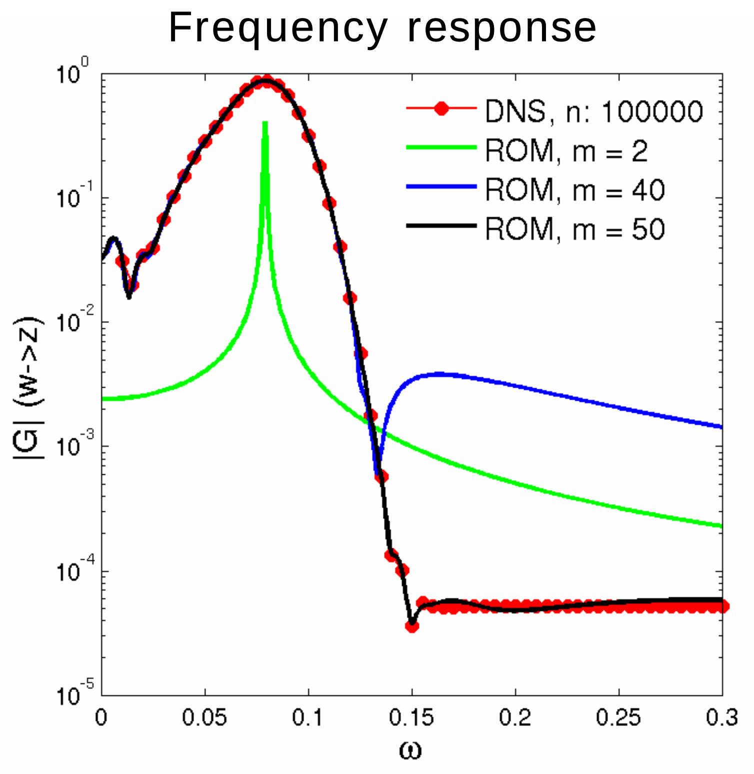
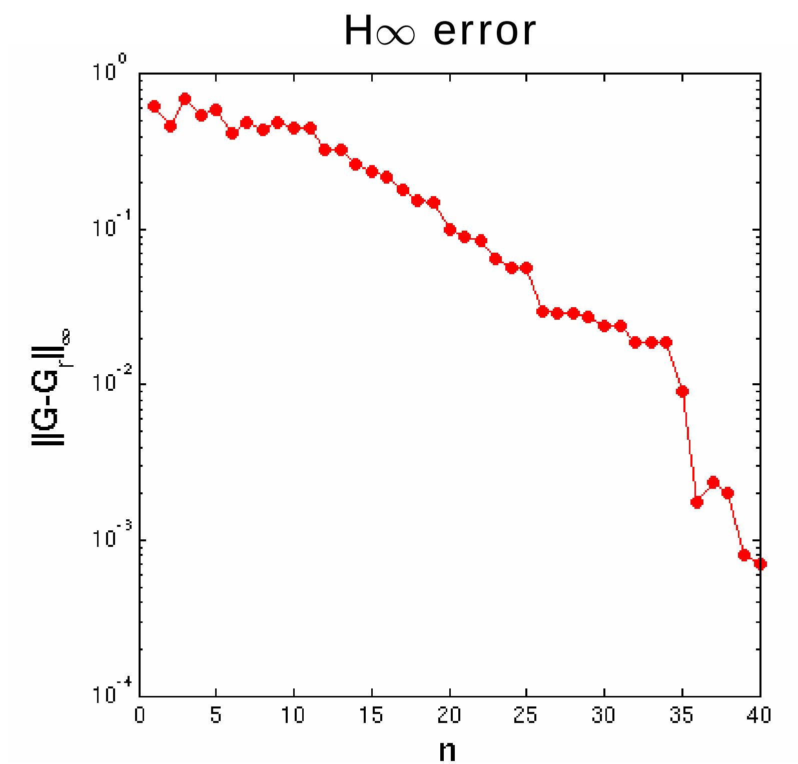
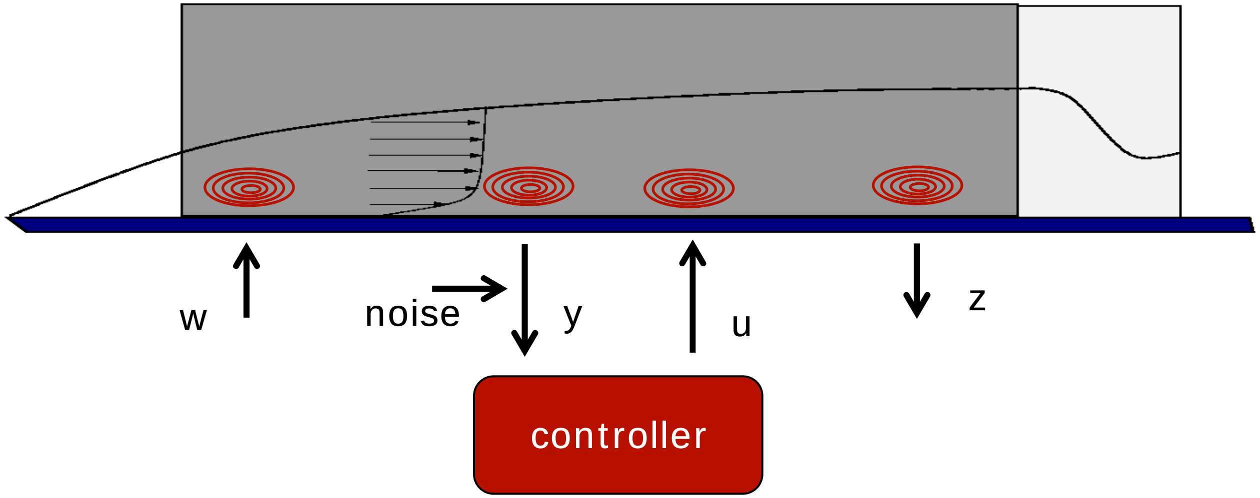




- The reduced model
  - m = 2 has its peak value for the same frequency as the full model.
  - m = 50 gives nearly the exact frequency response.
- Fast decay of model reduction error $\|G - G_r\|_\infty$

Optimal feedback control – H_2



- H_2 Problem:
  - Find a control signal **u(t)** based on the measurements **y(t)** such that the influence of external disturbances **w(t)** and **g(t)** on the output **z(t)** is *minimized*.

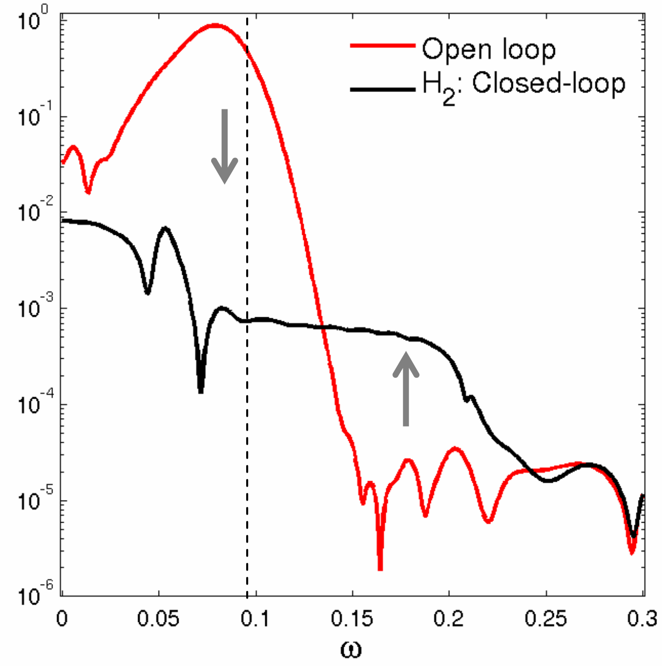
- Control Objective:

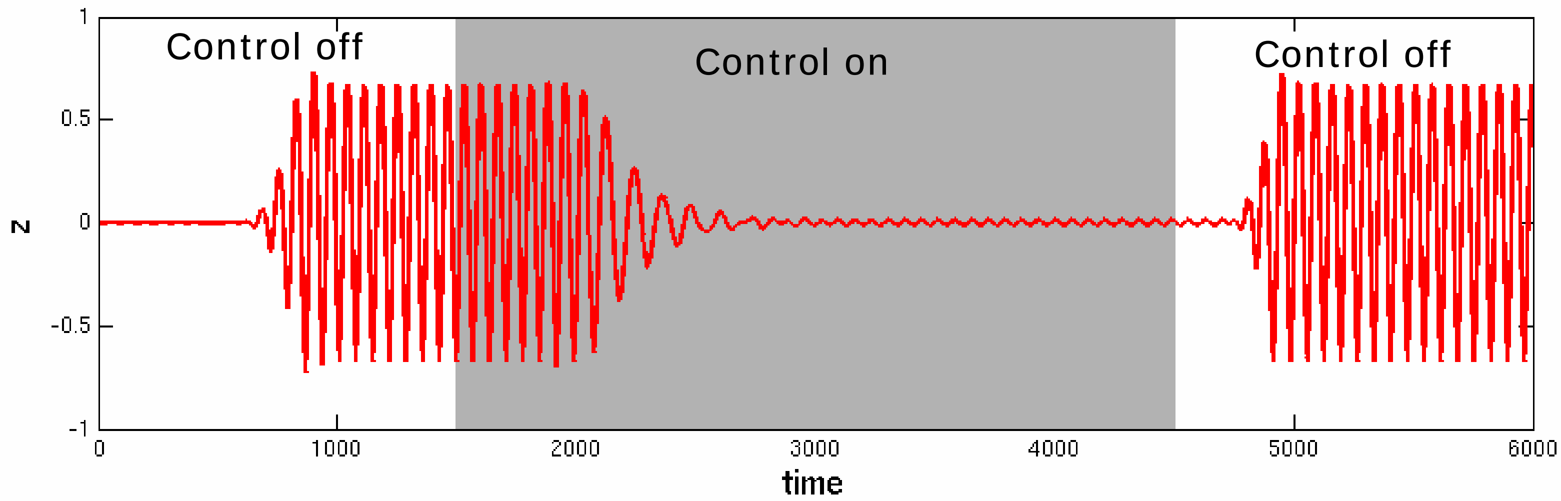
$$\mathcal{J} = \|z\|_2^2 = \int_0^T q^T C_1^T C_1 q + l^2 u^T u \; \mathrm{d}t$$

Closed-loop: harmonic forcing

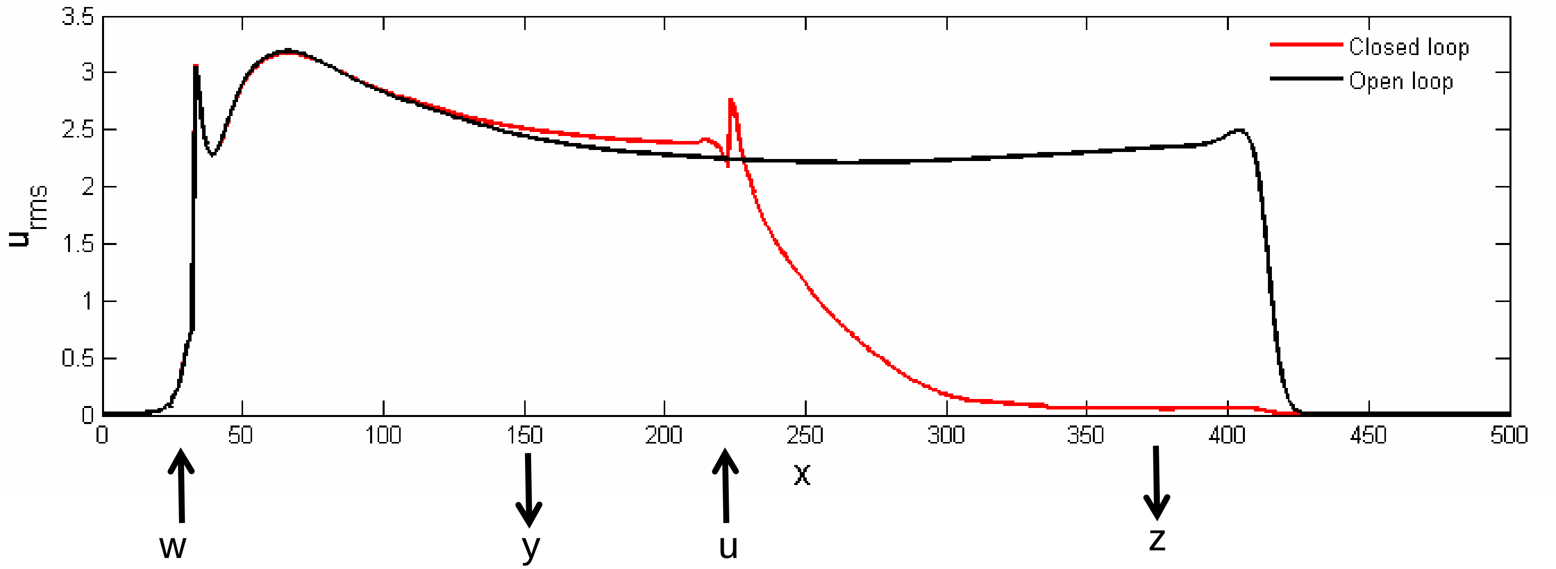
- Control design parameters:
  - control penalty $l=10^{-4}$
  - noise contamination $\alpha=0.01$

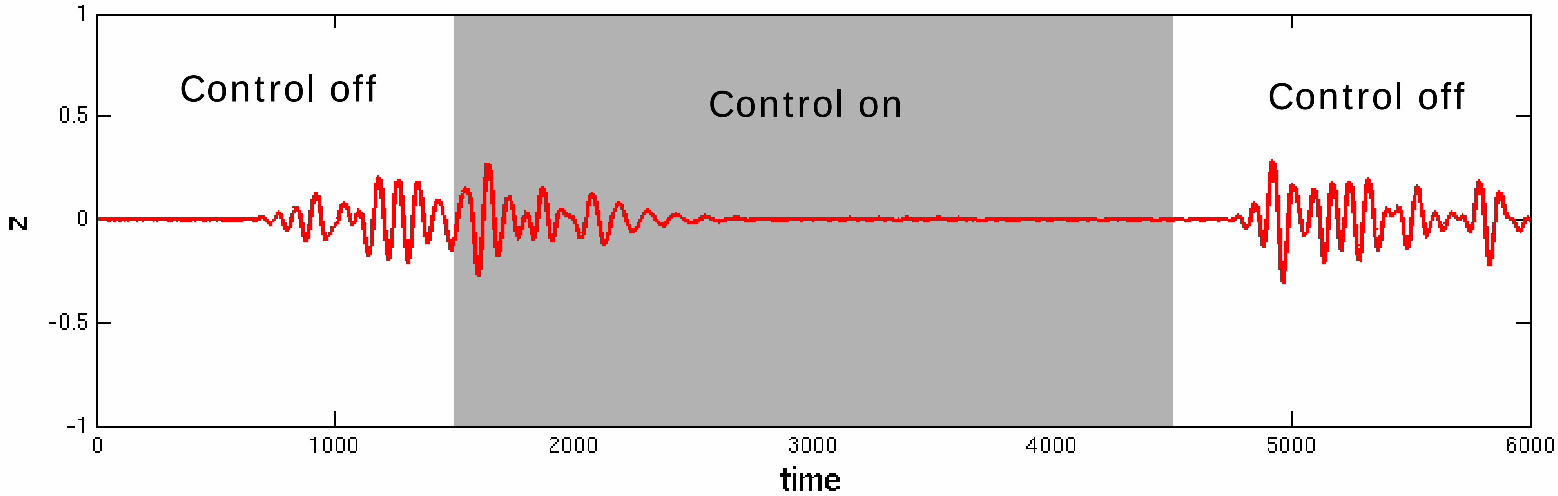
- The closed-loop:
  - dampens the dominant frequencies
  - amplifies certain high frequencies.





Closed-loop: stochastic forcing





Conclusions and outlook

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- The reduced balanced system with less than 50 dimensions captures in the input-output behavior of the 2D flat plate with 100 000 dimensions.
- The reduced model can be used to design modern control strategies, such as H_2 and H_∞ to efficiently damp the linear growth of disturbances.
- Extend to 3D Blasius boundary layer.
- Realistic actuators (blowing and suction at the wall), sensors (skin friction), disturbances (free-stream turbulence)...
- Delay transition to turbulence and perform experiments!