Global Stability of a Jet in Crossflow

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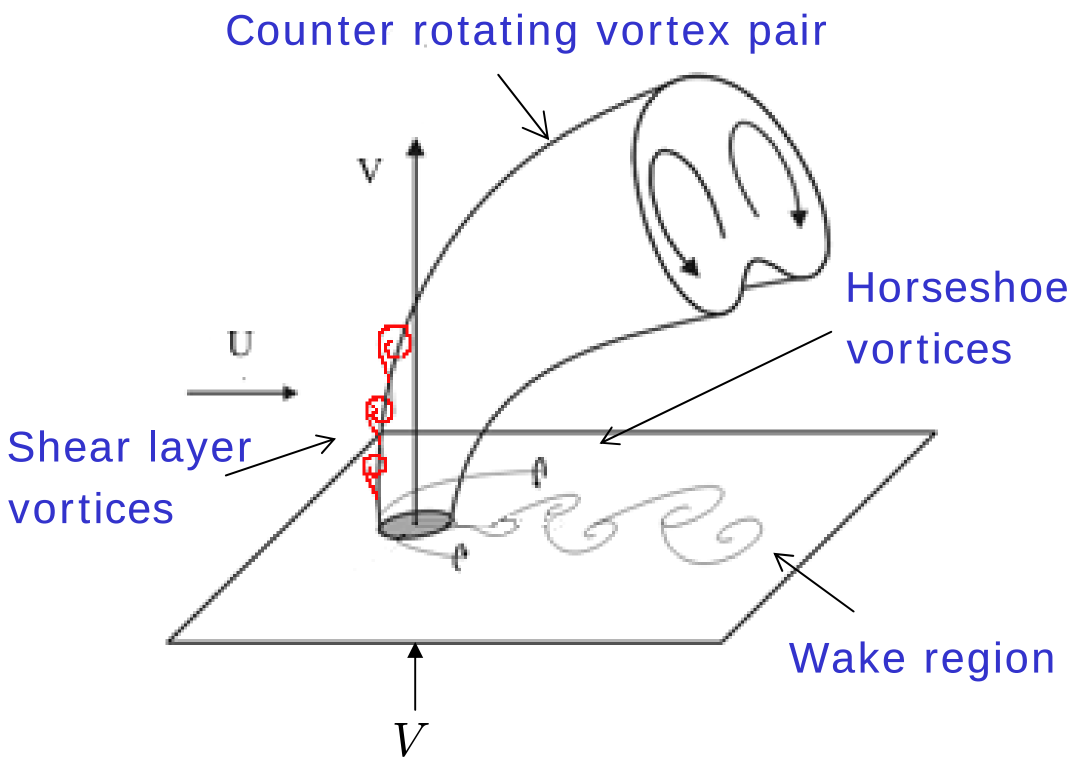
7th Euromech Fluid Mechanics Conference
University of Manchester
September 14-18, 2008

Jet in Crossflow



$$R = \frac{V}{U}$$

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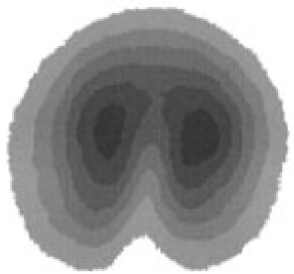
- Is the flow linearly globally unstable?
- What is the type of instability?

History & Applications

- Previous work:

Smith & Mungal *(1996)*

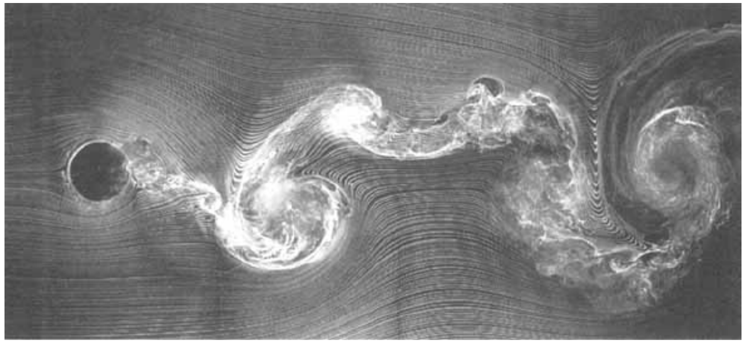


Counter-rotating vortices

Kelso *et al. (1996)*

Shear layer vortices

Fric & Roshko *(1994)*



Horseshoe & Wake vortices

- Industrial applications:

V/STOL

Smoke stacks

Fuel injection/ Film cooling

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Stability Analysis via timestepping technique

1. Simulate flow with DNS: Identify structures and regions

$$\mathbf{u}(\mathbf{x},t) = \mathcal{T}\mathbf{u}_0(\mathbf{x}) \qquad \mathbf{x} = (x,y,z)$$

2. Compute baseflow: Steady-state solution

$$\mathbf{u}_s(\mathbf{x}) = \mathcal{T}\mathbf{u}_s(\mathbf{x})$$

3. Compute impulse response of baseflow: Globally unstable

$$\mathbf{u}(\mathbf{x},t) = \mathcal{T}_{linear}(\mathbf{u}_s)\mathbf{u}_0(\mathbf{x},t)$$

4. Compute global spectrum of baseflow: Growth rates/ frequency

$$\mathbf{u}(\mathbf{x})\lambda = \mathcal{T}_{linear}(\mathbf{u}_s)\mathbf{u}(\mathbf{x})$$

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Barkley *et al.* (JFM, 2002)

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Observations from DNS

- DNS:
  - Fully spectral (Fourier/Chebychev) & parallelized (MPI/OpenMP)
  - Parabolic jet profile imposed as boundary condition
  - R=3 & Re=165
- Unsteady structures:
  - Shear layer vortices
  - Wake vortices

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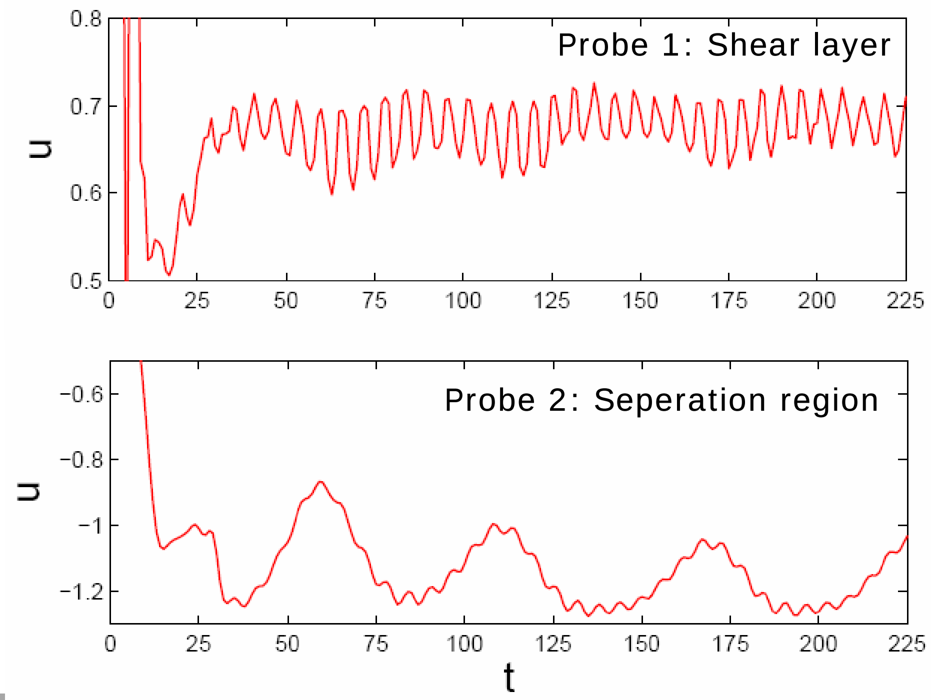
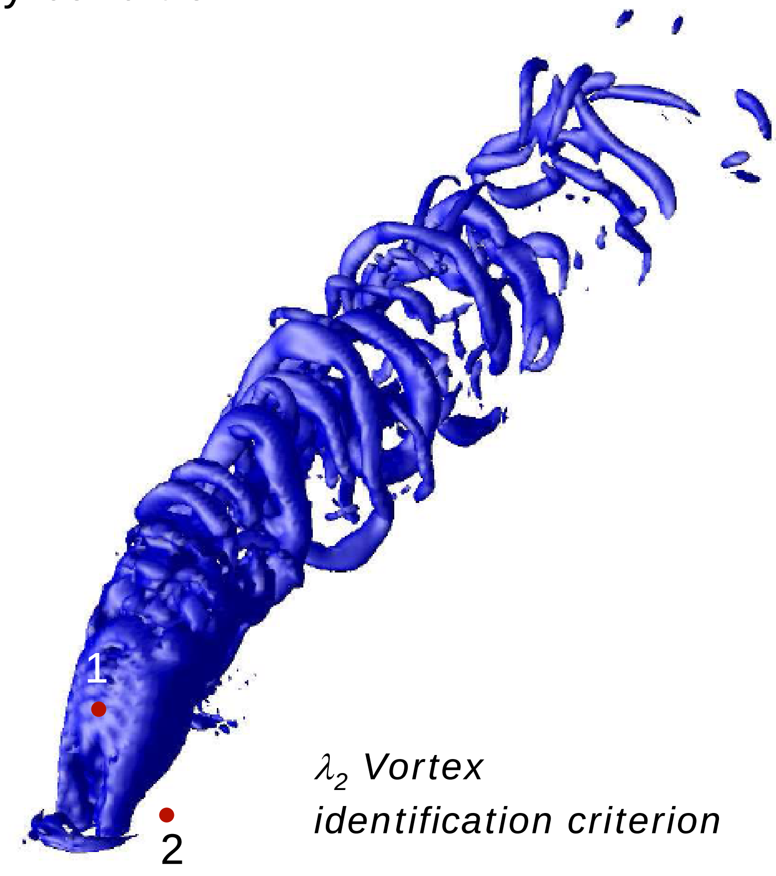


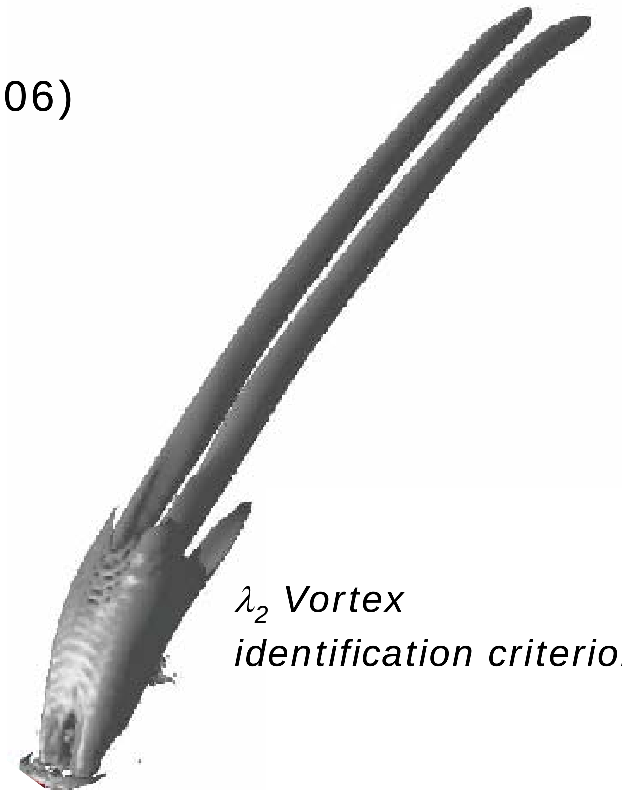


λ_2 Vortex identification criterion

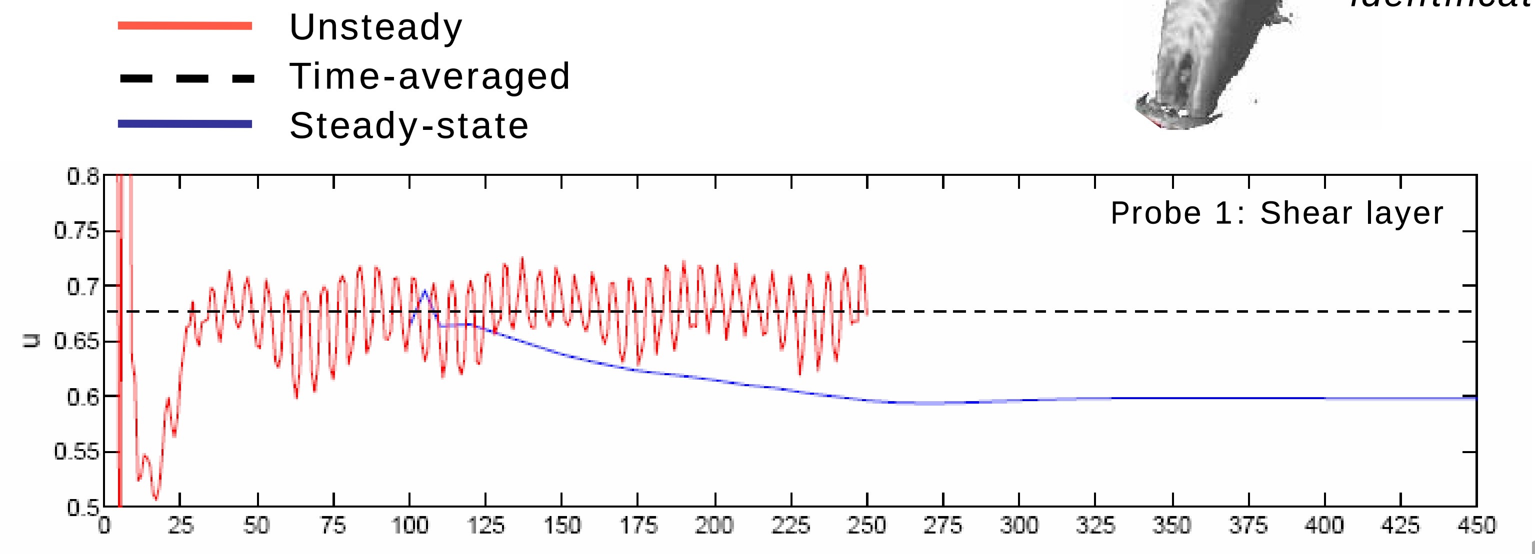
Three-Dimensional Base Flow

- SFD:
  - Selective frequency damping (Åkervik et al. 2006)
  - Solution of the steady Navier-Stokes eqs
  - Alternative to Newton's Method
  - Damp unstable frequencies

- Steady structures:
  - Counter-rotating vortex pair (CVP)
  - Horseshoe vortices



λ_2 *Vortex identification criterion*



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Linear Impulse Response

- Initial pulse:
  - Gaussian type inside boundary-layer upstream of jet
- Response of base flow:
  - Formation wave packet traveling on shear layer
- Linearly globally unstable
  - Asymptotic energy growth of perturbation

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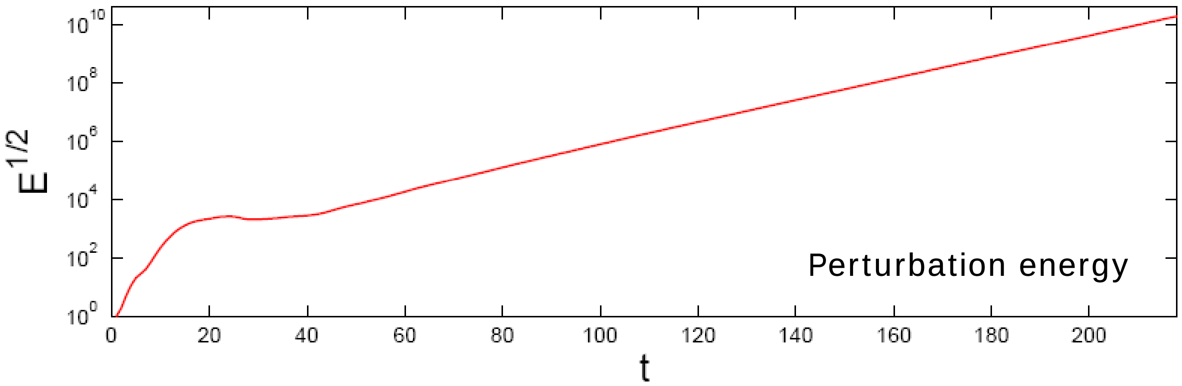
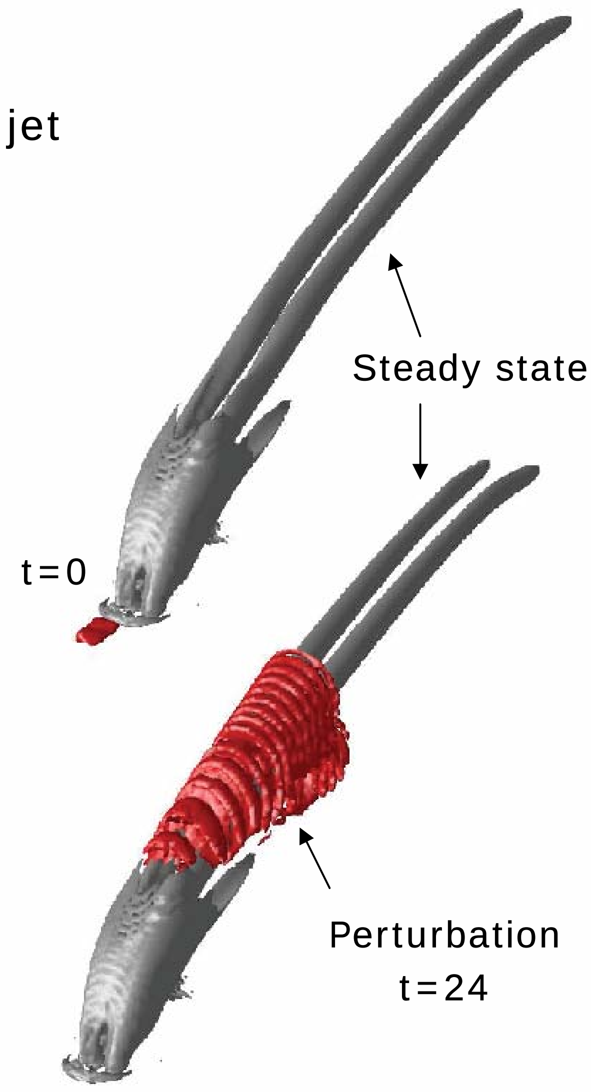
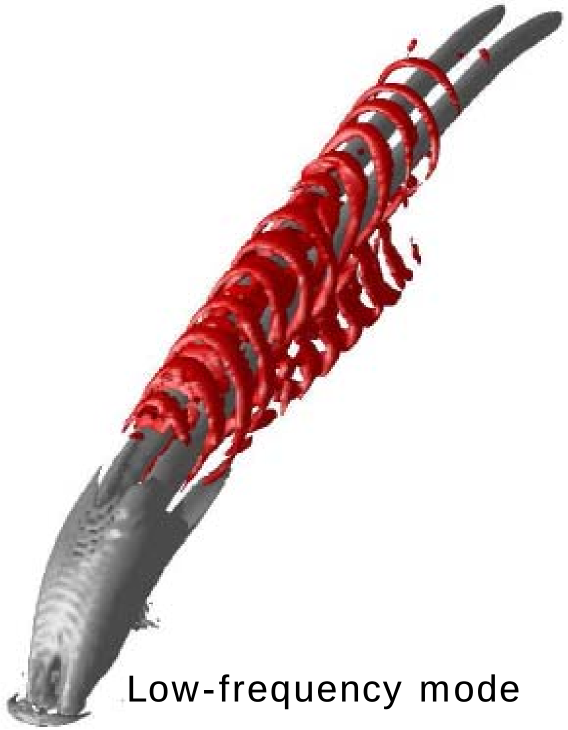




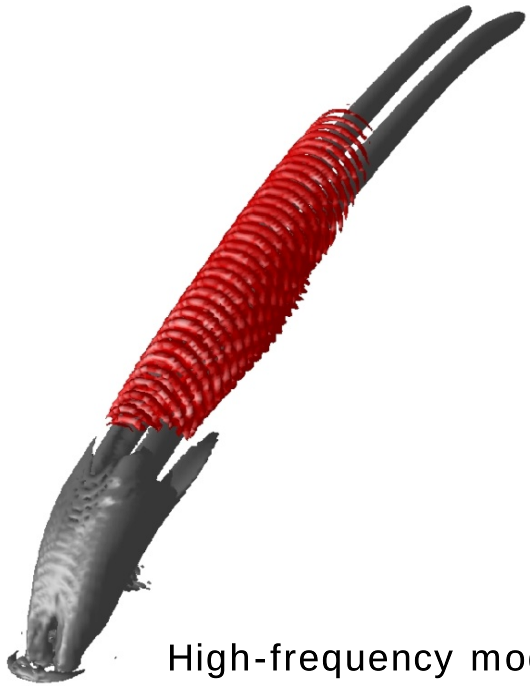
Global Eigenmodes

- Timestepper approach:
  - Matrix-free & Arnoldi method: DNS + ARPACK Library
  - Three inhomogenous directions
  - Storage of Jacobian matrix: 360 Terabyte

- Global eigenmodes:
  - Localized wavepackets wrapped around CVP



Low-frequency mode



High-frequency mode

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Global Spectrum

- 20 first Global eigenmodes:
  - Highly unstable
  - Shear-layer modes
  - Fully three-dimensional

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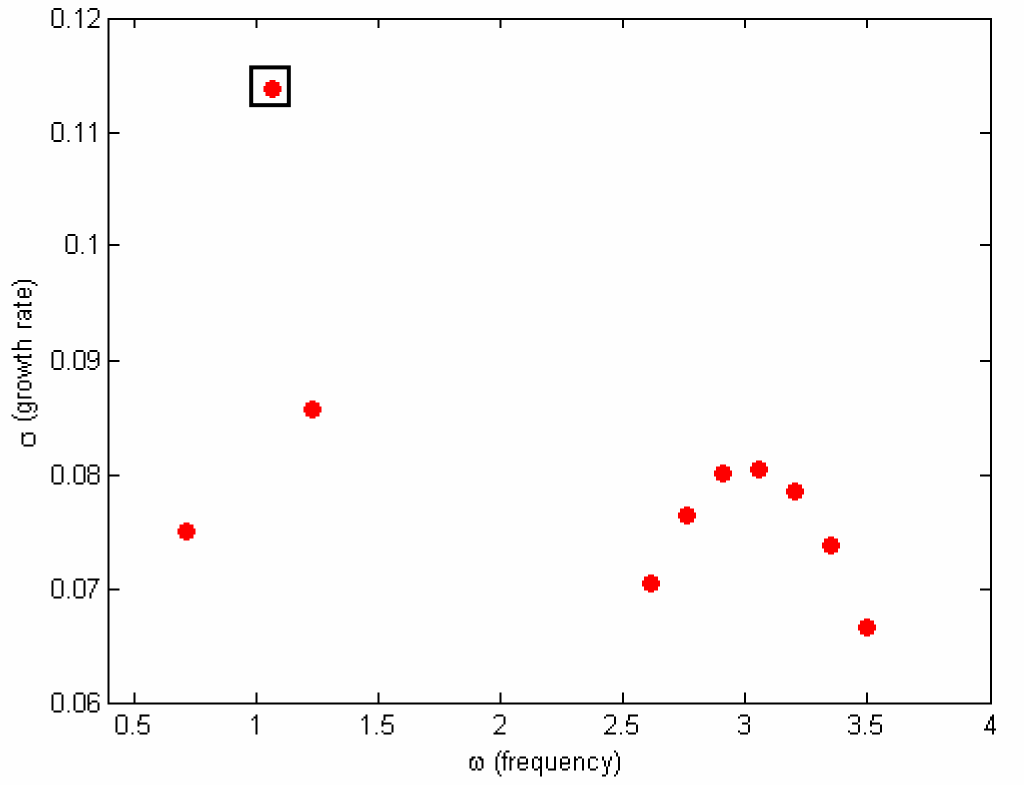

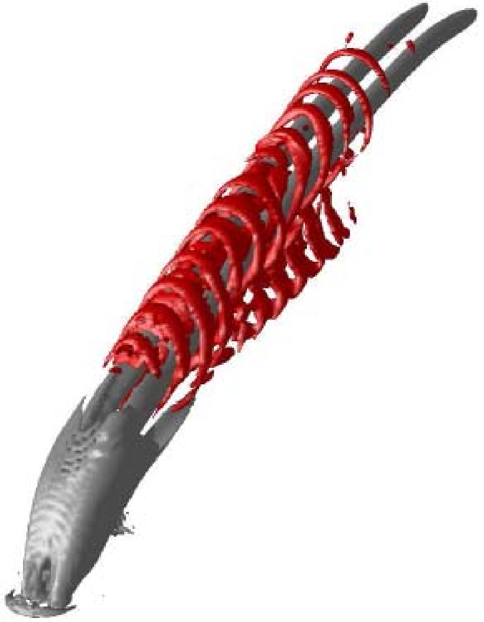

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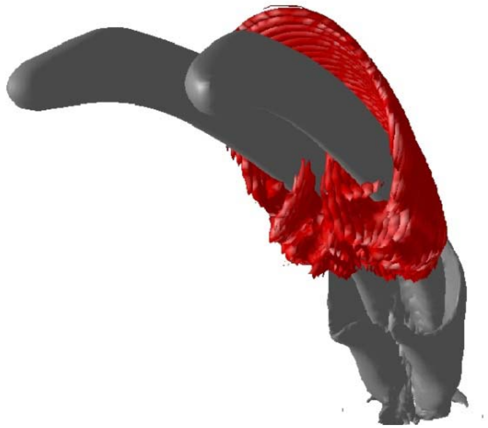

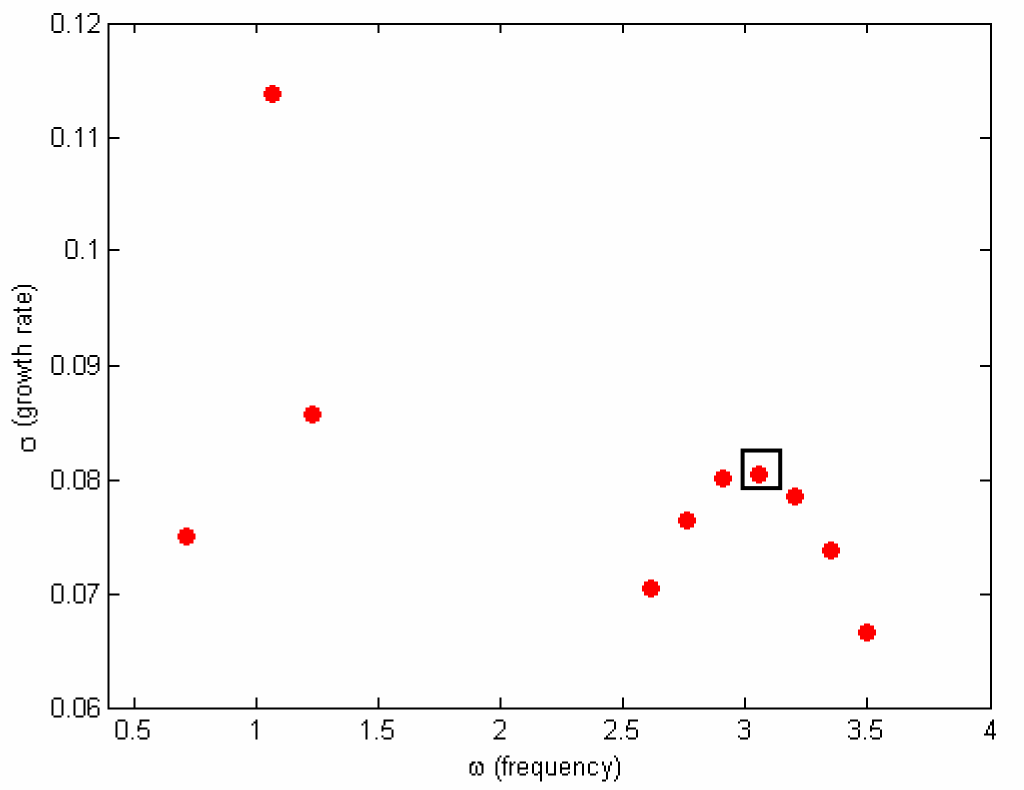

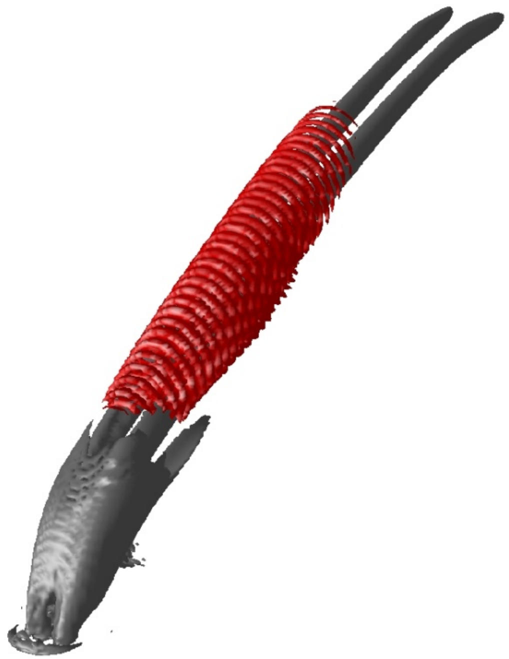

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Outlook & Conclusions

- We found self-sustained synchronized oscillations at R=3:
  - Observed in Direct Numerical Simulation
  - Linear impulse response of steady-state base
  - Linear 3D global stability analysis

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- Future work:
  - Bifurcation analysis: Find critical velocity ratio
  - Sensitivty to forcing (adjoint global modes)
  - Optimal disturbances