Global Stability of a Jet in Crossflow

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Jet in Crossflow

- Is the flow linearly globally unstable?
- What is the type of instability?

\[ R = \frac{V}{U} \]
History & Applications

- Previous work:
  - Smith & Mungal (1996)
  - Kelso et al. (1996)
  - Fric & Roshko (1994)

- Industrial applications:
  - V/STOL
  - Smoke stacks
  - Fuel injection/ Film cooling

Counter-rotating vortices
Shear layer vortices
Horseshoe & Wake vortices
Stability Analysis via timestepping technique

1. Simulate flow with DNS: Identify structures and regions
   \[ u(x, t) = T u_0(x) \]

2. Compute baseflow: Steady-state solution
   \[ u_s(x) = T u_s(x) \]

3. Compute impulse response of baseflow: Globally unstable
   \[ u(x, t) = T_{linear}(u_s) u_0(x, t) \]

4. Compute global spectrum of baseflow: Growth rates/ frequency
   \[ u(x) \lambda = T_{linear}(u_s) u(x) \]
Observations from DNS

- **DNS:**
  - Fully spectral (Fourier/Chebychev) & parallelized (MPI/OpenMP)
  - Parabolic jet profile imposed as boundary condition
  - $R=3$ & $Re=165$

- **Unsteady structures:**
  - Shear layer vortices
  - Wake vortices

$\lambda_2$ Vortex identification criterion
Three-Dimensional Base Flow

- **SFD:**
  - Selective frequency damping (Åkervik et al. 2006)
  - Solution of the steady Navier-Stokes eqs
  - Alternative to Newton’s Method
  - Damp unstable frequencies

- **Steady structures:**
  - Counter-rotating vortex pair (CVP)
  - Horseshoe vortices

\[ \lambda_2 \text{ Vortex identification criterion} \]

![Probe 1: Shear layer](image.png)
Linear Impulse Response

- **Initial pulse:**
  - Gaussian type inside boundary-layer upstream of jet

- **Response of base flow:**
  - Formation wave packet traveling on shear layer

- **Linearly globally unstable**
  - Asymptotic energy growth of perturbation

![Graph showing perturbation energy over time](image)

- Perturbation energy vs. time
- Steady state
- Perturbation at $t=0$ and $t=24$
Global Eigenmodes

- **Timestepper approach:**
  - Matrix-free & Arnoldi method: DNS + ARPACK Library
  - Three inhomogenous directions
  - Storage of Jacobian matrix: 360 Terabyte

- **Global eigenmodes:**
  - Localized wavepackets wrapped around CVP

Low-frequency mode

High-frequency mode
Global Spectrum

- 20 first Global eigenmodes:
  - Highly unstable
  - Shear-layer modes
  - Fully three-dimensional
Global Spectrum

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Outlook & Conclusions

- We found self-sustained synchronized oscillations at R=3:
  - Observed in Direct Numerical Simulation
  - Linear impulse response of steady-state base
  - Linear 3D global stability analysis

- Future work:
  - Bifurcation analysis: Find critical velocity ratio
  - Sensitivity to forcing (adjoint global modes)
  - Optimal disturbances