Stability analysis and control design of spatially developing flows

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KTH Mechanics

Shervin Bagheri

Supervisor
Dan Henningson

Co-advisors
Luca Brandt and Philipp Schlatter

Collaborators
Peter Schmid, Ardeshir Hanifi and Jerome Hoeppfner

Delaying transition

- Laminar flows are ordered and predictable
- Turbulent flows are chaotic and unpredictable
- Drag-force on surface is smaller for laminar than turbulent flows

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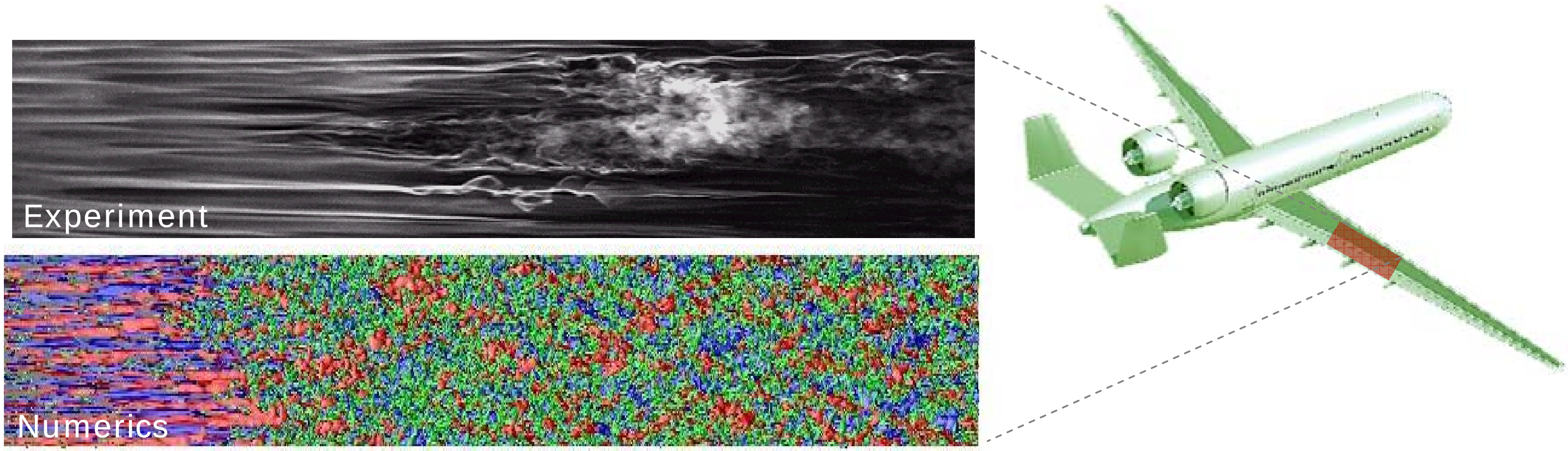

Laminar ⟶ Transition ⟶ Turbulent

Promoting transition

- Fluid injected through orifice (jet flow)
- Turbulent jets are more efficient in mixing jet fluid with ambient fluid than laminar flows

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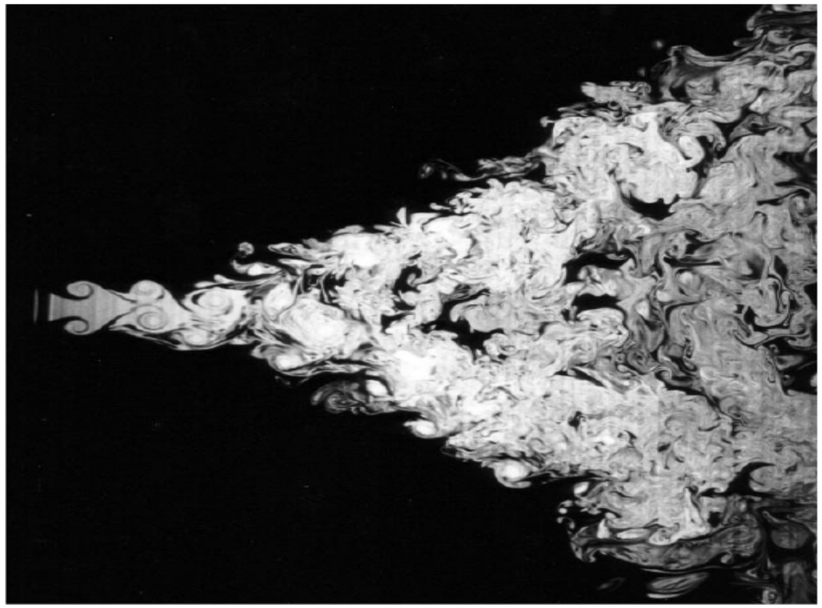

Laminar → Transition → Turbulent

Stability analysis

- Behavior of small-amplitude disturbances in space and time

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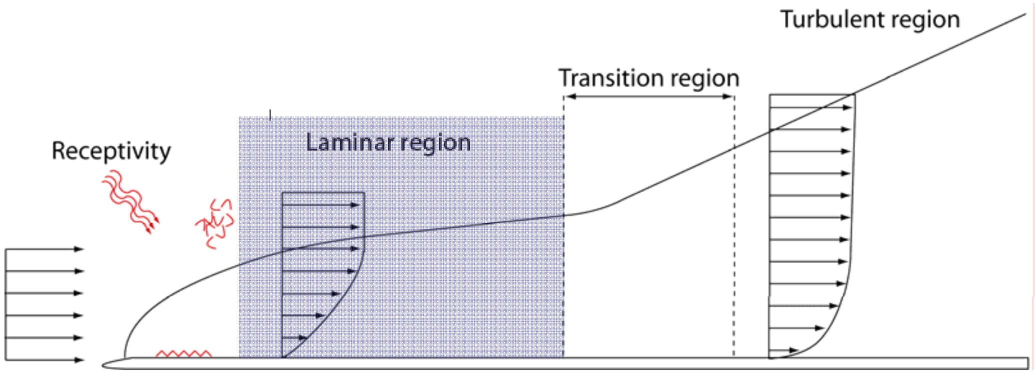
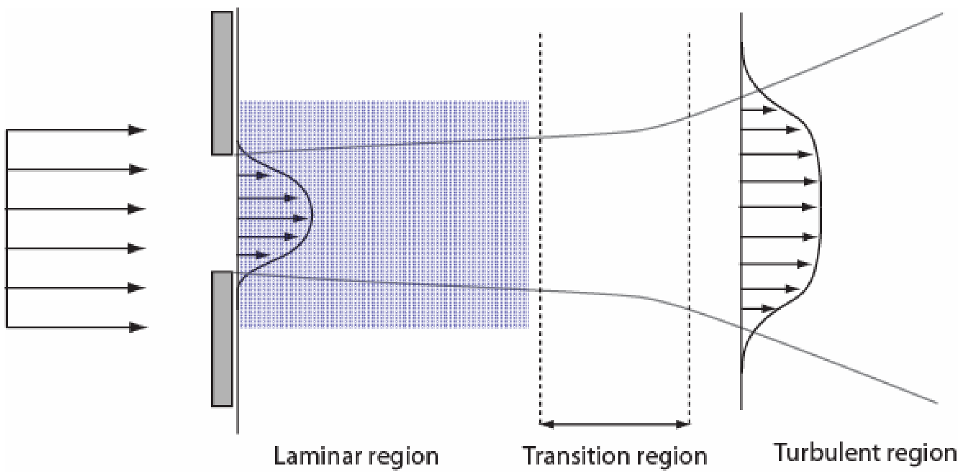
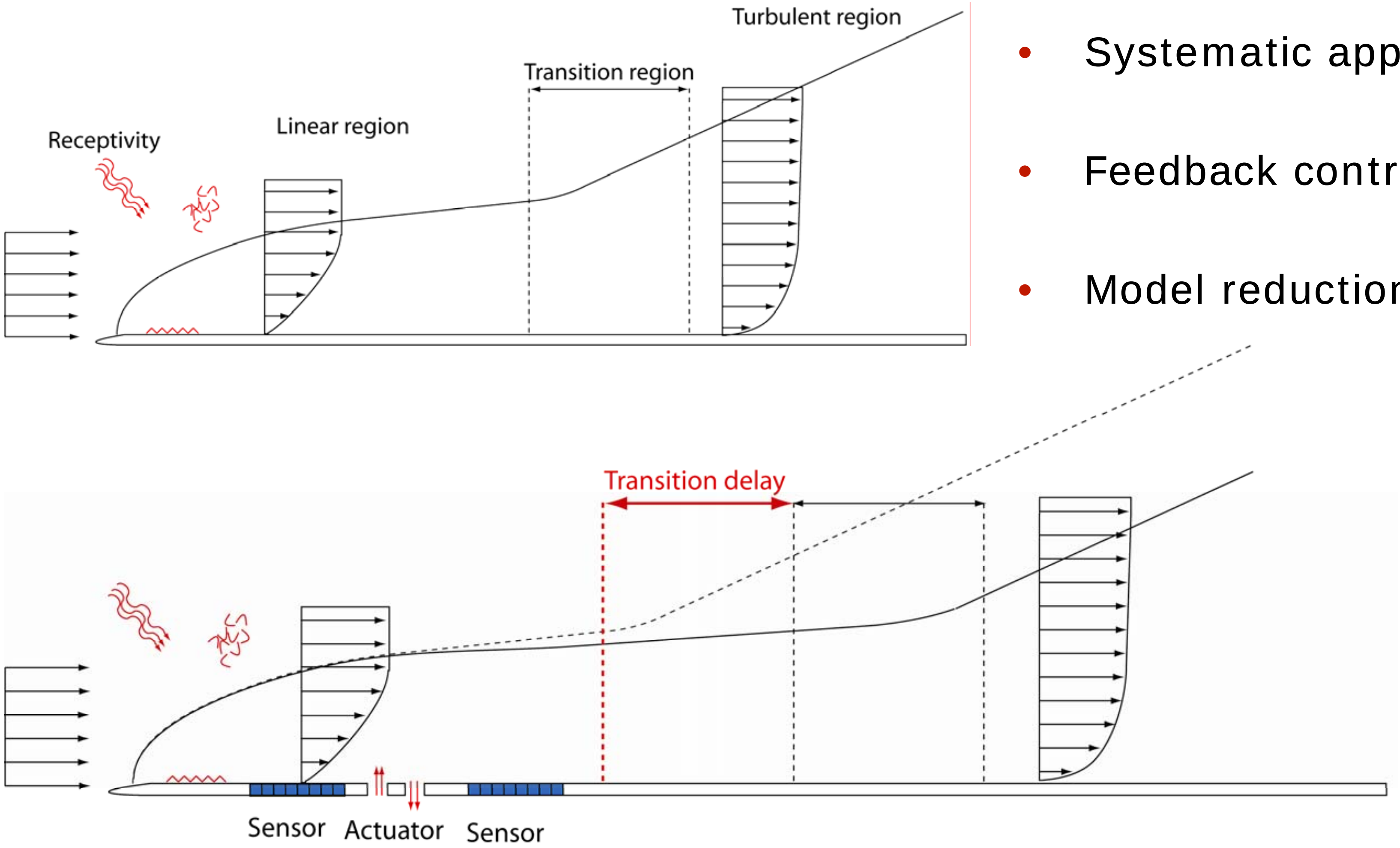




1. Receptivity
2. Disturbance behavior
3. Breakdown/transition
4. Turbulence

Control design



- Systematic approach
- Feedback control
- Model reduction

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Outline

I. Stability analysis

Global modes

Optimal disturbances

II. Control design

Model reduction

Control perfomance

Spatially developing flows:

Ginzburg-Landau equation (1D)

Flat-plate boundary layer (2D)

Jet in Crossflow (3D)

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Stability analysis

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Disturbance behavior

Amplifiers:
- Sensitive to disturbances
- Flat-plate, jets

Oscillators:
- Self-sustained oscillations
- Cylinder, hot/swirling jets, cavity

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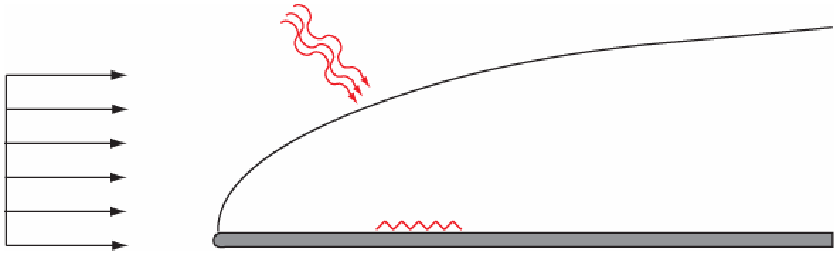
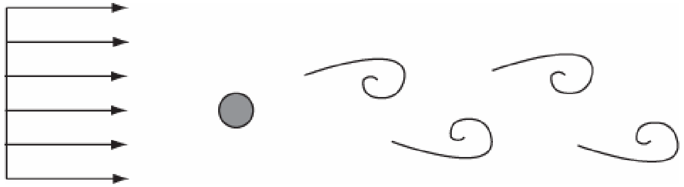
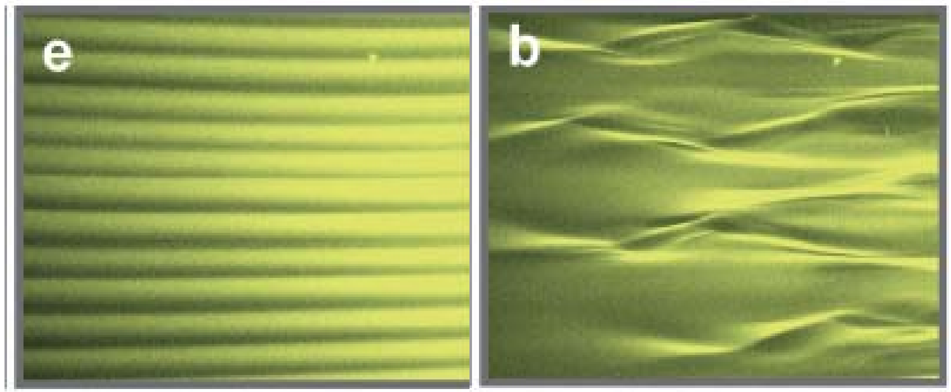
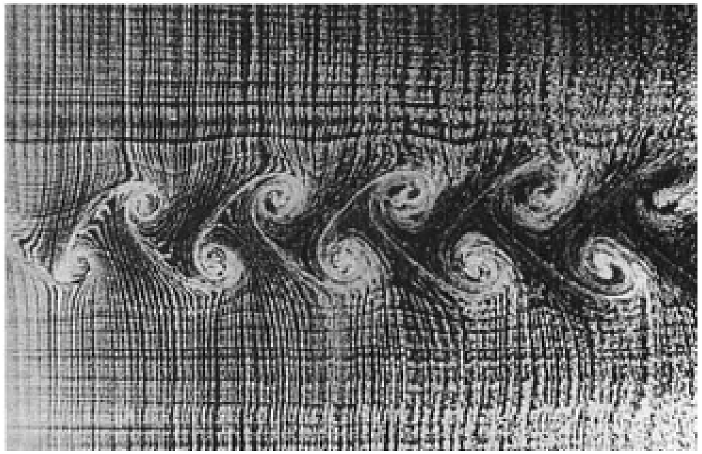






Large roughness height | Acoustic waves + small roughness height



Global approach

- Linearized Navier-Stokes equations

$$\begin{aligned} \frac{\partial \boldsymbol{u}}{\partial t} + (\mathbf{U}\cdot\nabla)\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\mathbf{U} &= -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\boldsymbol{u}, \\ \nabla\cdot\boldsymbol{u} &= 0, \\ \boldsymbol{u}(\mathbf{x},0) &= \boldsymbol{u}_0, \end{aligned}$$

- Initial value problem

$$\begin{aligned} \frac{\partial \boldsymbol{u}}{\partial t} &= A\boldsymbol{u} \\ \boldsymbol{u}(0) &= \boldsymbol{u}_0, \end{aligned}$$

- Solution

$$\boldsymbol{u}(t) = e^{At}\boldsymbol{u}_0$$

- Investigate the properties of matrix exponential
- Matrix exponential is computationally expensive to evaluate

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Dimension of discretized system

	Base Flow	Inhomogeneous direction(s)	Dimension of $\boldsymbol{u}(t)$	Storage of A
Ginzburg-Landau	$\boldsymbol{U}(x)$	1D	10^2	1 MB
Blasius	$\boldsymbol{U}(x, y)$	2D	10^5	25 GB
Jet in crossflow	$\boldsymbol{U}(x, y, z)$	3D	10^7	500 TB

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- Matrix A very large for spatially developing flows
- Use Navier-Stokes solver (DNS) or any CFD code to approximate the action of exponential matrix:

$$\mathbf{u}(t+T) = e^{A(t+T)}\mathbf{u}(t)$$

- Time-stepper technique:
  Never store matrices
  Use only velocity fields at different times

Quest for eigenmodes

- Asymptotic behavior:

$$e^{At}\mathbf{u}_j = \sigma_j \mathbf{u}_j$$

$|\sigma_1| > 1$ globally unstable

$|\sigma_1| \leq 1$ globally stable

→ u_j called global eigenvectors

- Transient behavior:

$$e^{A^*t}e^{At}\mathbf{u}_j = \sigma_j \mathbf{u}_j$$

$|\sigma_1| > 1$ convectively unstable

$|\sigma_1| \leq 1$ convectively stable

→ u_j called optimal disturbances

- Time-stepper and iterative methods to compute modes

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Previous work

- Global modes
  - Backward-facing step (Barkley *et.al.*, 2002)
  - Flat-plate boundary layer (Ehrenstein *et.al.*, 2005)
  - Smooth cavity (Åkervik *et.al.*, 2007)
  - Cylinder wake (Gianetti & Luchini, 2007)
  - Recirculation bubble (Marquet *et.al.*, 2008)

- Optimal disturbances
  - Swept Hiemens flow (Guegan *et.al.*, 2007)
  - Backward-facing step (Barkley *et.al.*, 2008)
  - Recirculation bubble (Marquet *et.al.*, 2008)

- Current/future work: Jet in crossflow

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Jet in crossflow

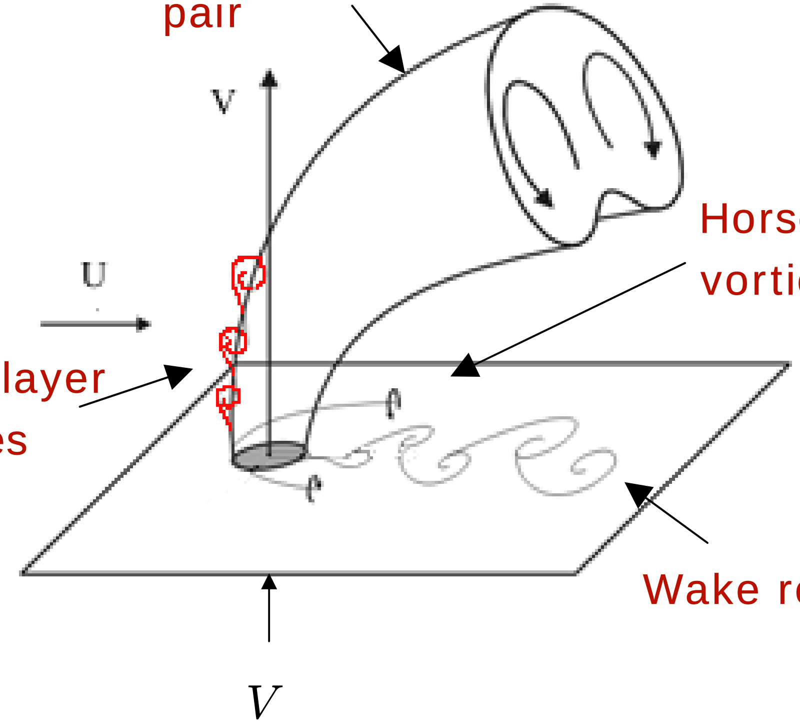
- Most of work experimental and no stability analysis
- Velocity Ratio

$$R = \frac{U}{V} = 3$$

- Reynolds number

$$\mathrm{Re} = \frac{U\delta_0}{\nu} = 165$$

- Smoke stacks,
  film cooling etc.



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Stability analysis – 4 steps

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1. Simulate flow with DNS: Identify structures and regions
2. Compute baseflow: Steady-state solution
3. Compute impulse response of baseflow:

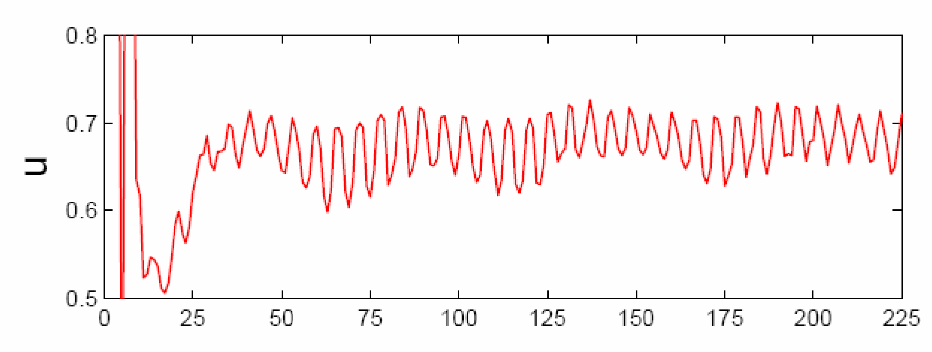
$$\boldsymbol{u}(t) = e^{At}\boldsymbol{u}_0$$

4. Compute global modes of baseflow:

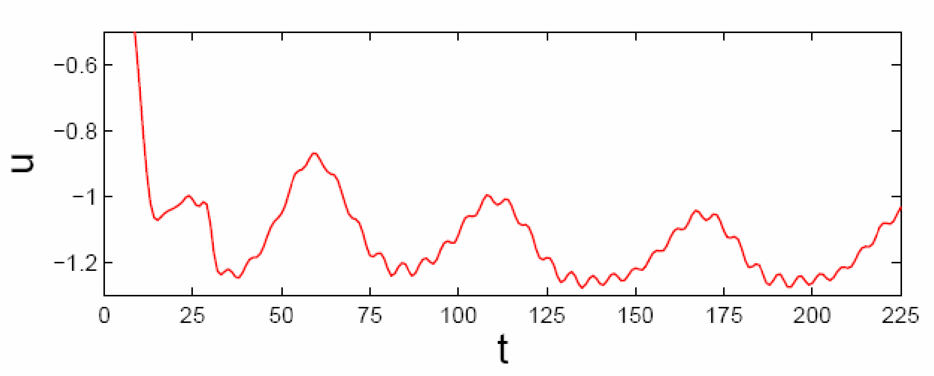
$$e^{At}\mathbf{u}_j = \sigma_j \mathbf{u}_j$$

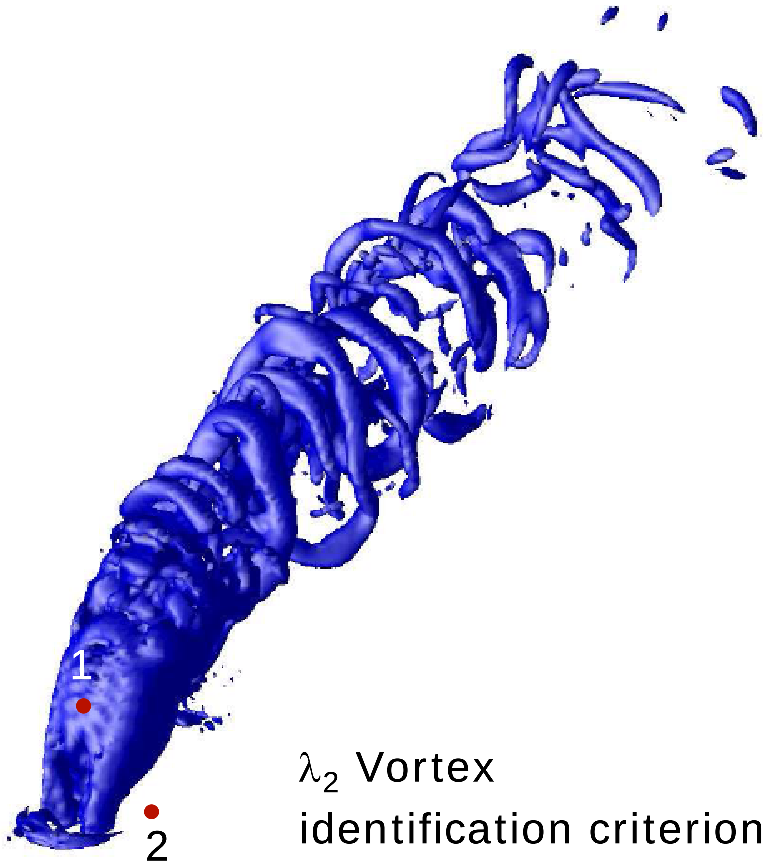
Direct numerical simulations

- DNS: Fully spectral and parallelized
- Self-sustained global oscillations
- Probe 1– shear layer



- Probe 2 – separation region





λ_2 Vortex identification criterion

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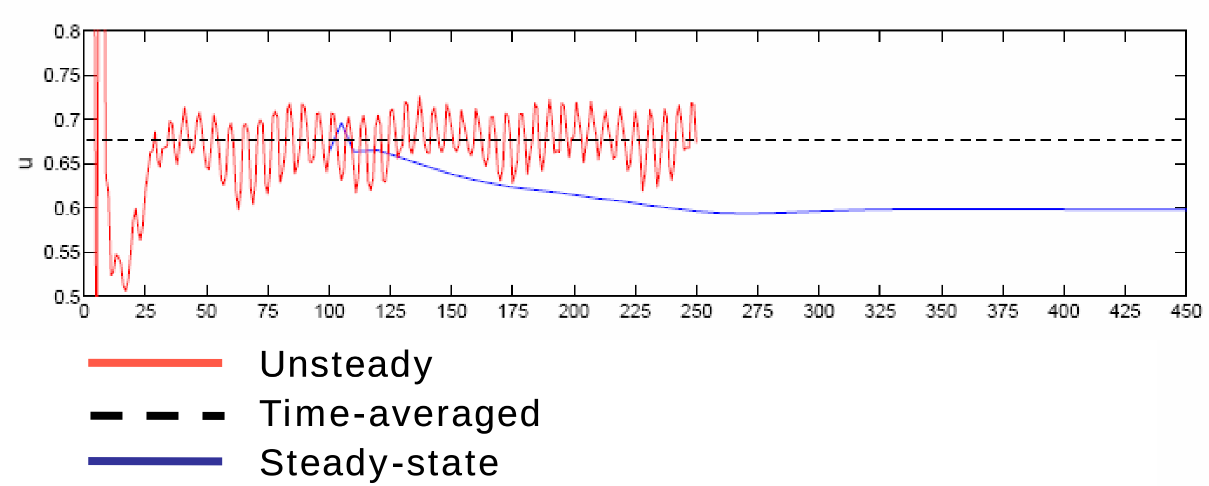
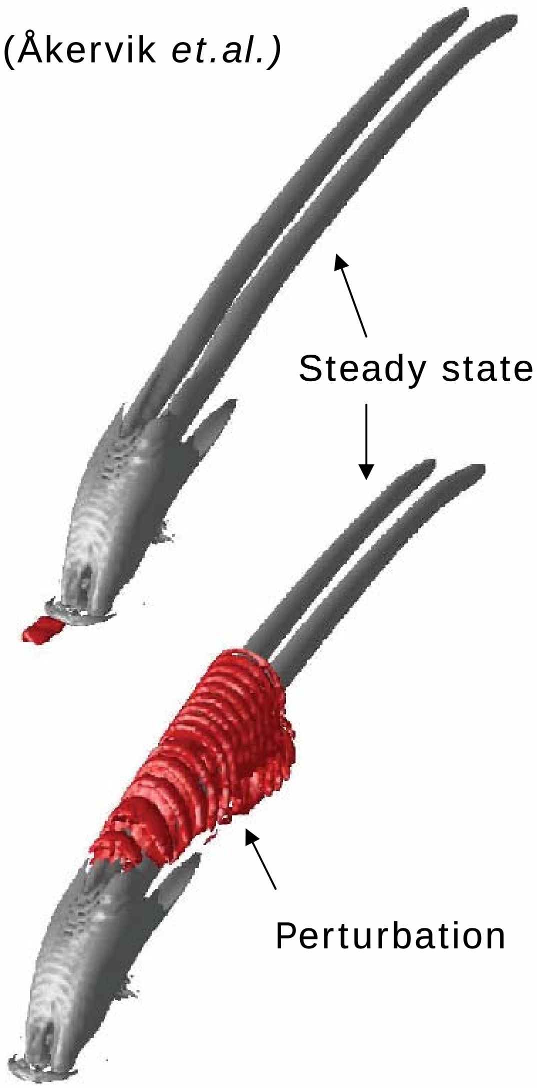
Impulse response

- Steady state computed using the SFD method (Åkervik *et.al.)*





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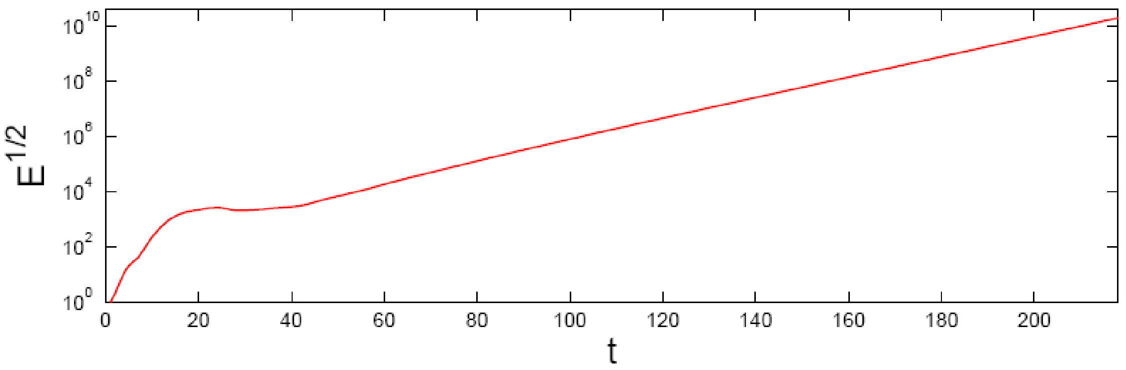
- Asymptotic energy growth of perturbation



Global eigenmodes

- Global eigenmodes computed using ARPACK
- Growth rate: 0.08 $(\sigma_1 = 1.17)$
- Strouhal number: 0.16

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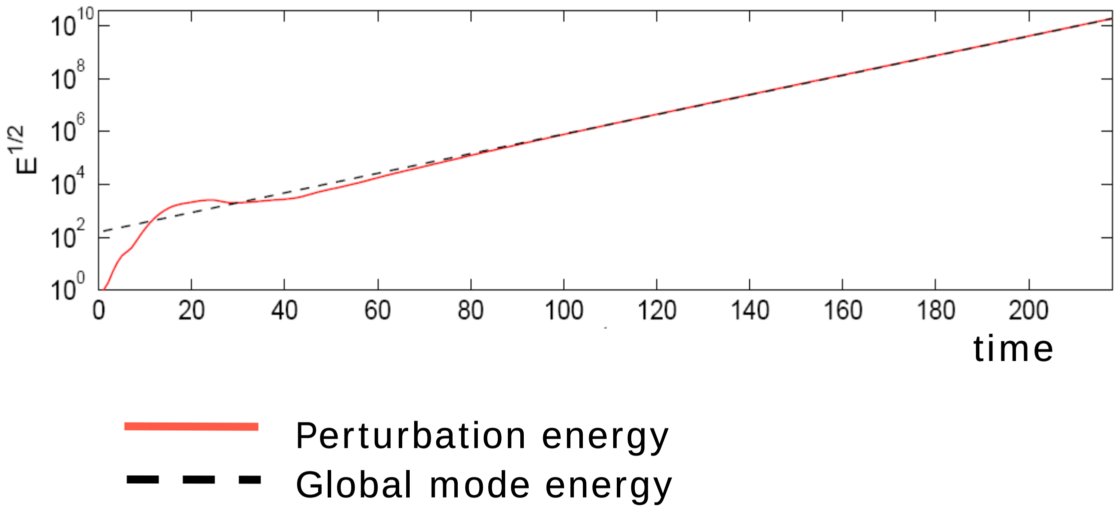
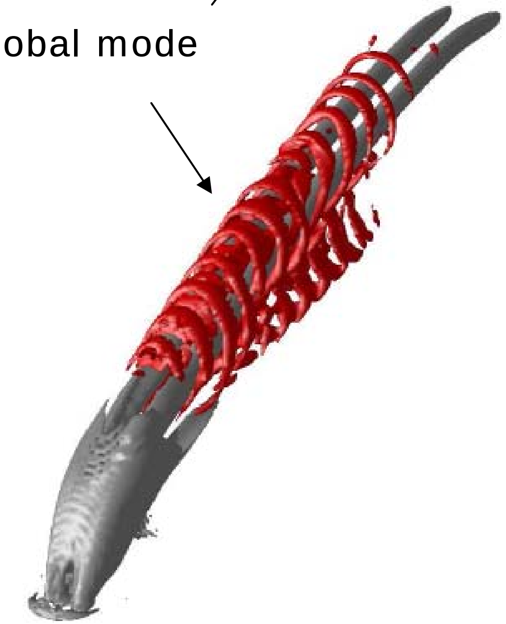


1st global mode



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Control design

Flat-plate boundary layer

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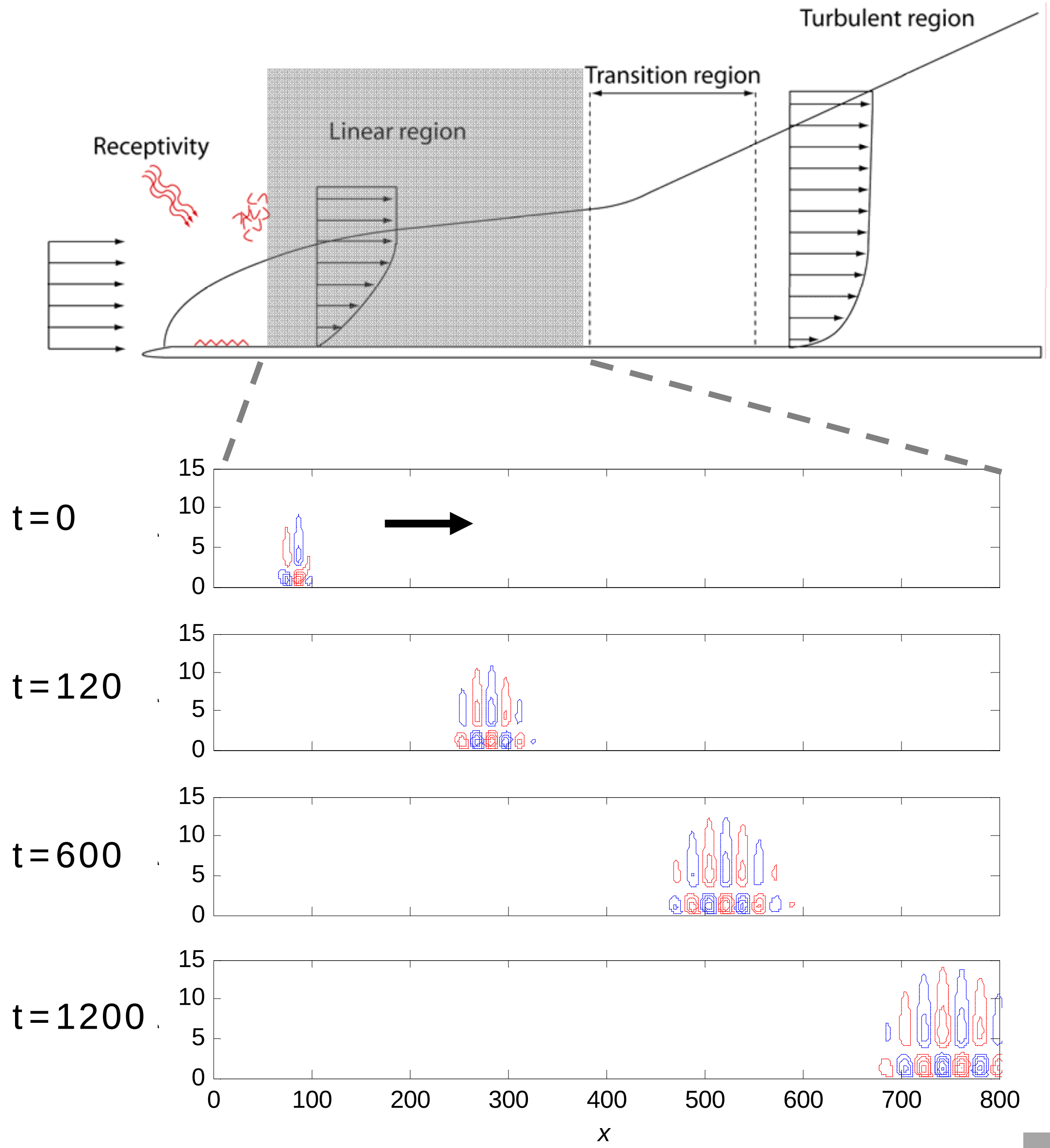
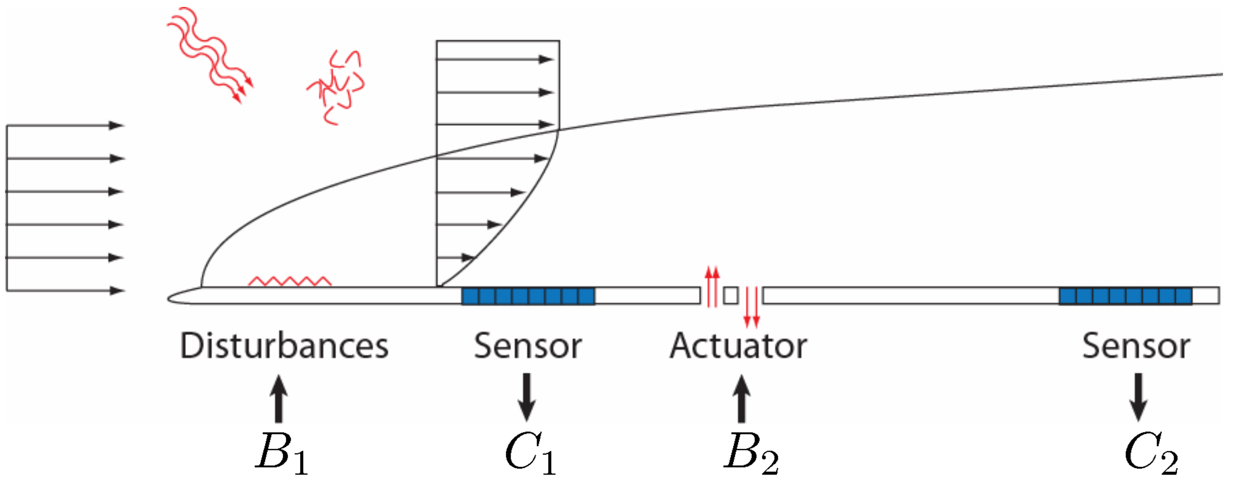
- 2D disturbances (TS-type/wavepackets)
- Convectively unstable (amplifier)
- Reynolds number:

$$\mathrm{Re} = \frac{U\delta_0}{\nu} = 1000$$



Inputs and outputs



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- Inputs:

  Rougness, free-stream turbulence, acoustic waves, blowing/suction etc.

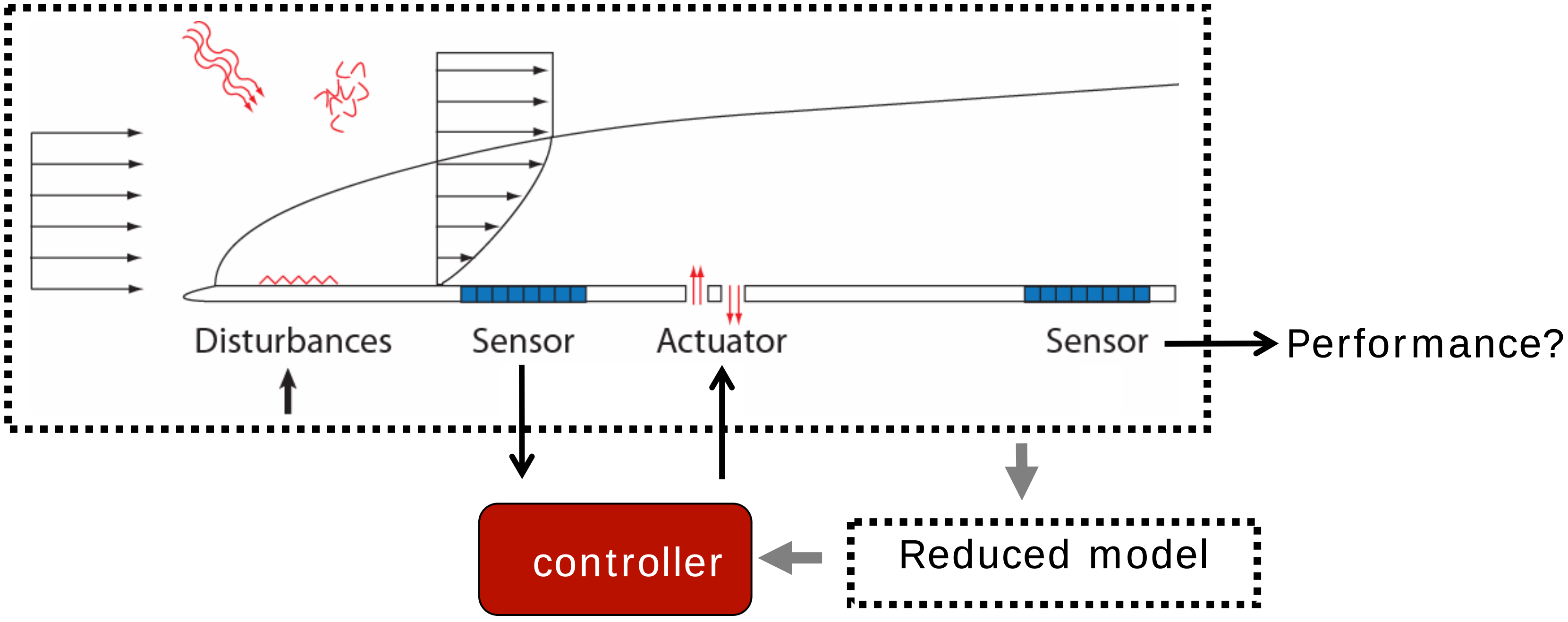
- Outputs:

  Measurements of pressure, skin friction etc.

- Setup for modern control design:

  Minimize effects of disturbances on second sensor using first sensor and actuator

Control design – 5 steps



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1. Construct **plant**: Flow, inputs, outputs
2. Construct **reduced model** from the plant using balanced truncation
3. Design **controller** using the reduced-model (LQG/H2)
4. **Closed-loop**: Connect sensor to actuator using the reduced controller
5. Run small controller **online** and evaluate closed-loop perfomance

Input-output behavior

- State-space formulation:

$$\begin{aligned} \dot{\boldsymbol{u}} &= A\boldsymbol{u} + Bw \\ z &= C\boldsymbol{u}. \end{aligned}$$

- Solution:

$$z = \int_0^t Ce^{A(t-\tau)} Bw(\tau)\, d\tau$$

B – Inputs
C – Outputs
w – Input signal
z - Output signal

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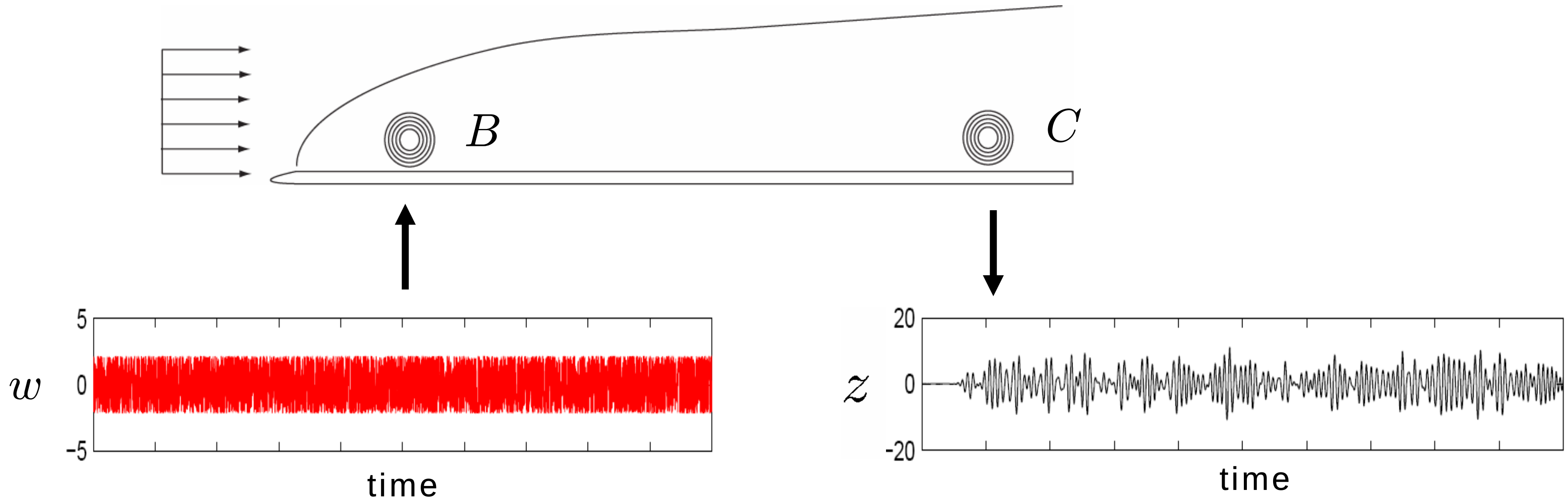

Model reduction

- Approximate the large system

$$\begin{aligned}\dot{\boldsymbol{u}} &= A\boldsymbol{u} + Bw \\ z &= C\boldsymbol{u}.\end{aligned}$$

$n > 10^5$

with a small system

$$\begin{aligned}\dot{\kappa} &= \hat{A}\kappa + \hat{B}w \\ \hat{z} &= \hat{C}\kappa\end{aligned}$$

$m < 100$

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so that the I/O behavior is preserved:

$$\|z - \hat{z}\| \leq \epsilon \qquad \forall w$$

- One systematic approach is *balanced truncation* *(Moore 1981)*

Balancing

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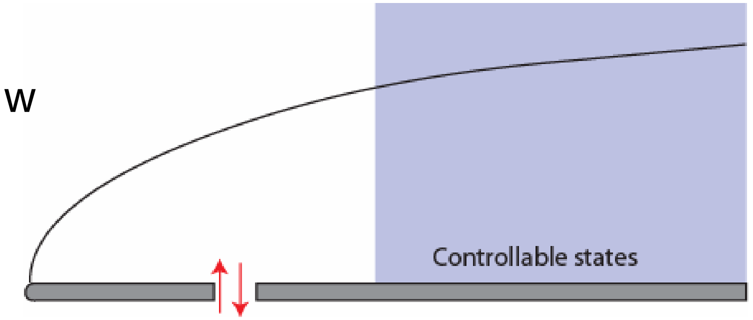
- Controllability

  Flow states most easily excited by input

  Solution: POD modes

  Diagonalize the correlation matrix of the flow

$$P = \int_0^\infty e^{A\tau} B B^* e^{A^*\tau} \mathrm{d}\tau$$



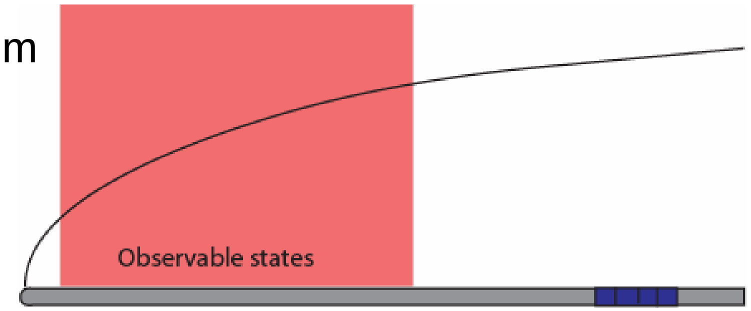
- Observability

  Flow states that will most easily excite output

  Solution: adjoint POD modes

  Diagonalize correlation matrix adjoint system

$$Q = \int_0^\infty e^{A^*\tau} C^* C e^{AT\tau} \mathrm{d}\tau$$



- Balanced modes

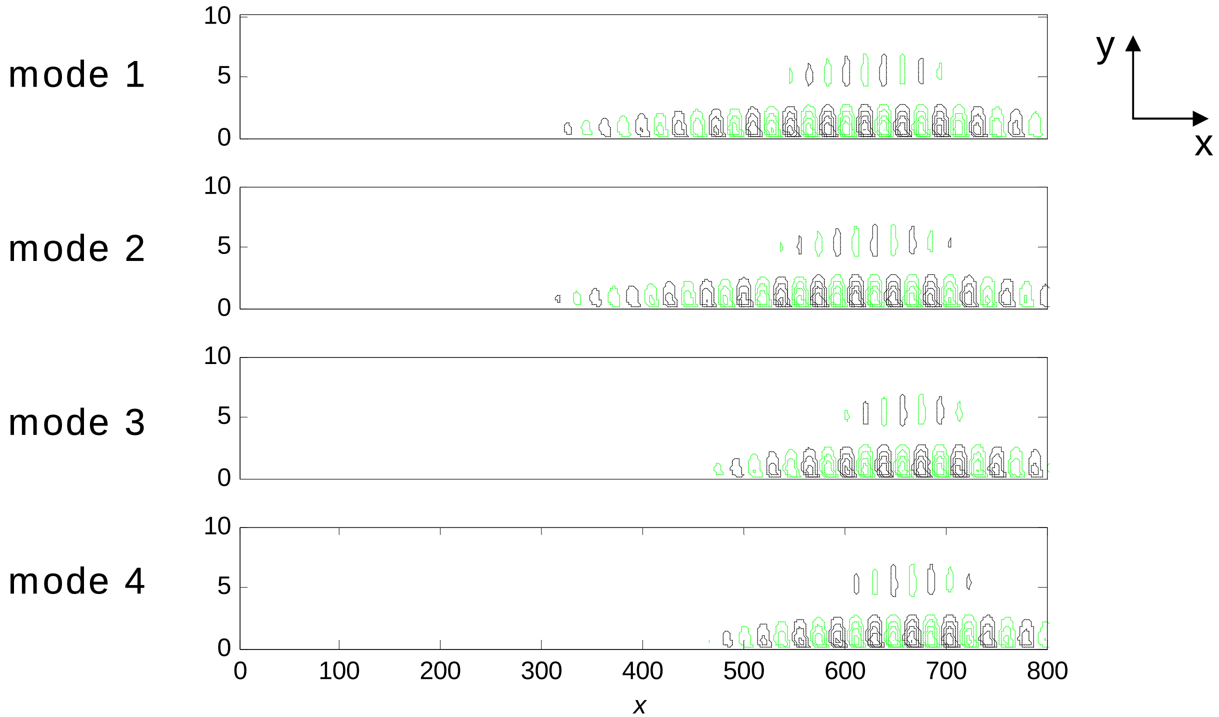
  Diagonalize both controllability and observability Gramian

$$PQ\mathbf{u}_j = \sigma_j \mathbf{u}_j \qquad (n \times n), \qquad n > 10^5$$

Balanced modes

- Computed using snapshot method *(Rowley 2005):*
  Collect snapshots of forward and adjoint simulation + small eigenvalue problem

- Balanced modes (u-velocity):



- Oblique projection onto balanced modes to obtain reduced system (ROM):

$$A, B, C \longrightarrow \hat{A}, \hat{B}, \hat{C}$$

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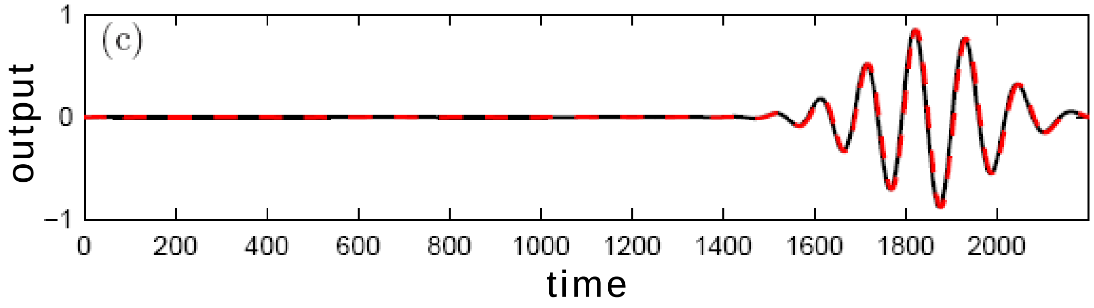
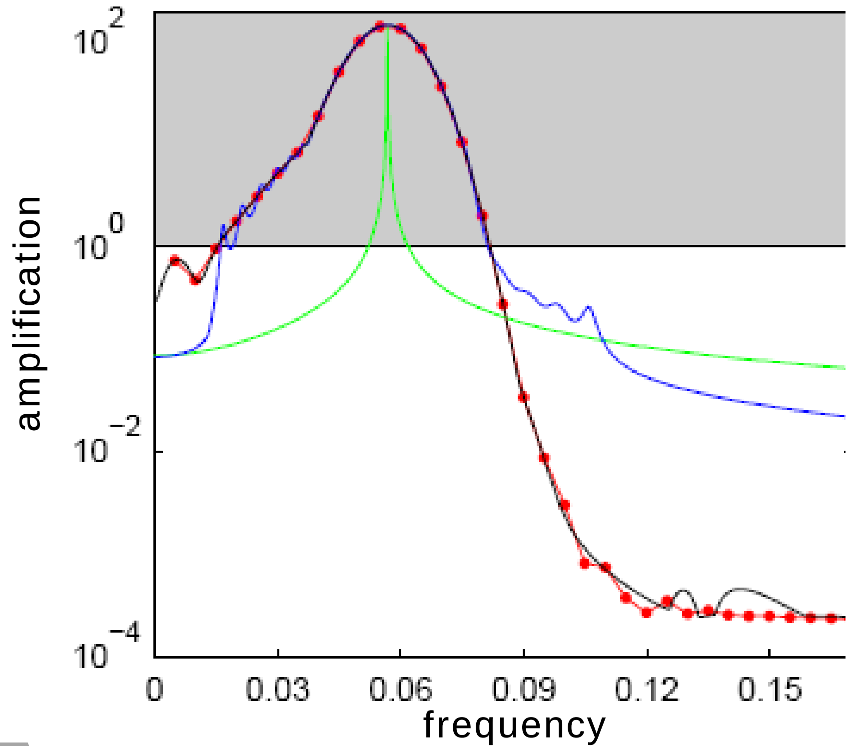
Reduced system vs. Full system



Impulse response

DNS: $n=10^5$

ROM: $m=50$



Frequency response

DNS: $n=10^5$

ROM: $m=80$

$m=50$

$m=2$

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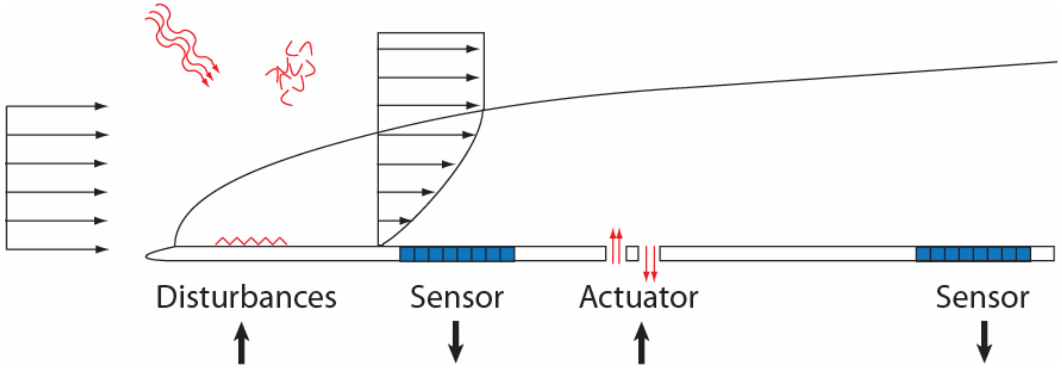
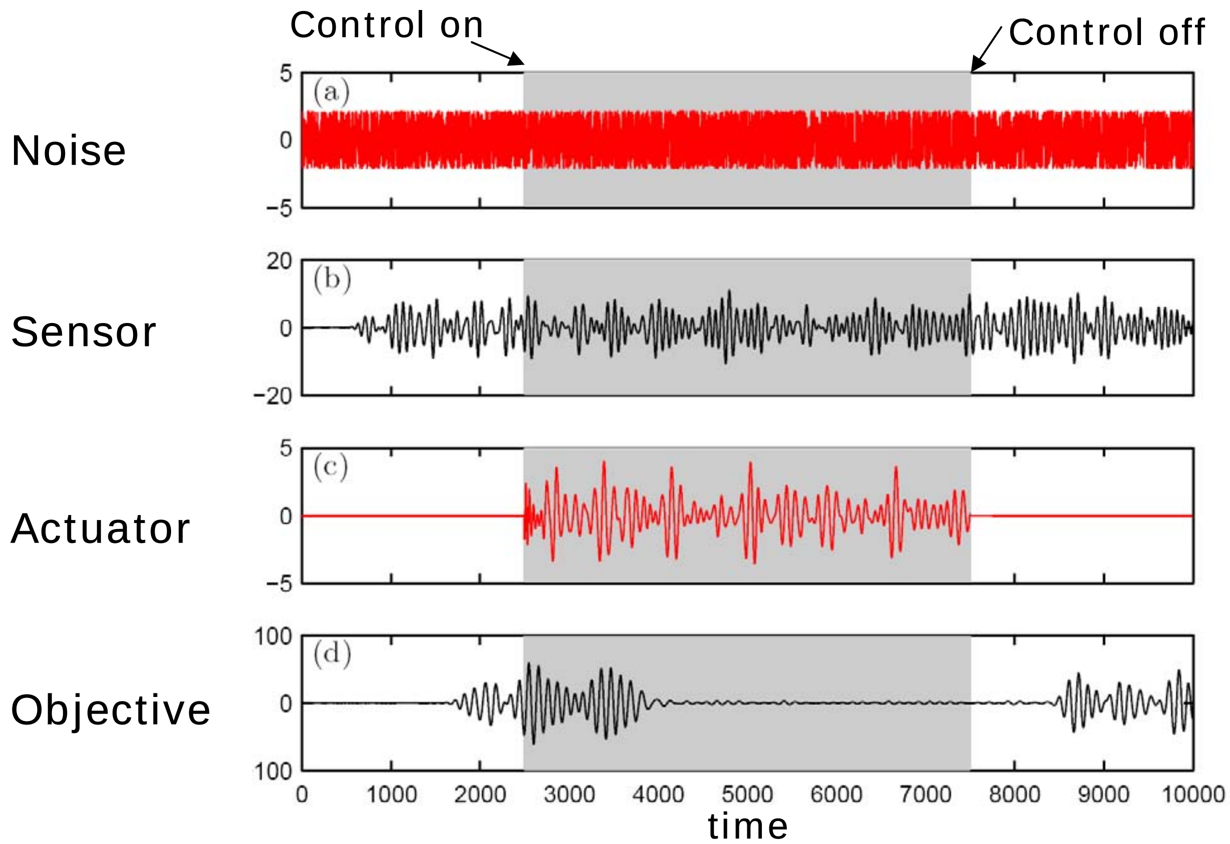
Performance of controlled system





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Performance of controlled system

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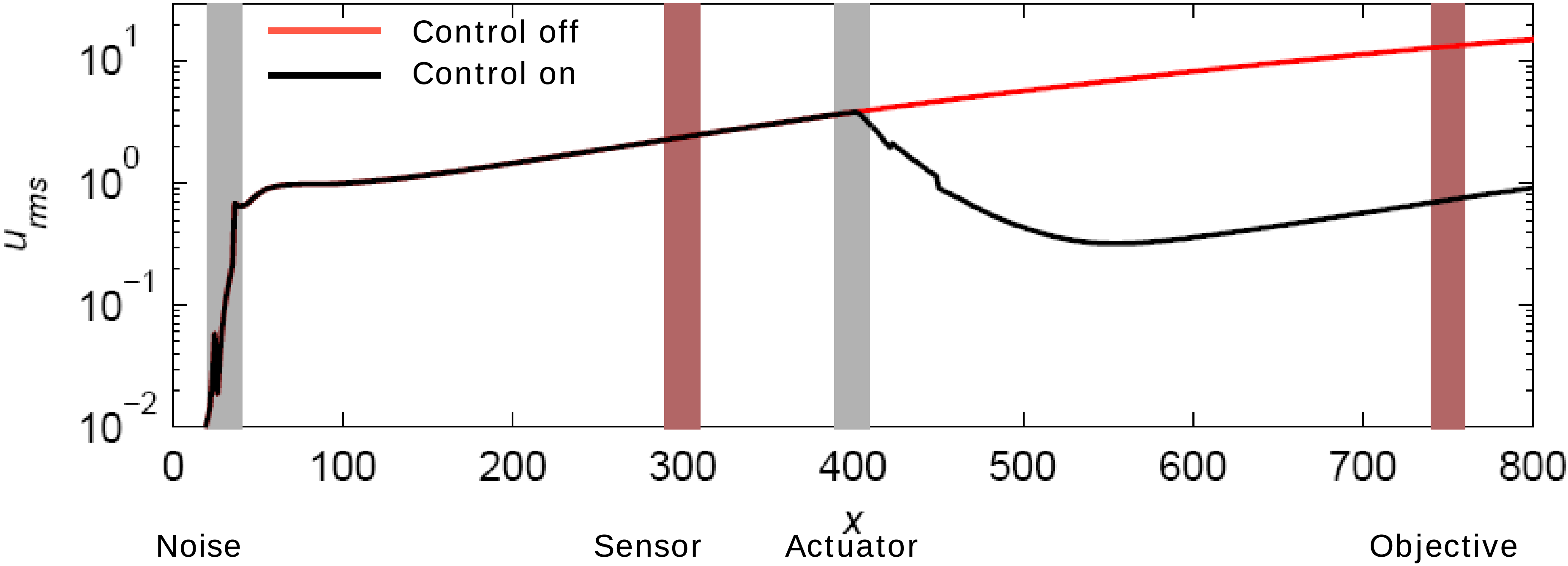

- Disturbance amplitude efficiently damped

Summary

- Time-stepper technique necessary for modern stability analysis and control design of complex flow systems
- Stability – Both long and short time analysis possible using time-steppers
- Model reduction – Input-output is preserved using balanced truncation
- Control design – Using reduced-order models optimal/robust control schemes become applicaple to complex flow systems

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Outlook

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- Jet in crossflow
  - Bifurcation analysis
  - Find critical velocity ratio
  - Sensitivty to forcing (adjoint global modes)
  - Optimal disturbances

- Flat-plate boundary layer
  - Three dimensional disturbance
  - Realistic actuators and sensors
  - Delay transition to turbulence