

Linear control of 3D disturbances on a flat-plate



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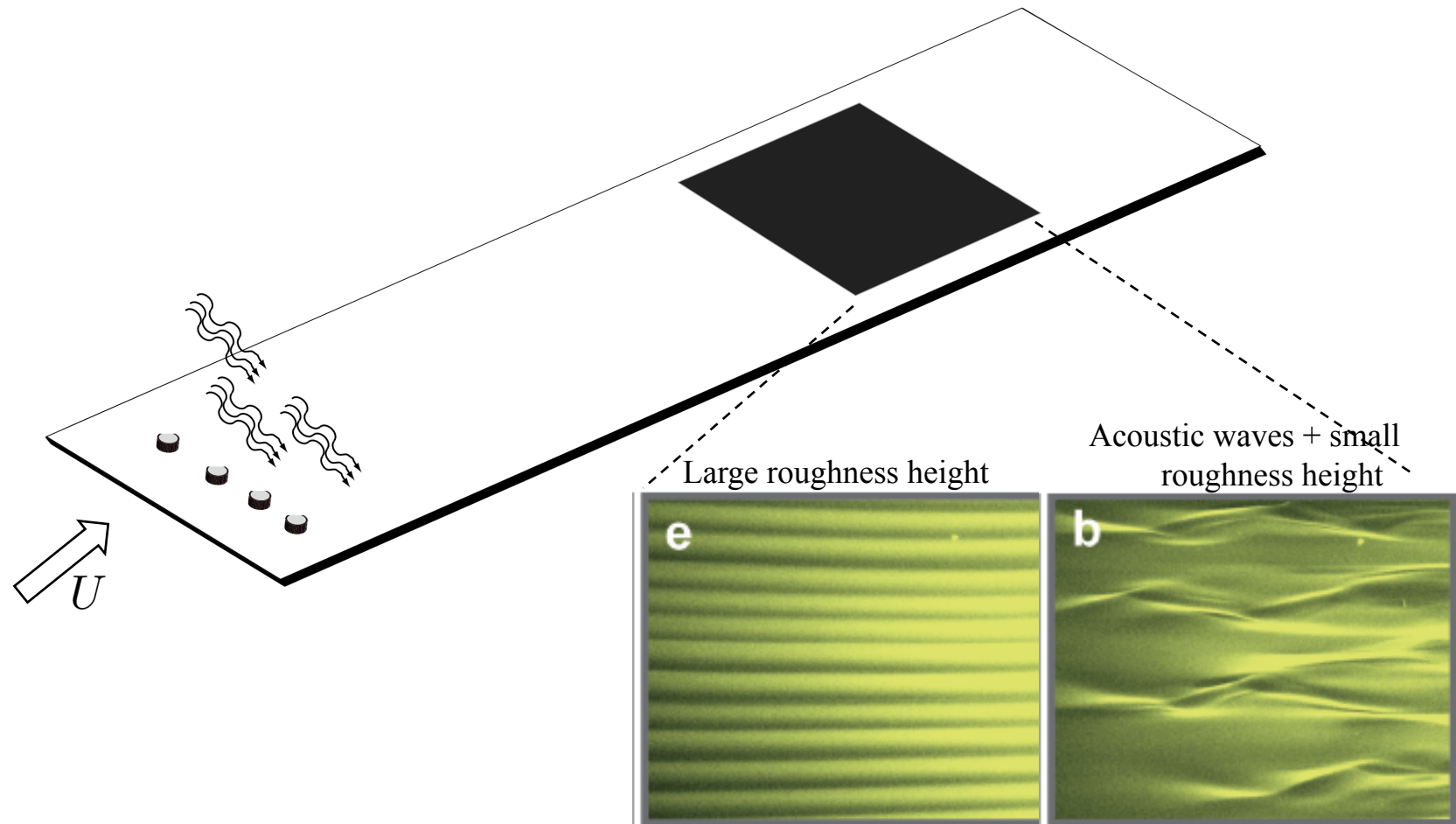
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7th IUTAM Symposium on Laminar-Turbulence Transition
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Transition control

- Sensitive dynamics \rightarrow response depends on initial conditions



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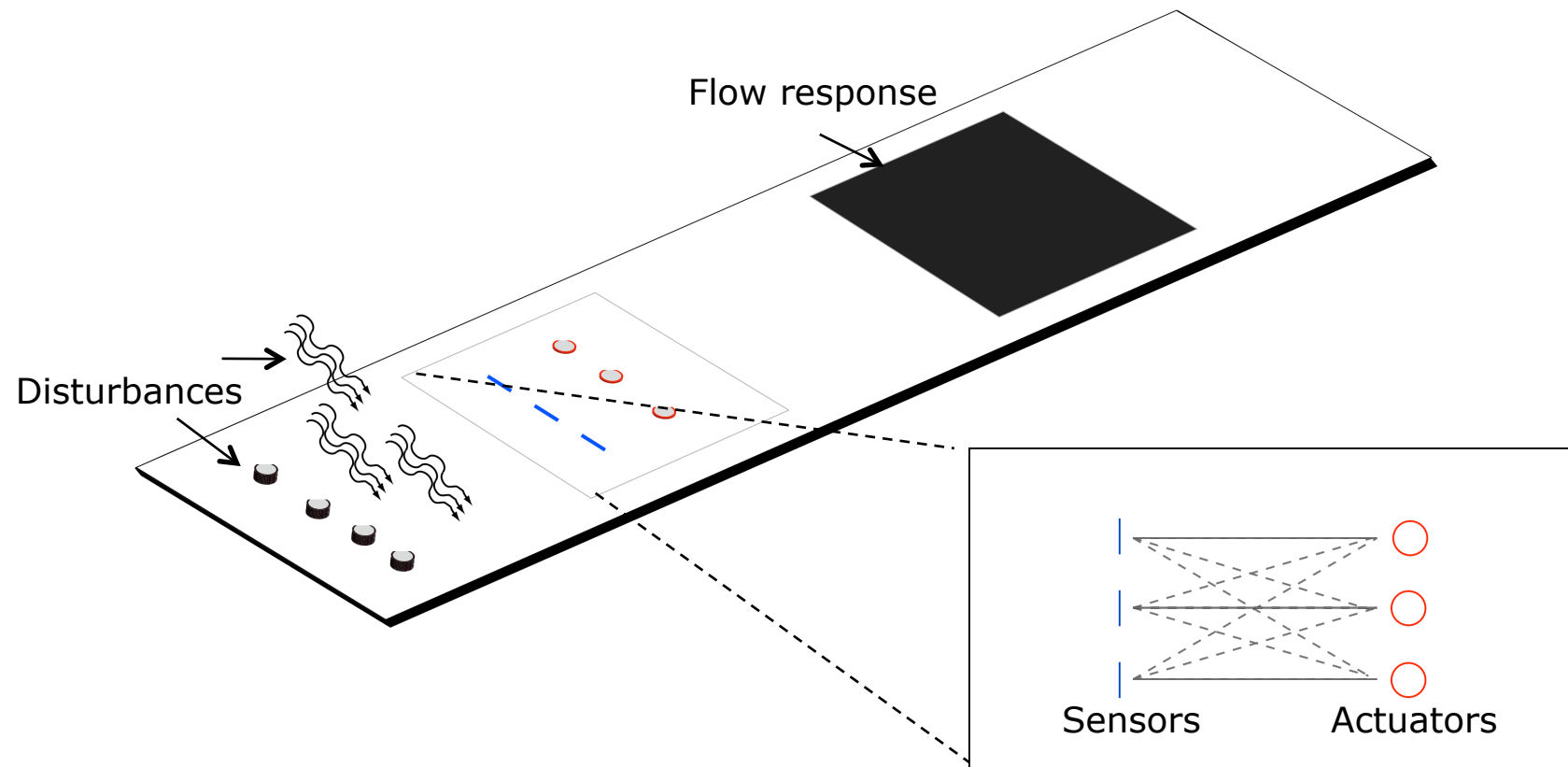
- Reduce sensitivity to external disturbances \rightarrow transition delay

Transition control

- No exact knowledge of disturbances → Feedback control
 - Inputs: disturbances & actuators
 - Outputs: sensors for estimation & performance



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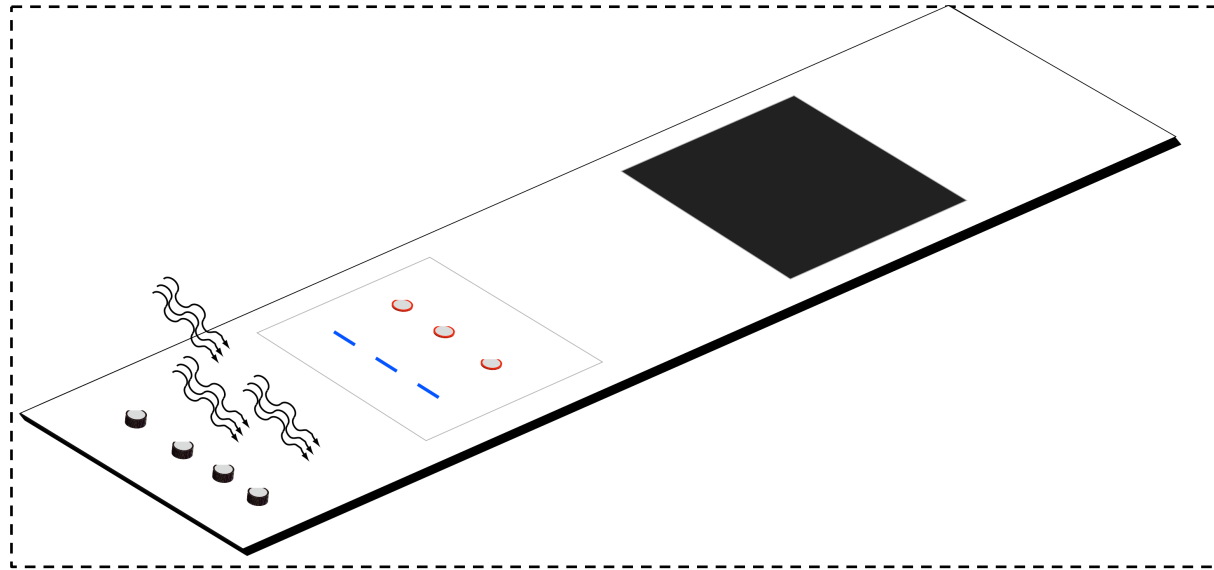


$$Re = U\delta_0^*/\nu = 1000$$

500 × 20 × 80 (Box)

384 × 81 × 80 (Grid)

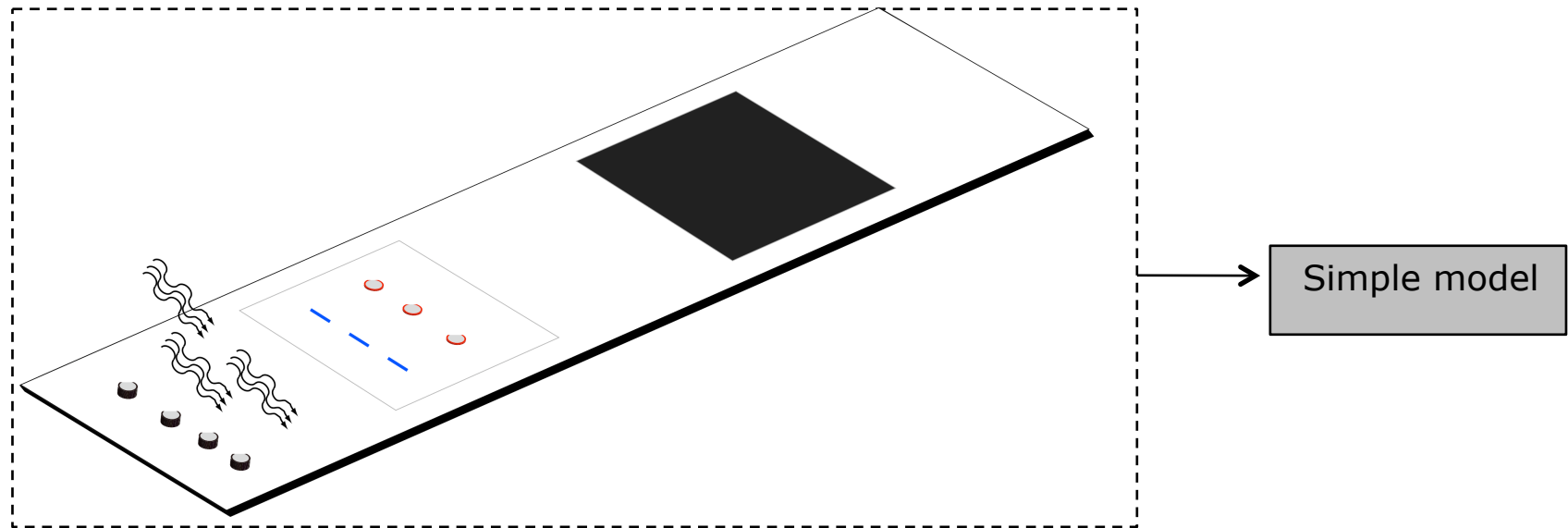
Control Design – 4 Steps



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1. Construct **plant**: Flow, inputs, outputs

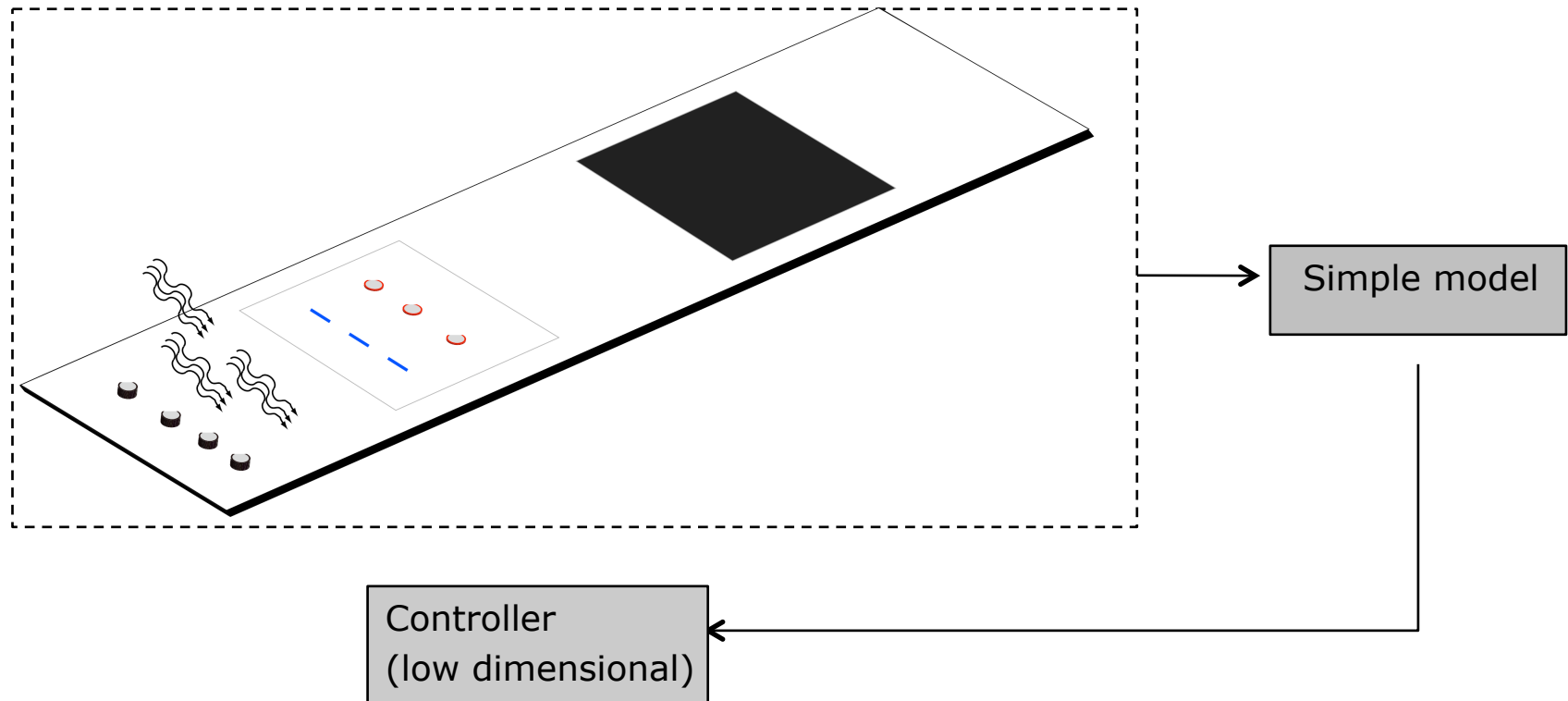
Control Design – 4 Steps



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1. Construct **plant**: Flow, inputs, outputs
2. Construct **reduced model** from the plant using balanced truncation

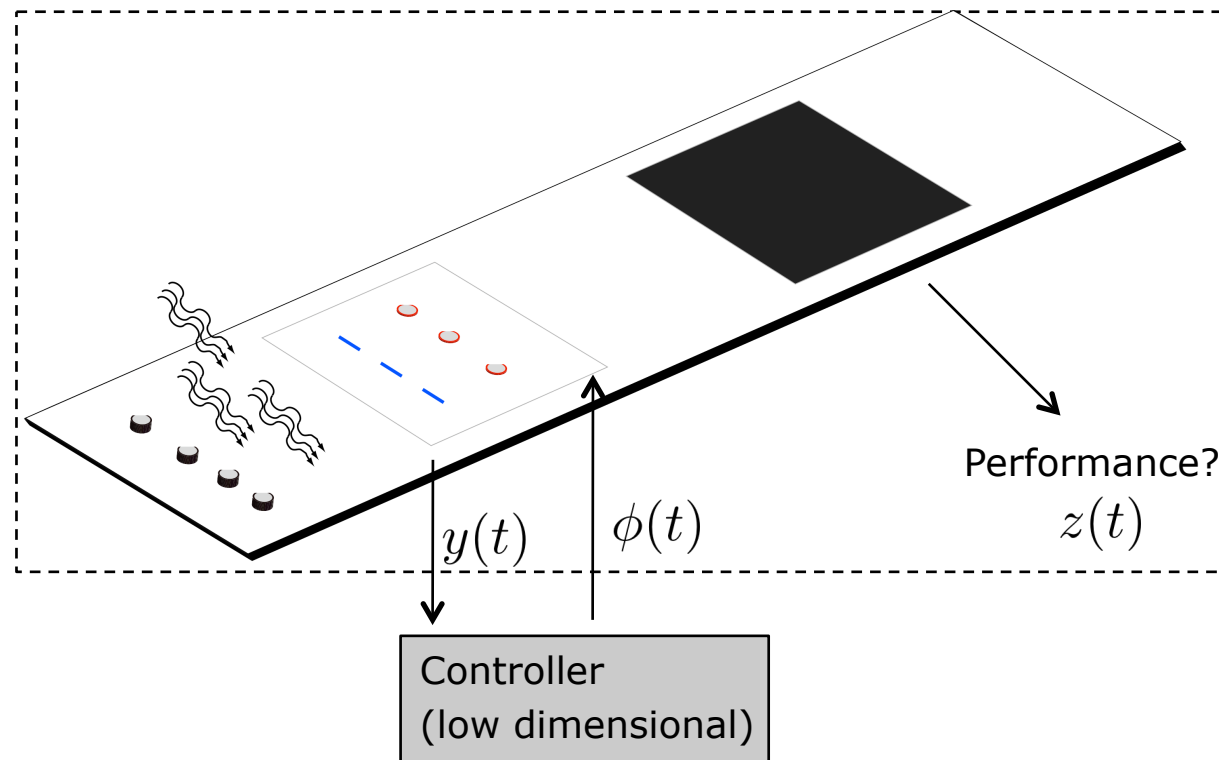
Control Design – 4 Steps



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1. Construct **plant**: Flow, inputs, outputs
2. Construct **reduced model** from the plant using balanced truncation
3. Design **controller** using the reduced-model

Control Design – 4 Steps



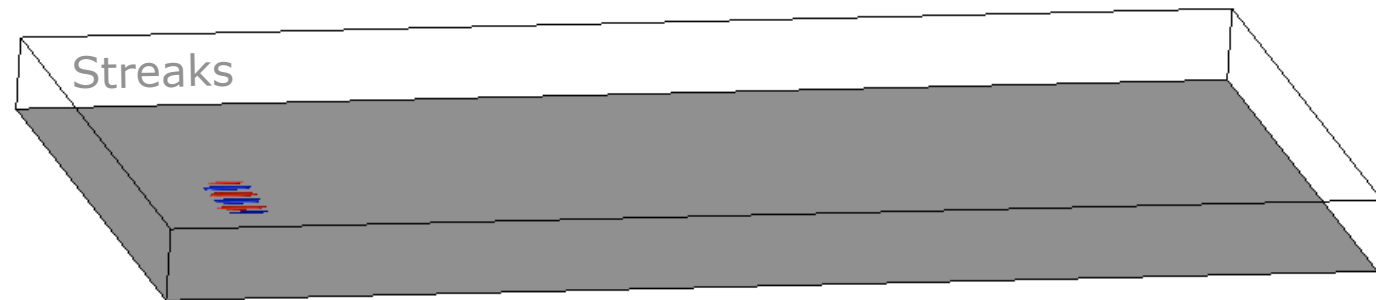
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1. Construct **plant**: Flow, inputs, outputs
2. Construct **reduced model** from the plant using balanced truncation
3. Design **controller** using the reduced-model
4. **Closed-loop**: Connect sensor to actuator using the reduced controller
Run small controller **online** and evaluate closed-loop performance

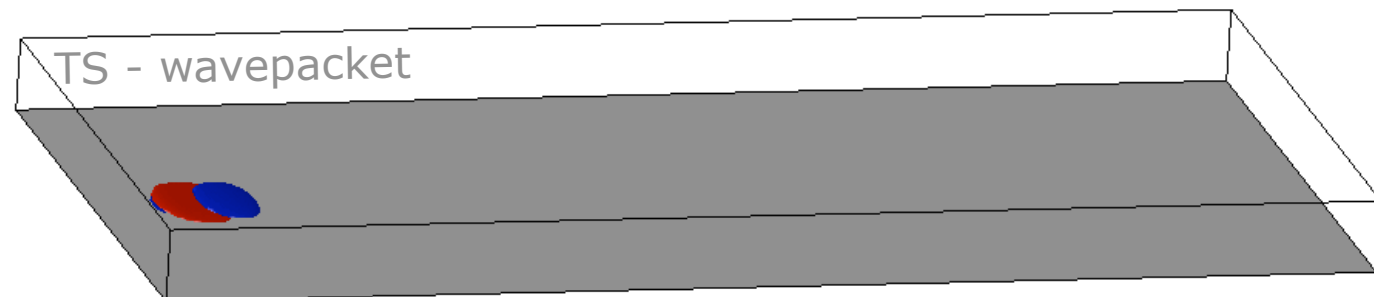
Inputs & Outputs

- **Inputs:** Optimal disturbances giving largest growth at time T (Ref: Monokrousos et. Al. (IUTAM 09, P30))

- Short time: Lift-up mechanism \rightarrow streaky structures



- Long time: Orr mechanism/TS wavepacket \rightarrow 2D wavepacket



- **Outputs:** leading POD modes \rightarrow most of energetic structures



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Positive u -vel
Negative u -vel

Projection onto balanced modes

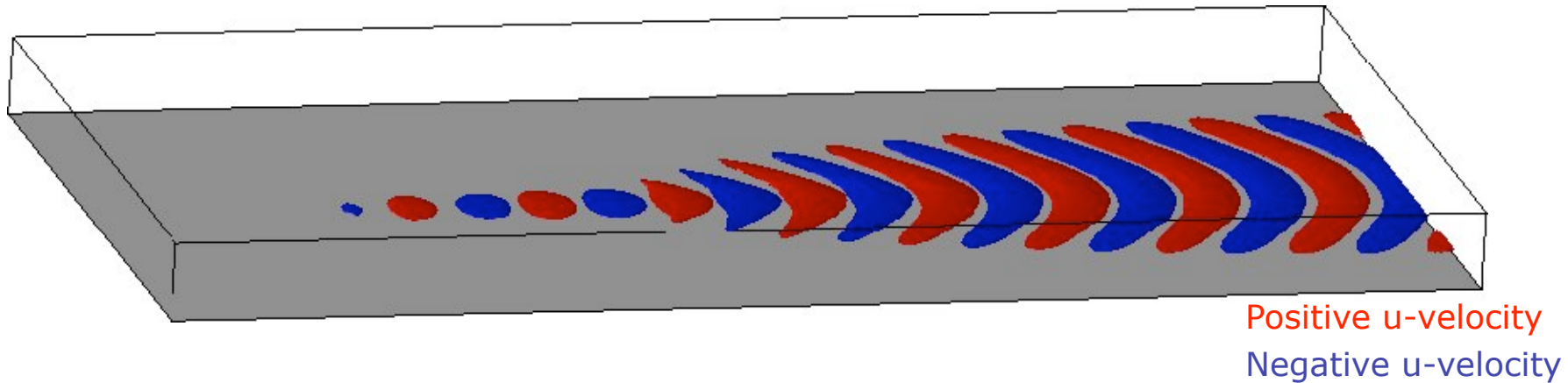
- Reduced-order model via a Galerkin (oblique) projection onto spatial modes
 - POD modes – captures most energetic structures
 - Global eigenmodes – captures instability dynamics
 - **Balanced modes – captures the input-output behavior**
- Snapshot method to compute balanced modes (Rowley, 2005)
 1. Collect flow field snapshots from impulse response simulation for each input
 2. Collect flow field snapshots from impulse response simulation of the adjoint equation for each output
 3. One small SVD (size: adjoint snapshots x forward snapshots)



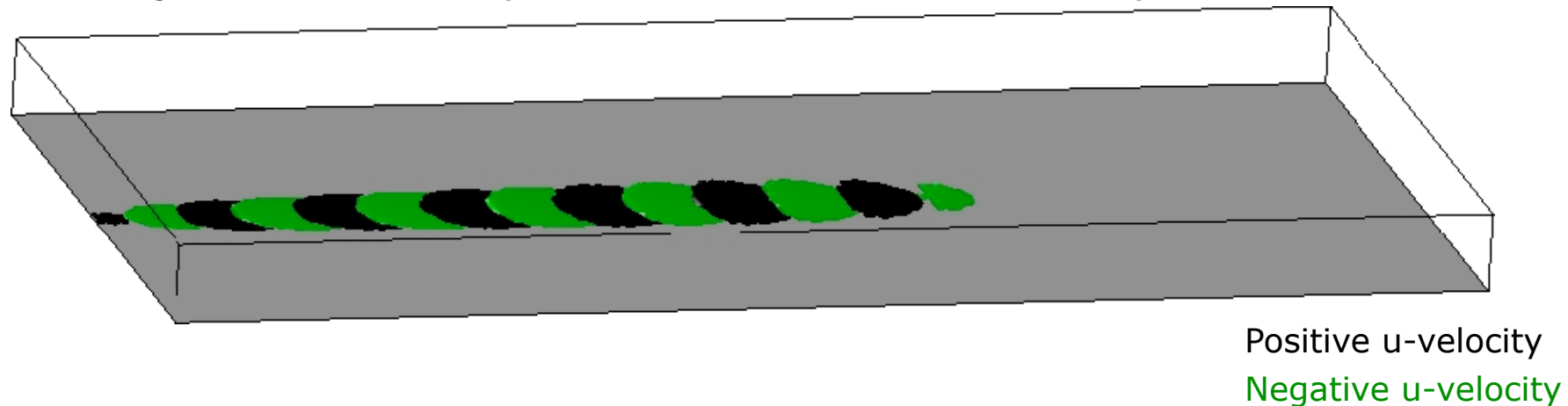
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Balanced mode #1: TS wave

- Direct mode – wavepacket downstream



- Adjoint mode – upstream tilted structures upstream



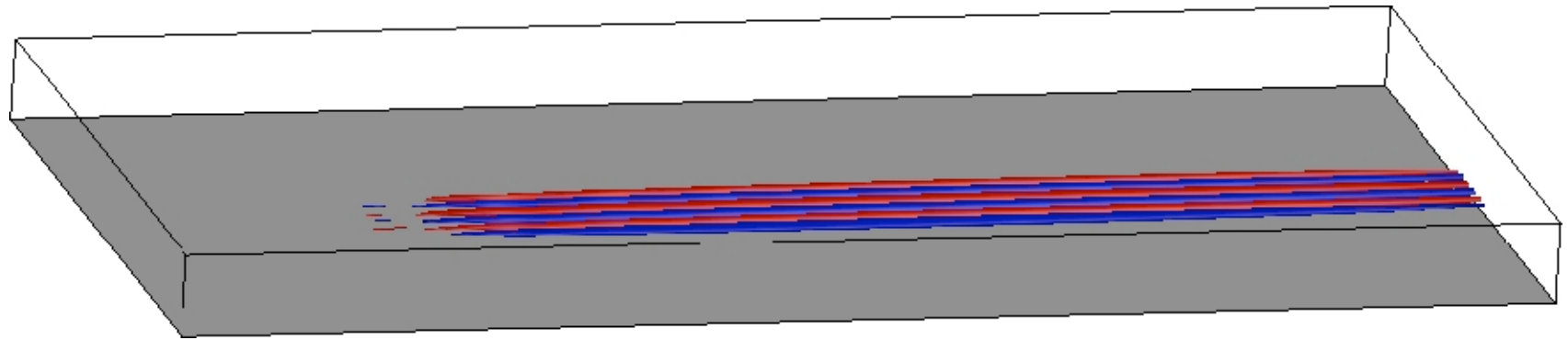
- Streamwise non-normality



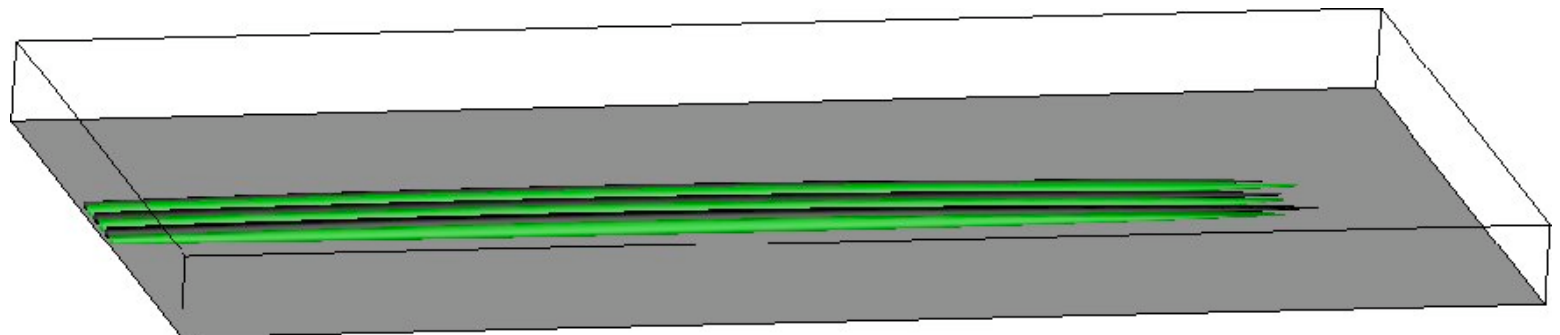
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Balanced mode #11: Streak

- Direct mode – streaky structures in u



- Adjoint mode – streamwise vortices (v, w)



- Component-wise non-normality

Positive u -velocity
Negative u -velocity

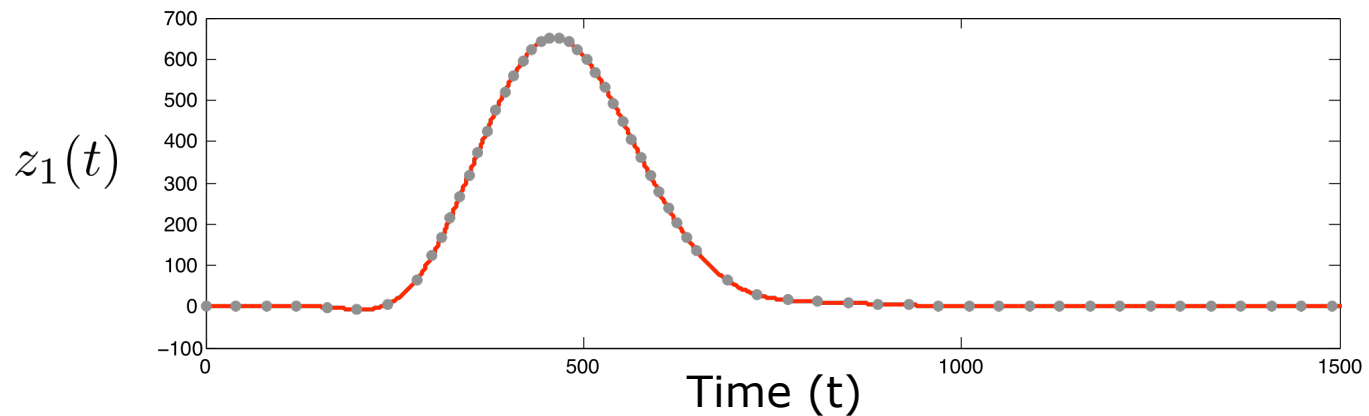


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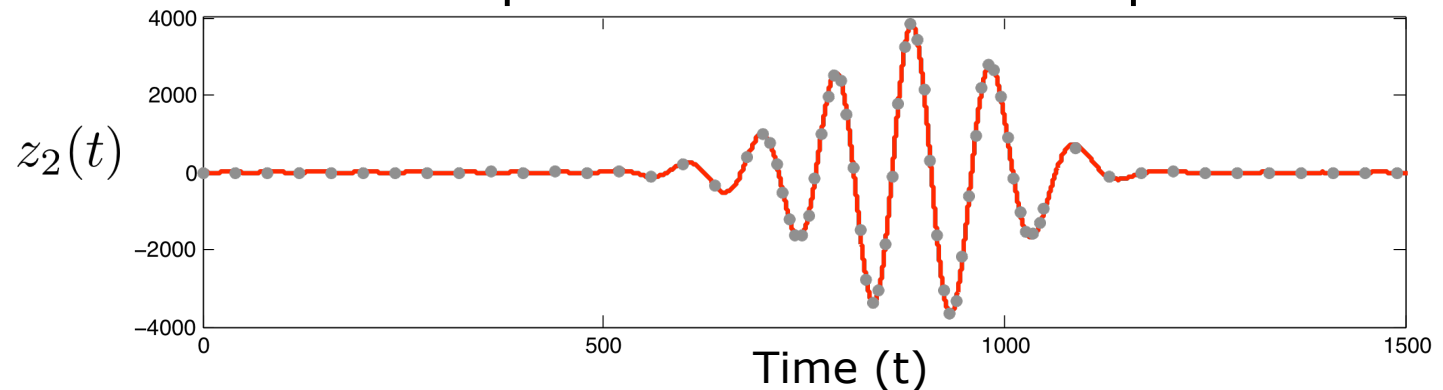
Positive v -velocity
Negative v -velocity

Reduced-order model (size=50)

- Compare **output signals** of DNS and small model for **impulsive input**
- Initial condition: Streamwise vortices



- Initial condition: Upstream tilted 2D wavepacket



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DNS: $n=7\,000\,000$
 ROM: $m=50$

Reduced-order controller

- B2/C2: actuators/sensors:
 - Localized wavepackets near the wall
- C1: objective functions
 - A number of localized spanwise Fourier function → defines a spatial region downstream
- Design H2/LQG controller
 - Small: 50 degrees of freedom
 - Optimal: one Riccati for estimation & one Riccati for control
 - Control penalty: $l=1$, degree of measurements noise: $\alpha = 0.1$



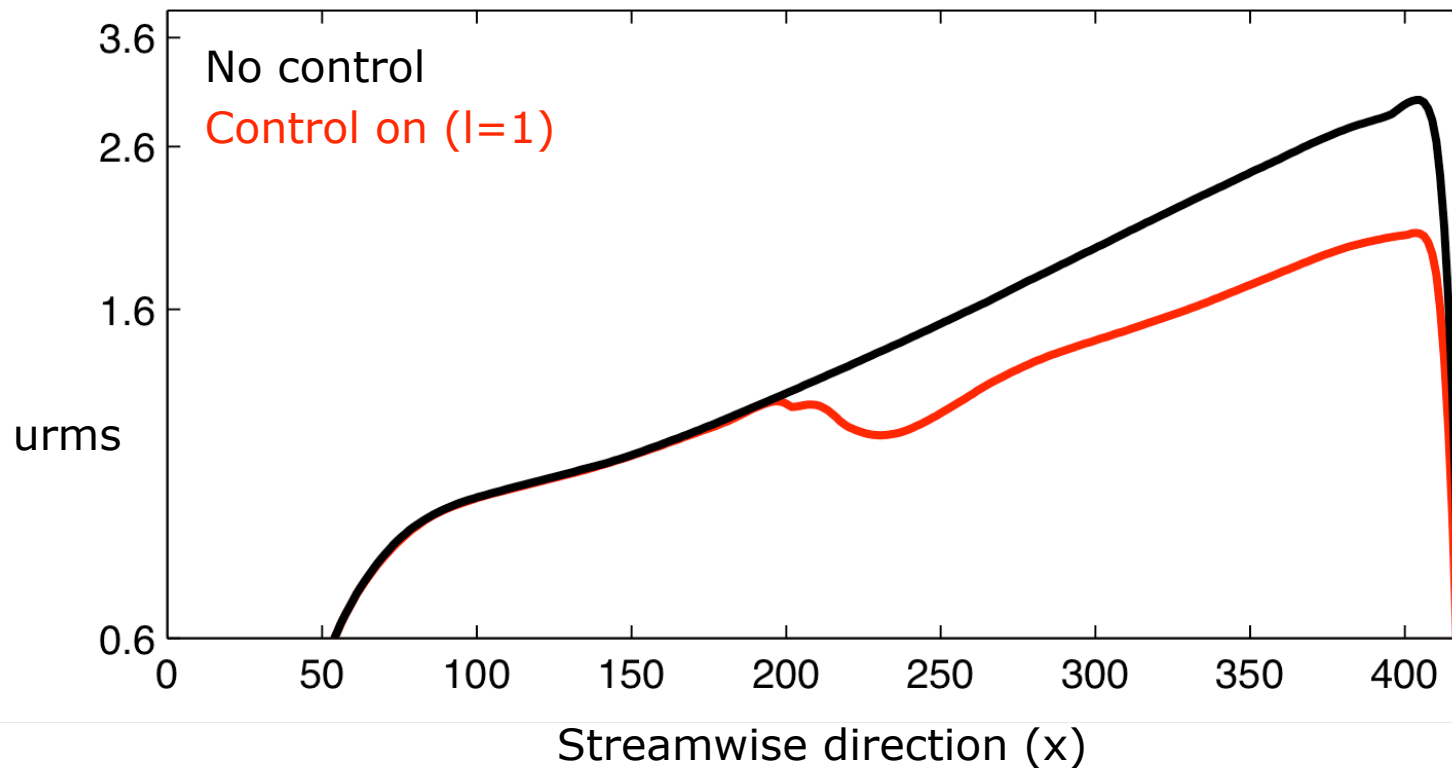
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Control performance

- Response to stochastic (white noise) excitation
- RMS of streamwise velocity component



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Outlook

- Design a controller that efficiently reduced disturbance growth for any disturbance
 - Design “realistic” actuators and sensors
 - Extend basis used for the objective function to span larger domain
- Compensate the nonlinear flow
- Move to wind-tunnel:
 - Use the computed low-dimensional controller
 - Use the numerical model for parametric studies (i.e. actuator/sensor shapes and locations, control configuration etc...)



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