Flow Control

Systematic Approach to Model Reduction and Control Design

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Outline

Part I: Background on Flow Control

- Physical vs. systematic approach
- Computational challenges and model reduction
- Concept of controllability and observability

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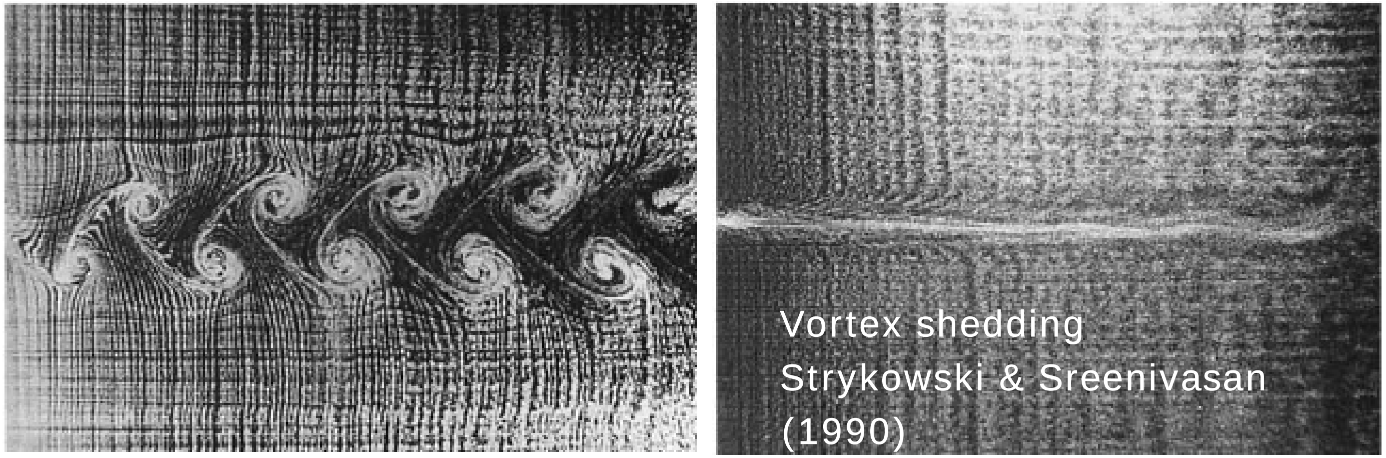
Part II: Flat-plate Boundary Layer

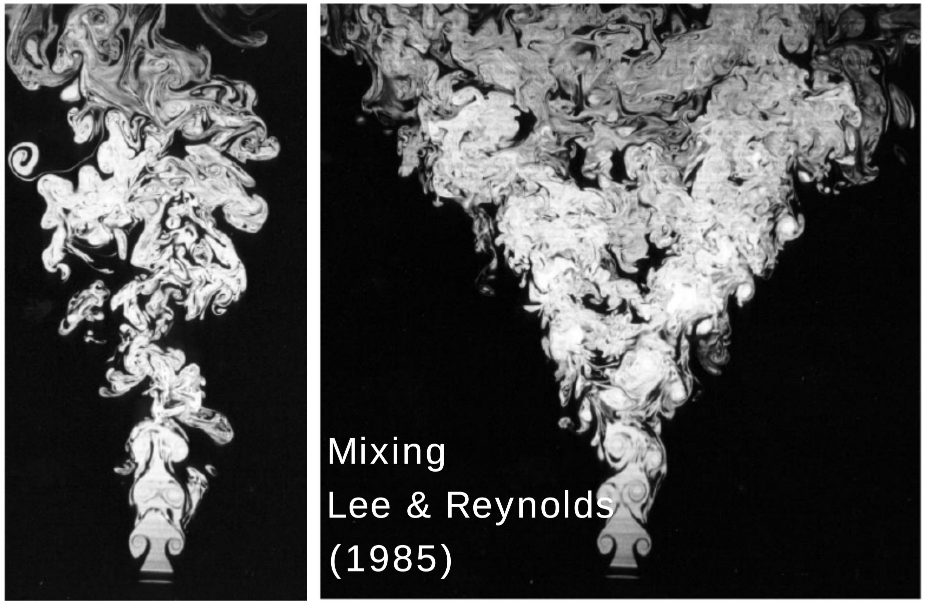
- Reduce Navier-Stokes to small system and design controller
- How good is the small model compared to the actual flow?
- What can be achieved with linear control?

Open-loop Control



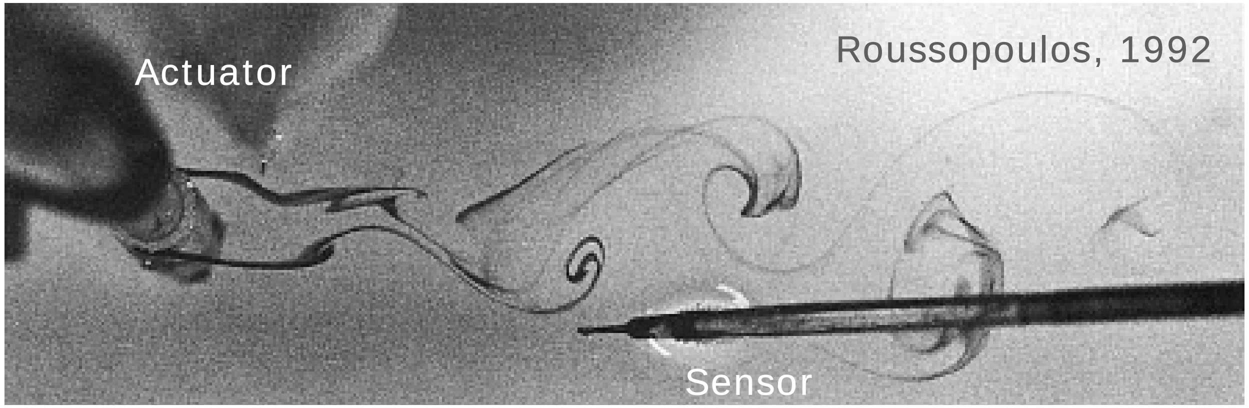
Vortex shedding
Strykowski & Sreenivasan
(1990)

- Changes to geometry
- Periodic forcing
- Steady/periodic suction
- Heating/cooling



Mixing
Lee & Reynolds
(1985)

Closed-loop Control



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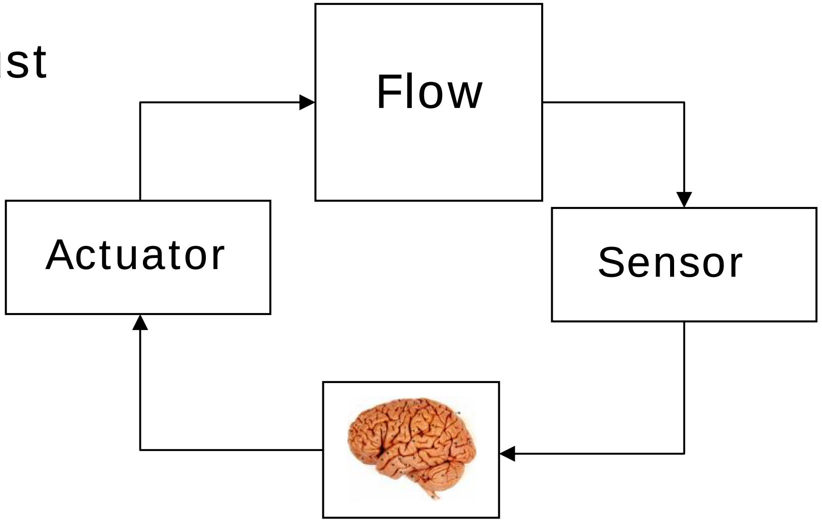
- Take measurements and adjust actuation accordingly
- Account for unknown variations:
  - Sensor noise
  - Modeling errors



Intuition vs. Theory

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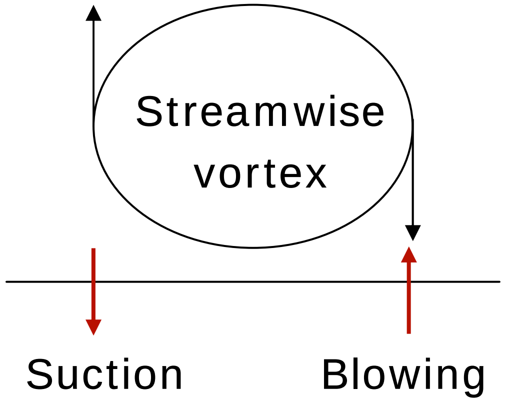
- Physics-based models
  - Controller based on physical intuition
  - Cheap, difficult to optimize
  - Example: opposition control



- Systematic methods
  - Derive controller from Navier-Stokes based on control theory
  - Example: LQG
  - Expensive, easy to optimize
  - Model reduction necessary

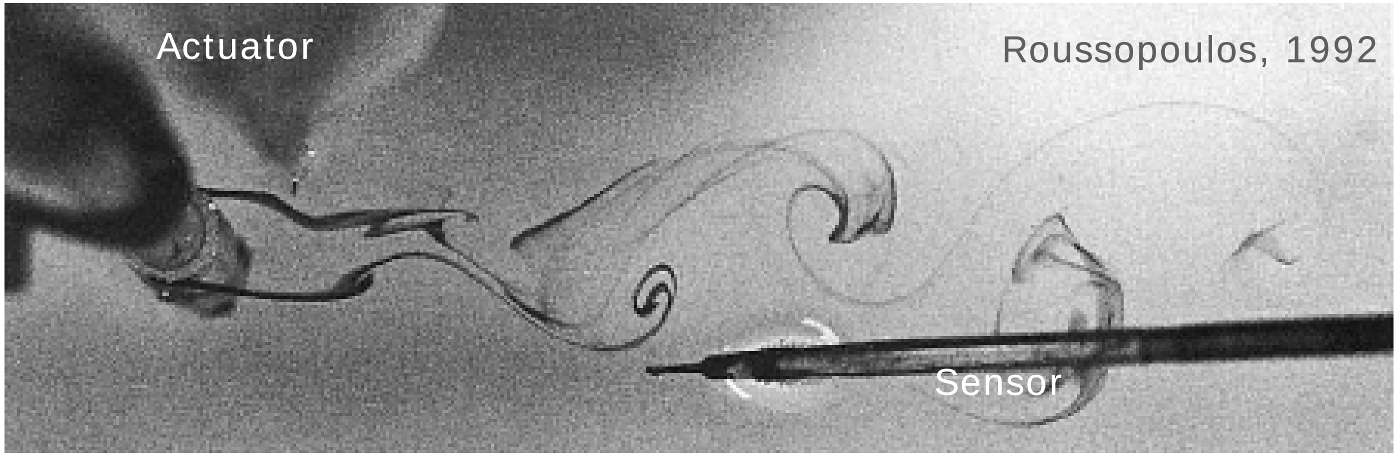
$$\underbrace{\dot{q} = f(q,u)}_{\text{n equations, too complex}} \longrightarrow \underbrace{\dot{a} = \hat{f}(a,u)}_{\text{m equations, useful model}}$$

n equations
too complex

m equations
useful model

Which flow components should we keep and which ones should we discard?

Controllability and Observability



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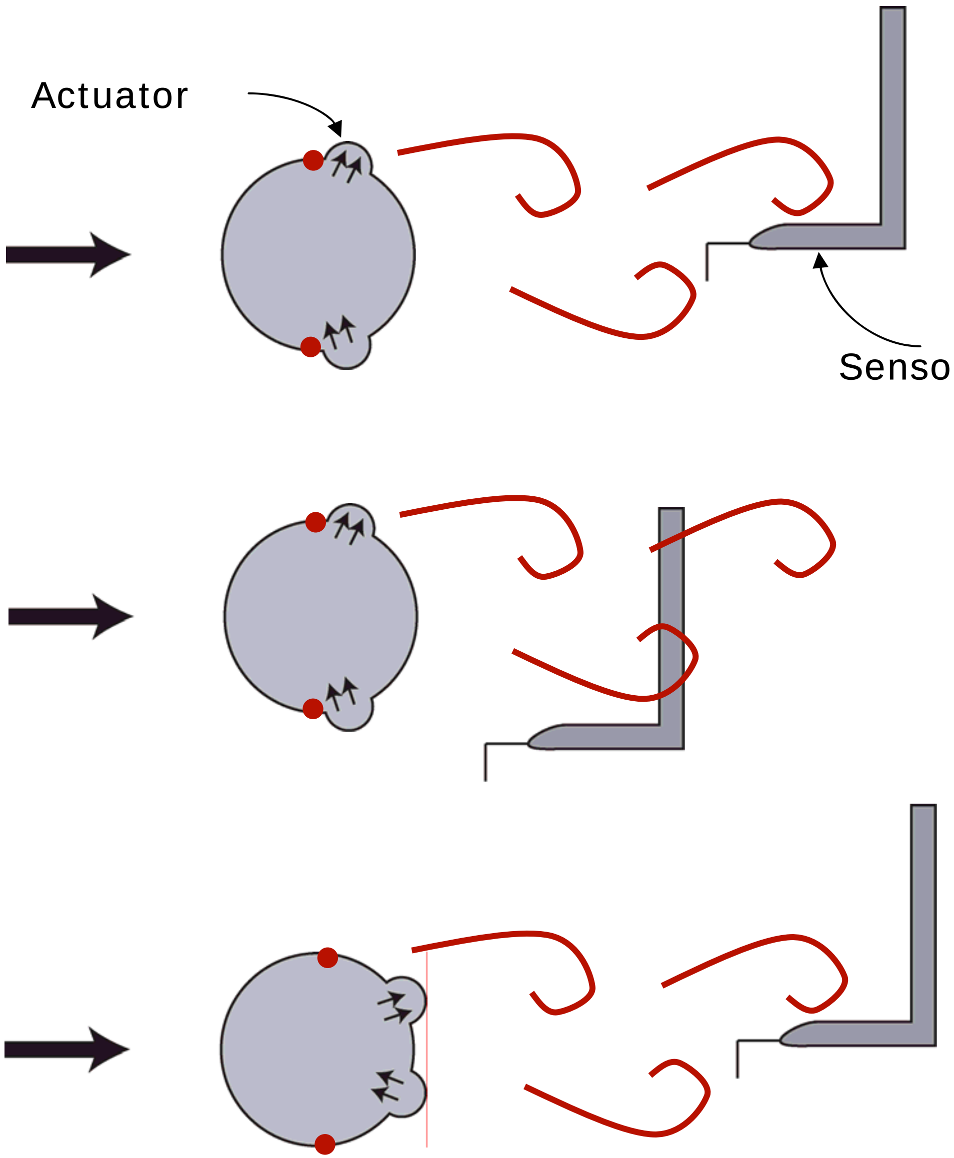
- Where on the cylinder should the actuator be located?
- Why does control work for a narrow range of the sensor location?

Controllability and Observability



Shedding observable and controllable
→ Control possible

Shedding is not observable
→ Control not possible

Shedding is not controllable
→ Control not possible

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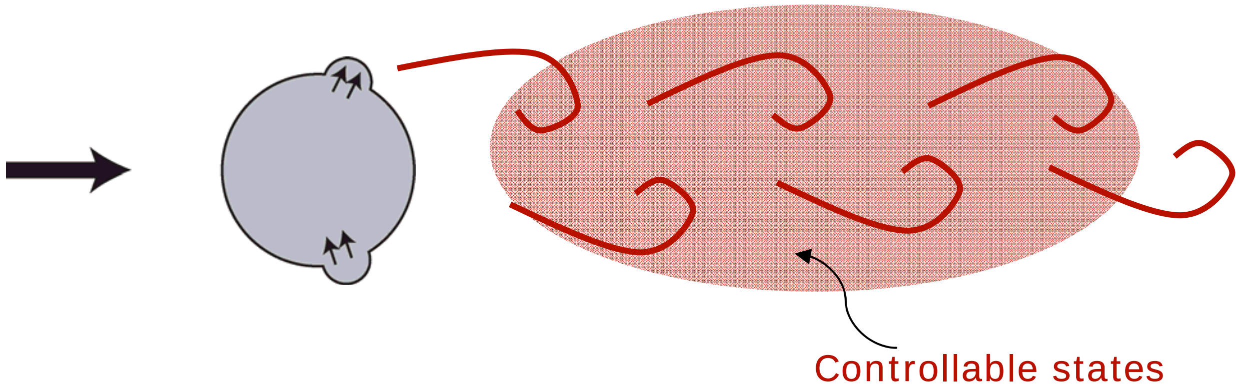
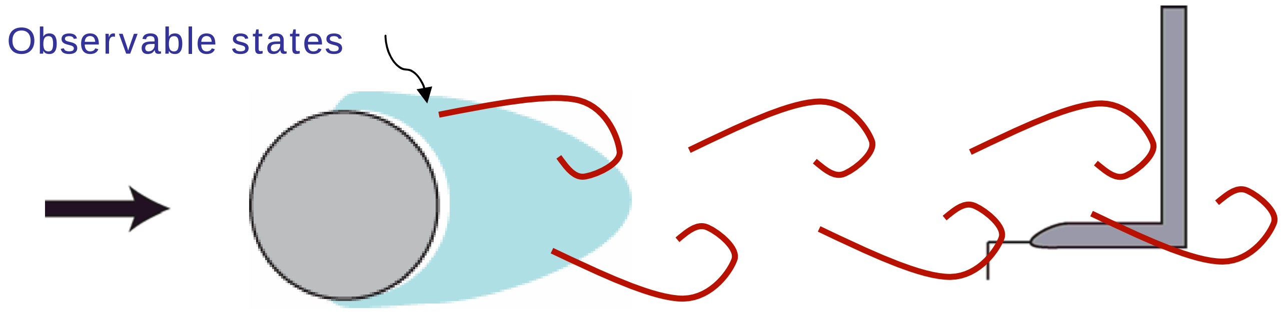
Balanced Truncation

- Which flow states most sensitive to input/actuator?



- Which initial flow states will produce largest output/sensor energy?



- Balanced trunction
  – keep only states both controllable and observable

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Part II: Flat-plate Boundary Layer

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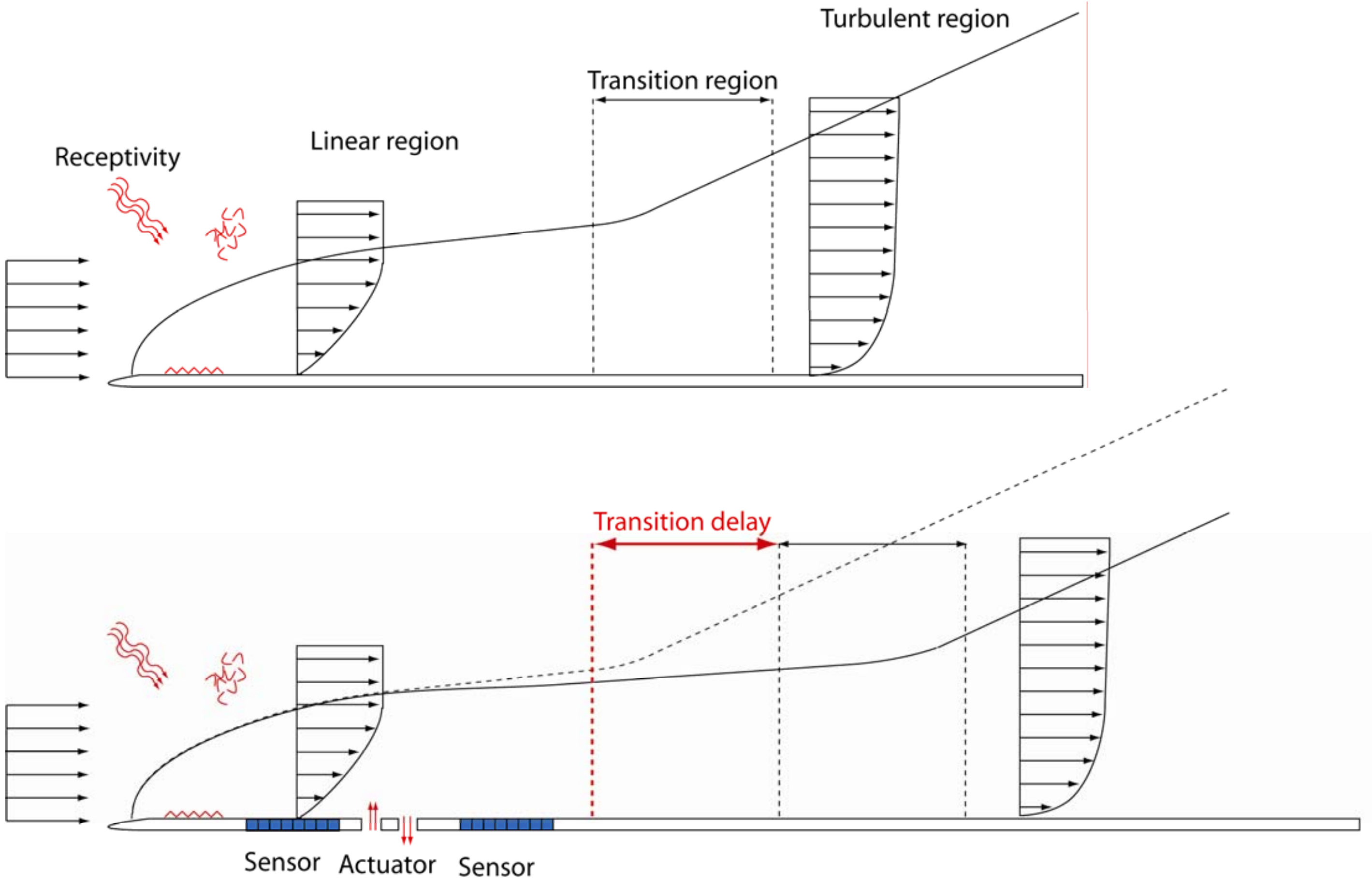
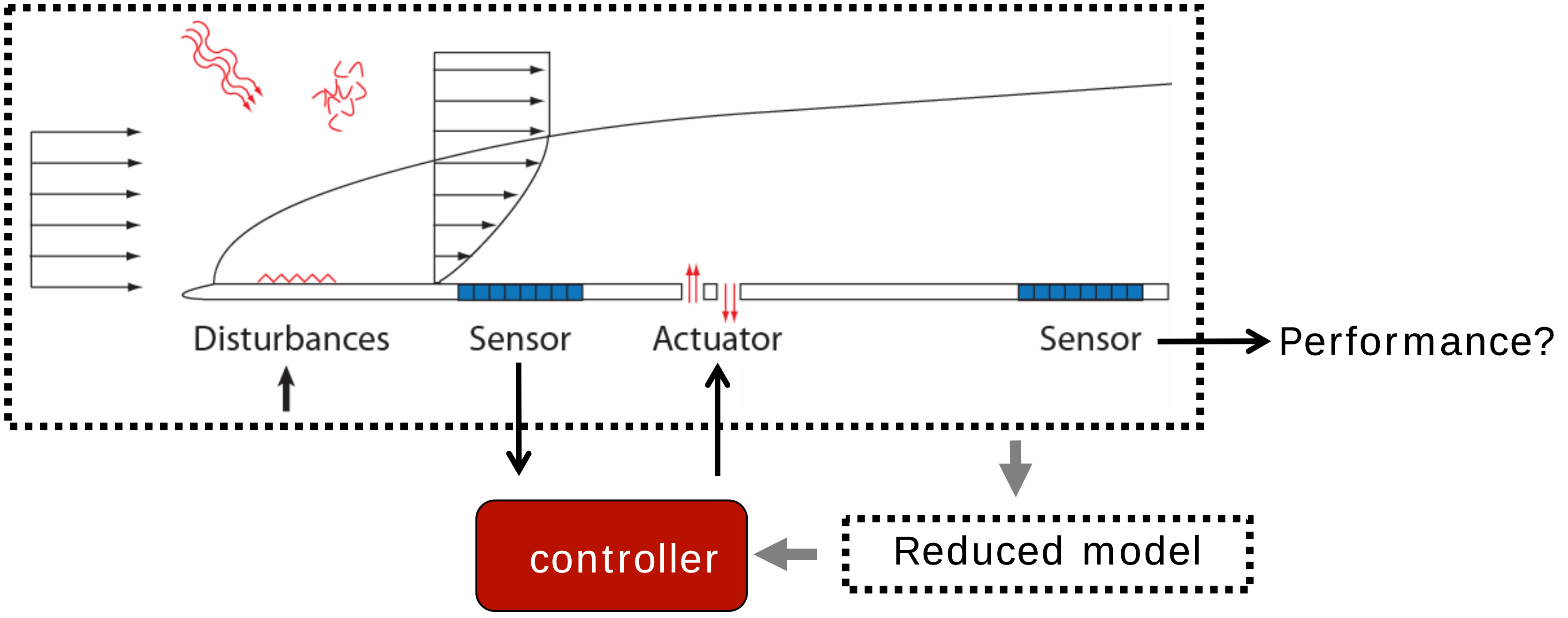


Objective: suppress the growth of small disturbances in the laminar region

Control Design – 5 Steps



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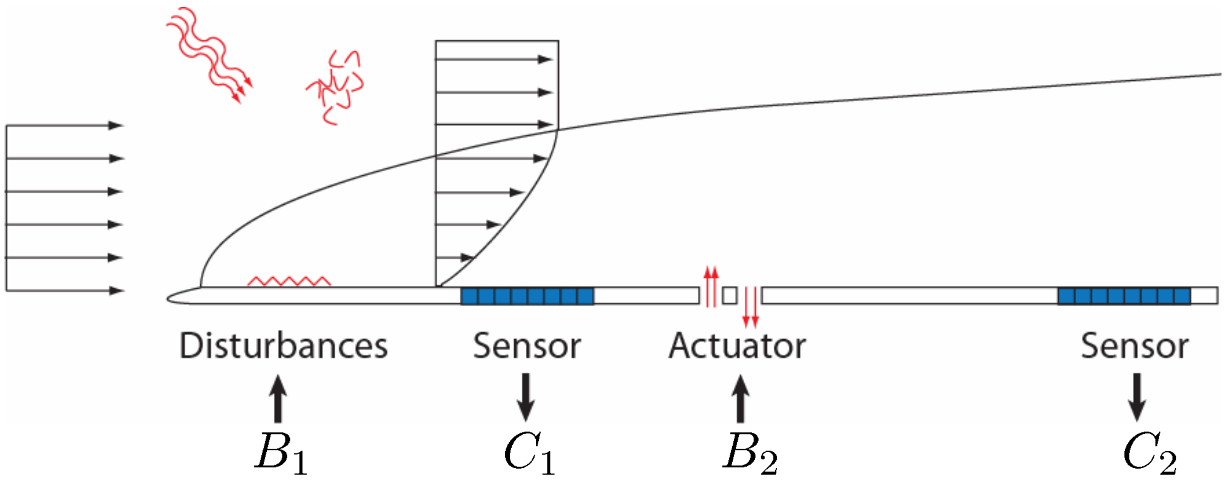
1. Construct Plant: Flow, inputs, outputs
2. Construct reduced model from the plant using balanced truncation
3. Design controller using the reduced-model
4. Closed-loop: Connect sensor to actuator using the reduced controller
5. Run small controller online and evaluate closed-loop perfomance

The Plant



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$$\begin{aligned} \dot{q} &= Aq + Bu \\ y &= Cq \end{aligned}$$

- A - Linearized Navier-Stokes
- B – Inputs
- C – Outputs
- q – Flow state
- u – Control signal
- y - Sensor signal

Model Reduction for Linear Systems

- Obtain a set of functions $T = \{T_i\}_{i=1}^m$ and project the state on this basis:

$$q = \sum_{i=1}^{m} T_i a_i = Ta$$

- Insert into the plant to obtain reduced model:

$$\begin{aligned} \dot{q} &= Aq + Bu \\ y &= Cq \end{aligned} \quad \longrightarrow \quad \boxed{\begin{aligned} \dot{a} &= T^{-1}ATa + T^{-1}Bu \\ y &= CTa \end{aligned}}$$

- T can be:
  - Global modes: leading eigenvectors of A
  - POD modes: most energetic modes
  - Balanced modes: preserves relation between inputs & outputs

Controllable and Observable States

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- Controllable states
  - Flow states most easily excited by input/actuator
  - Solution: POD modes
  - Diagonalize the correlation matrix of the flow

$$\dot{q} = Aq + Bu \quad \longrightarrow \quad X = \int_0^t q(t)q(t)^* \mathrm{d}t$$

- Observable states
  - Flow states that will most easily excite output/sensor
  - Solution: adjoint POD modes
  - Diagonalize correlation matrix of the adjoint system

$$\dot{q}^+ = A^* q^+ + C^* v \quad \longrightarrow \quad Y = \int_0^t q^+ (q^+)^* \mathrm{d}t$$

- Diagonalization of matrices too expensive

  → Use the method of snapshots

Snapshot-Based Balanced Truncation

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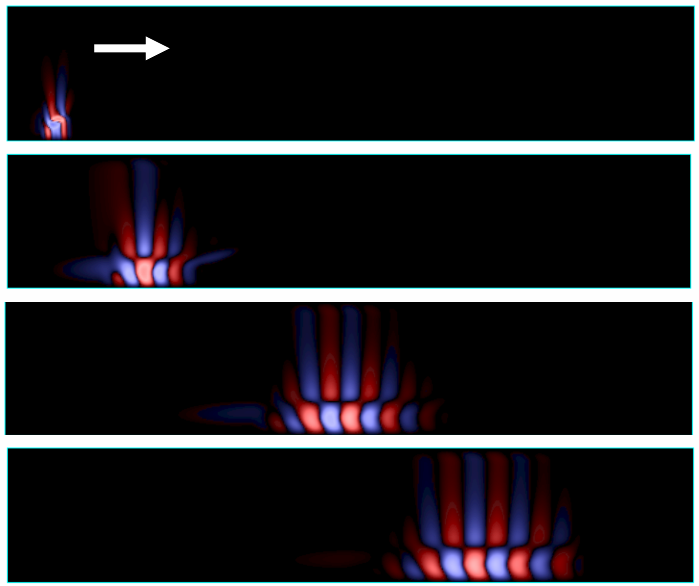
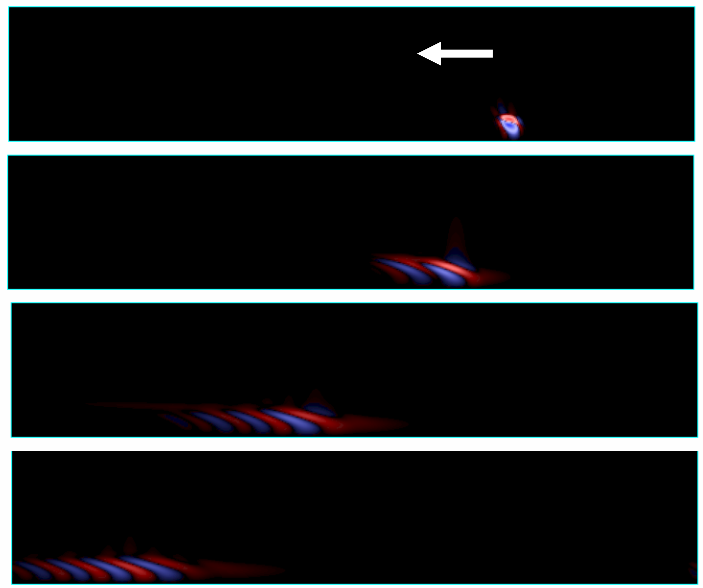
Snapshots of direct simulation:

$$X = [q(t_1) \dots q(t_m)]$$



Snapshots of adjoint simulation:

$$Y = [q^+(t_1) \dots q^+(t_m)]$$

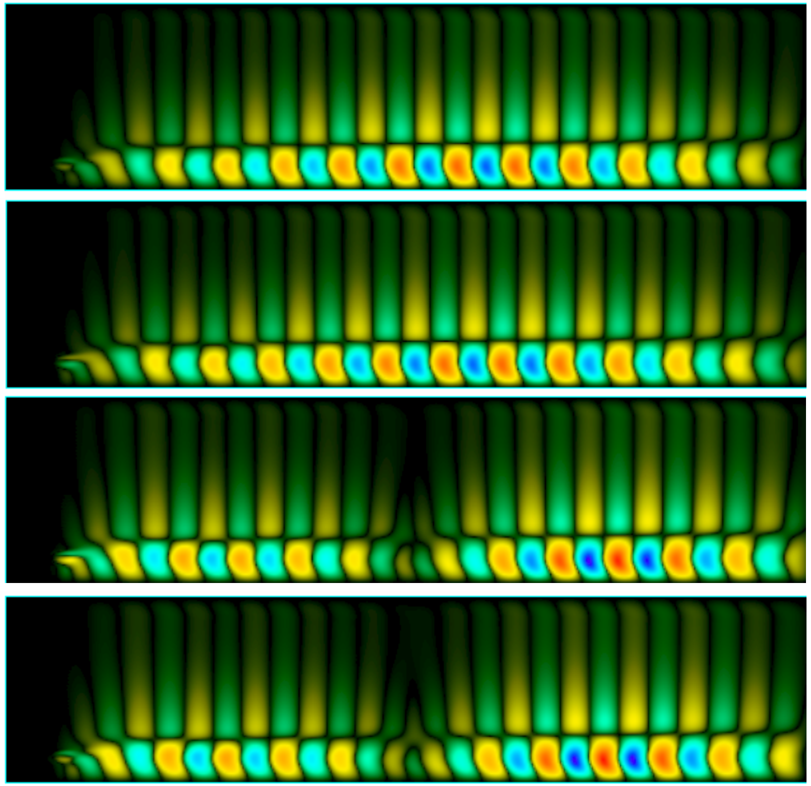


POD modes: SVD(X*X)

Adjoint POD modes: SVD(Y*Y)

Balanced Modes as Expansion Basis

- Combine snapshots of direct and adjoint simulation *(Rowley 2005)*
- Balanced modes: SVD(Y^*X)



- Projection onto balanced modes to obtain reduced system

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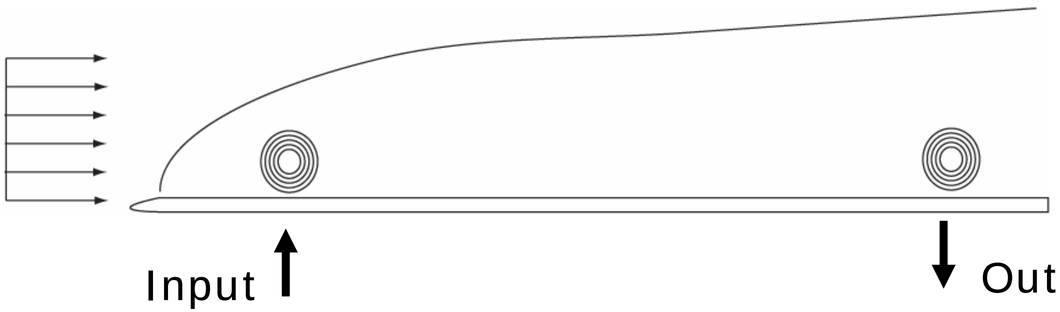
Comparison of Reduced Model with DNS



DNS: $n=10^5$

ROM: $m=50$

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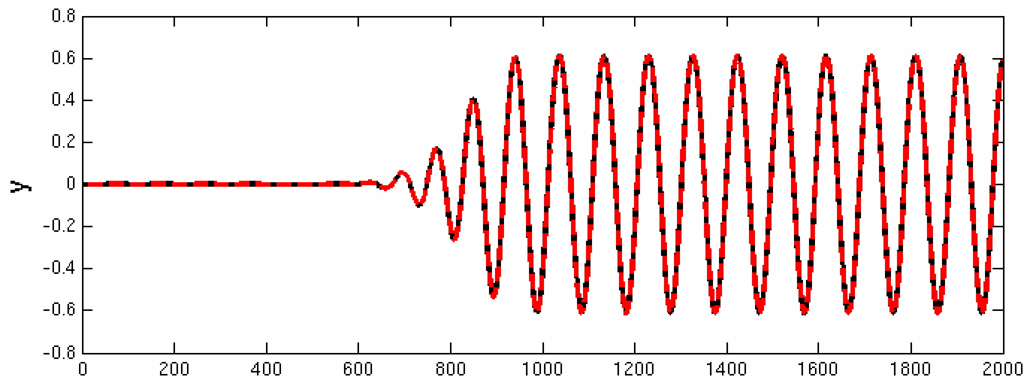
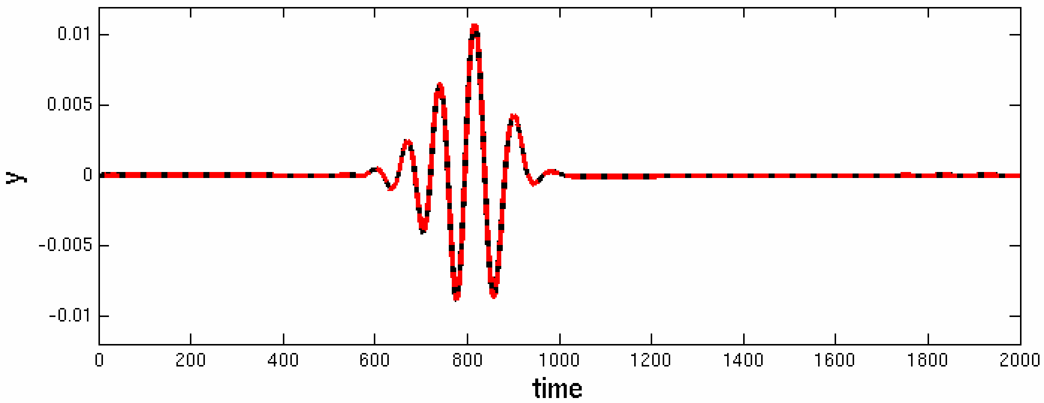
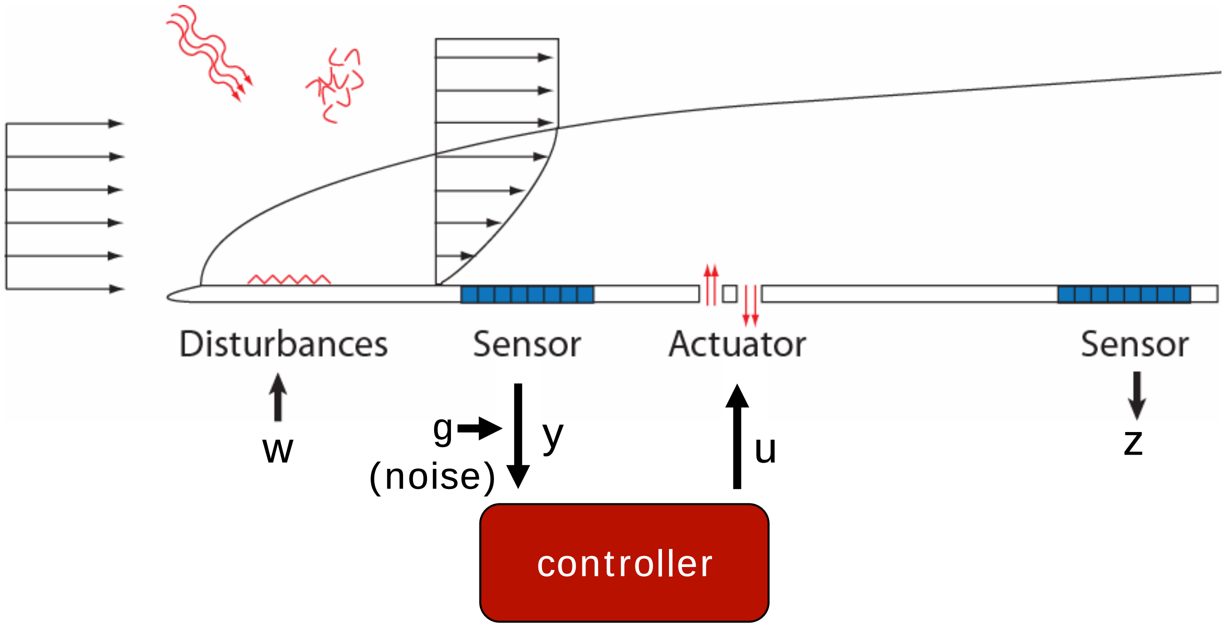
Frequency response



Impulse response



Optimal Feedback Control – LQG



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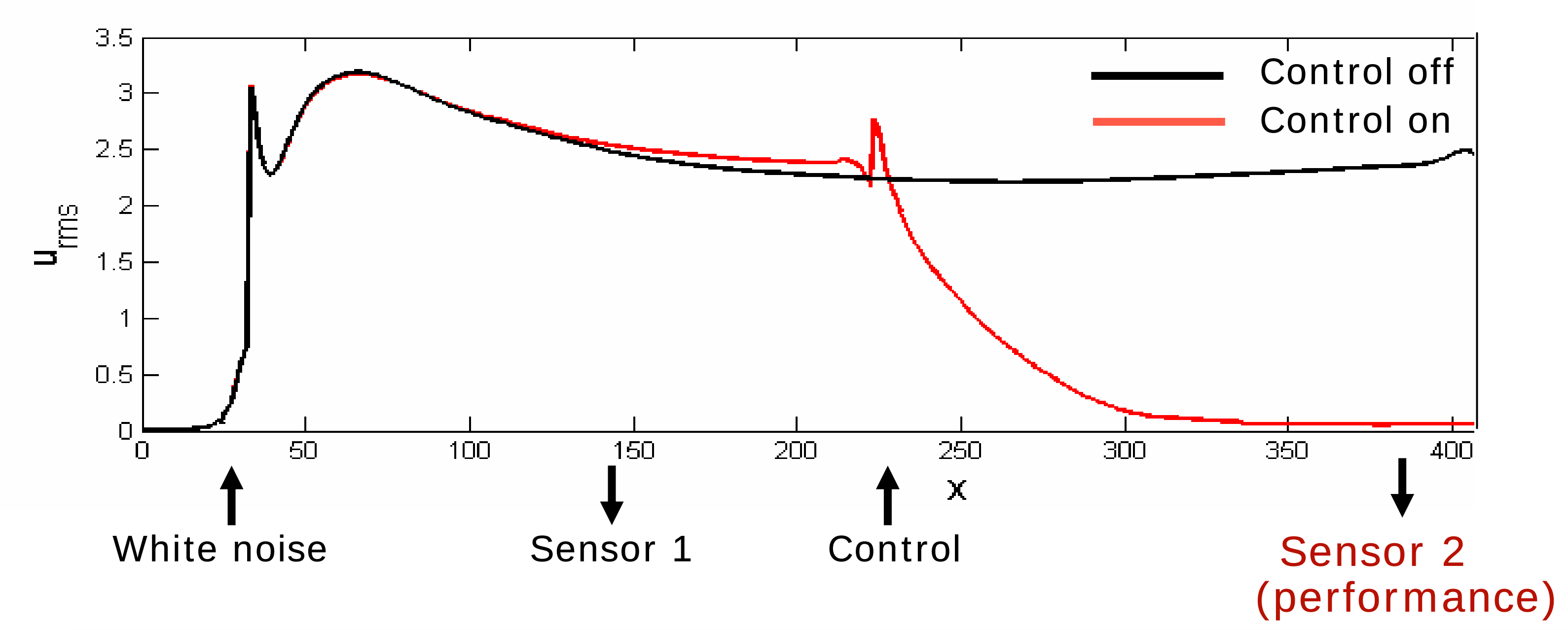
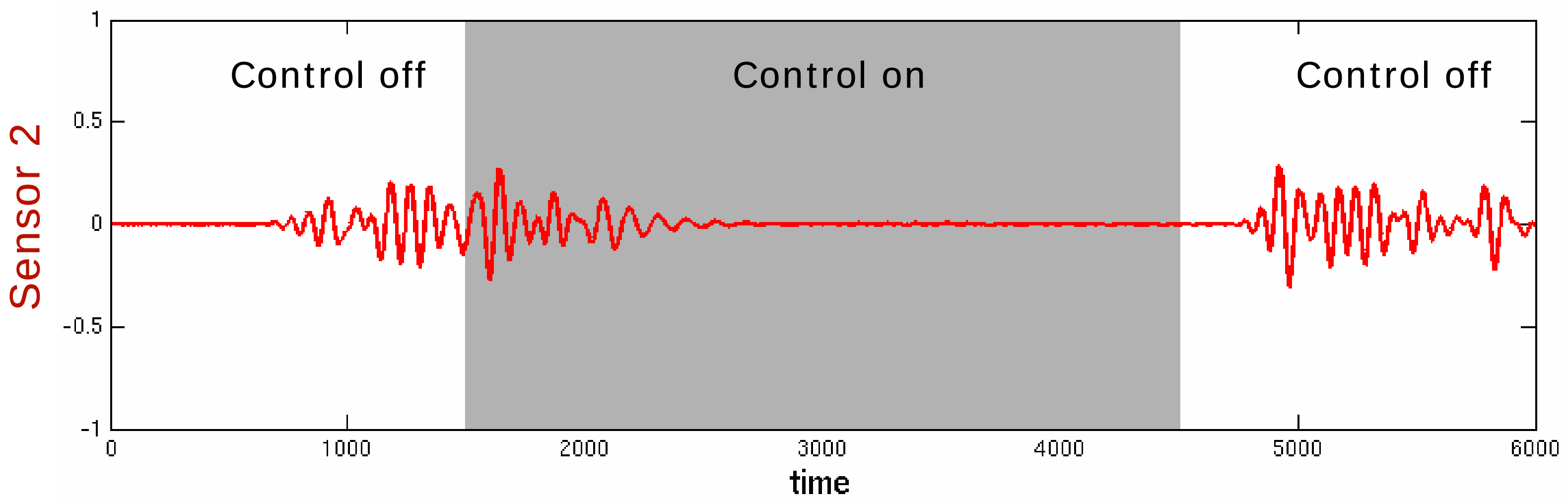
Find control signal *u(t)* based on the measurements *y(t)* such that the influence of external disturbances *w(t)* and *g(t)* on the output *z(t)* is minimized.

→ Solution: LQG/H2

Performance of controlled system





Conclusions

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- Intuition-based control design is useful for understanding the physics, but is often not the most efficient or robust method of control

- Model reduction and in particular balanced truncation enables the use of advanced tools from control theory in flow control