Nonlinear PSE:
Interaction between streamwise streaks and Tollmien-Schlichting waves

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Outline & Aim of Presentation

- Observation:

  The growth and breakdown of 2D Tollmien-Schlichting Waves are modified in the presence of streaky structures

  How do these two disturbances interact? Can one be used to control the other?

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- Outline:
  1. Background on transition to turbulence
  2. Summary of numerical & experimental results (Fransson, Brandt, Schlatter, de Lange, Talamelli, Cossu)
  3. Parametric study using nonliner PSE

Background

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Flat-Plate Boundary Layer

- Receptivity
- Laminar (linear) region
- Transition
  1. Classical transition: Tollmien-Schlichting (TS) waves
  2. By-pass transition: Streaks
- Turbulence

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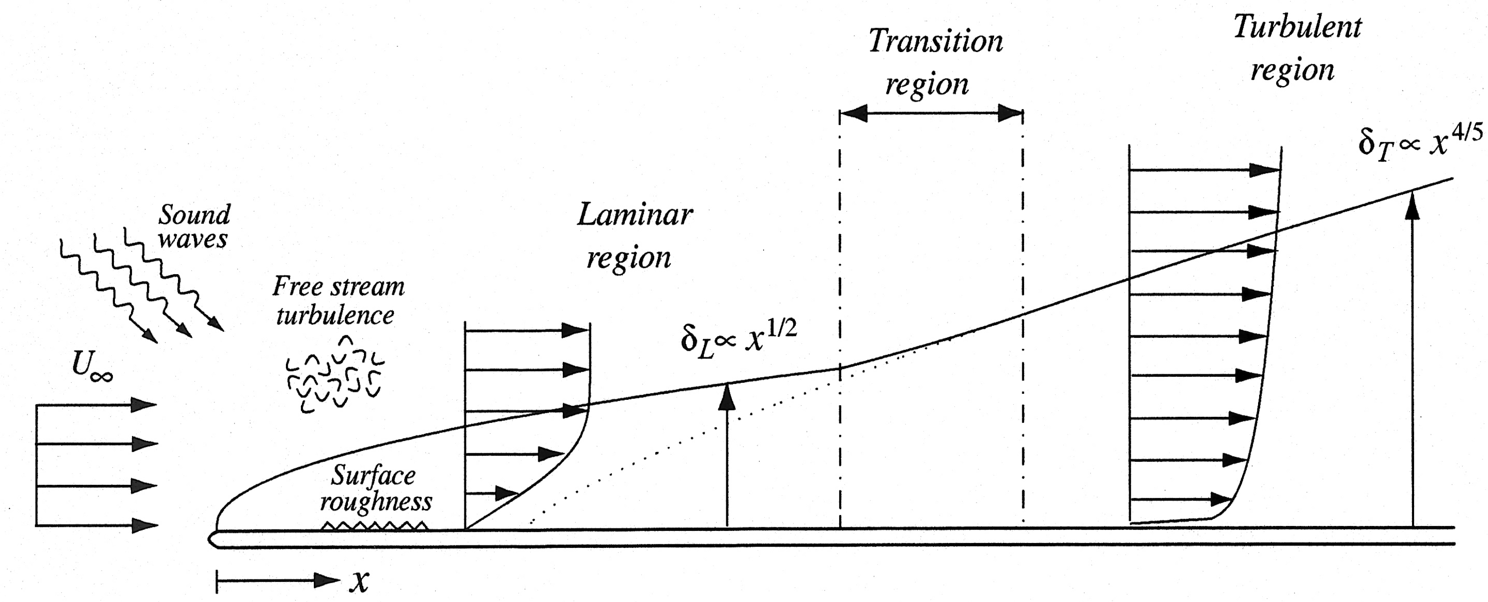
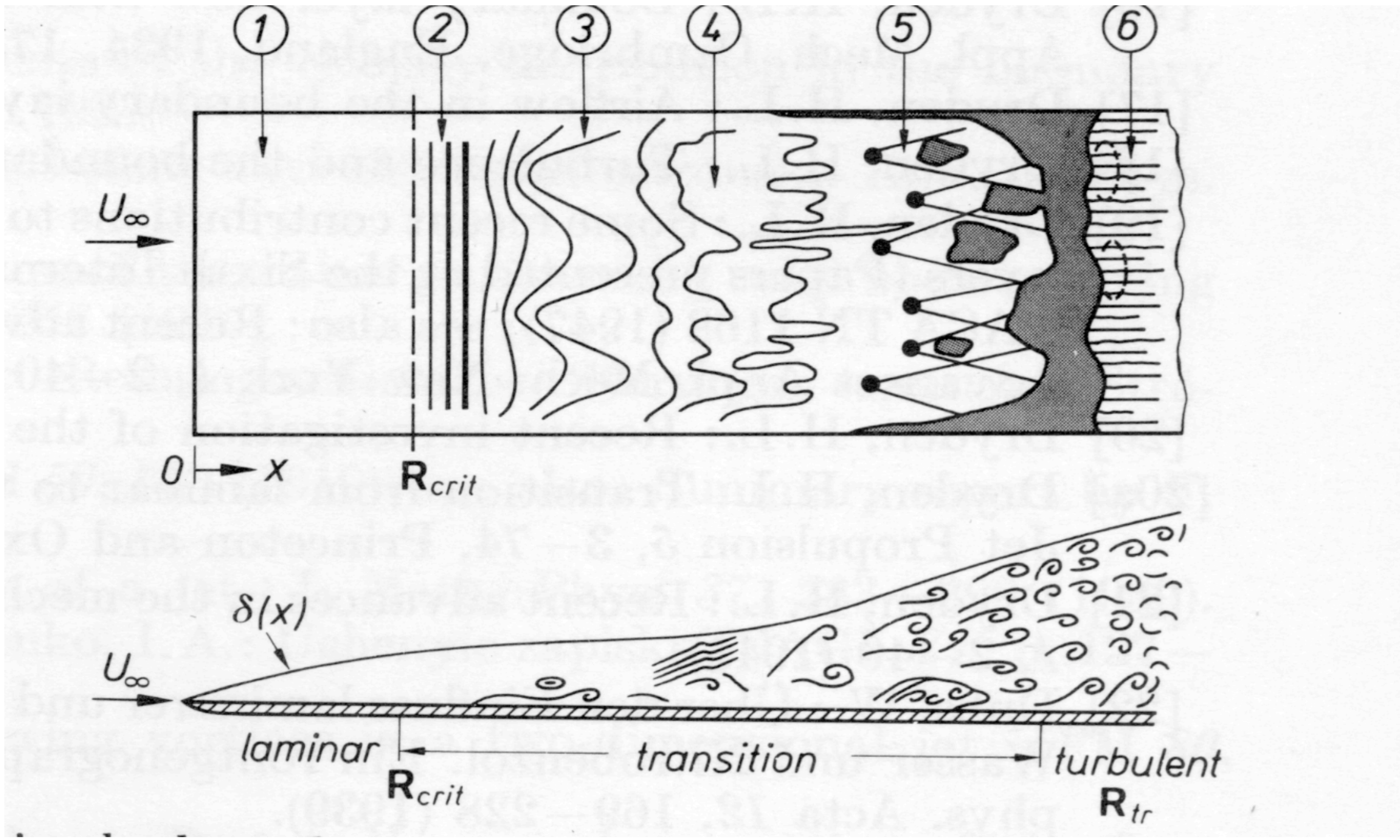


Sketch of a boundary layer developing on a flat plate

Classical Transition

- Low levels of background noise (<1%)

→ exponential modal growth



Schlichting (1977)

2D primary instability (TS waves)

↓

Secondary instability (K- & H-modes)

↓

Turbulent spots

↓

Turbulence

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Bypass Transition

- High levels of free-stream turbulence (>1%)

→ exponential growth of TS waves is "bypassed"

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Matsubara & Alfredsson (2001)

U1 →

Non-modal growth of 3D streaks

⬇

Secondary instability of streaks

⬇

Turbulent spots

⬇

Turbulence

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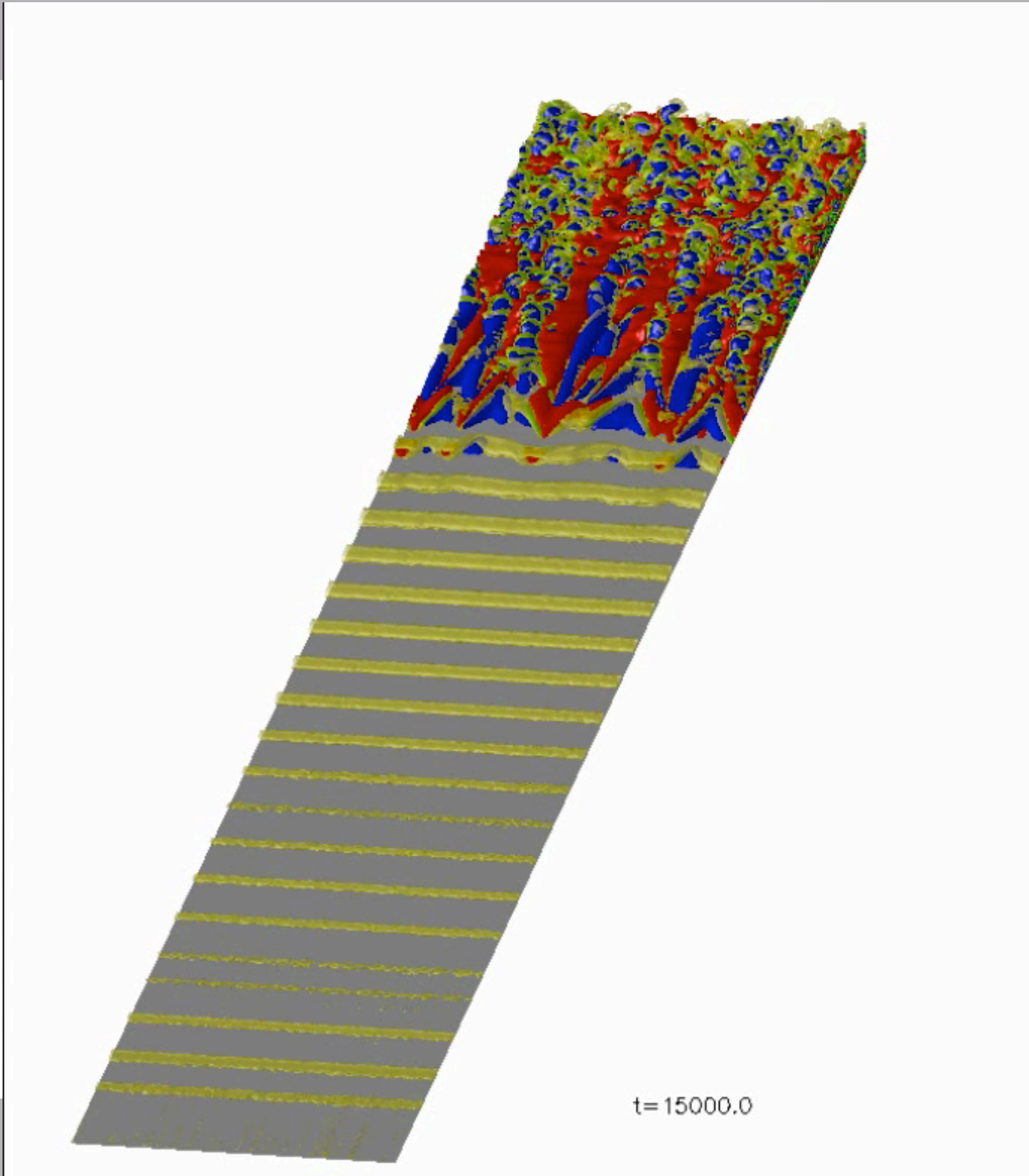

λ2 criterion
high dist. velocity
low dist. velocity

.ES: Performed by Philipp Schlatter

DNS: Stabilizing Effect

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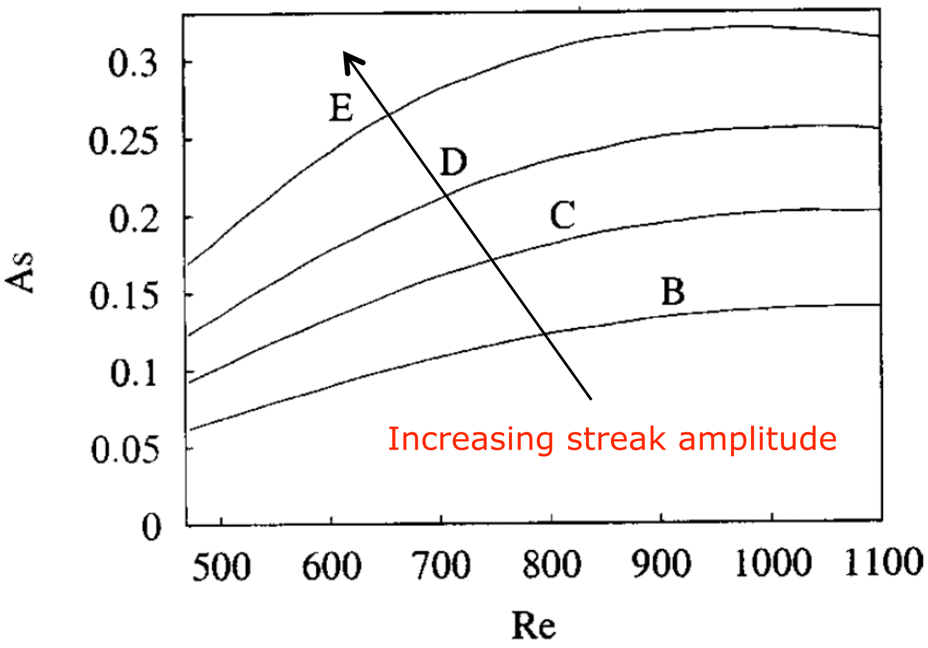
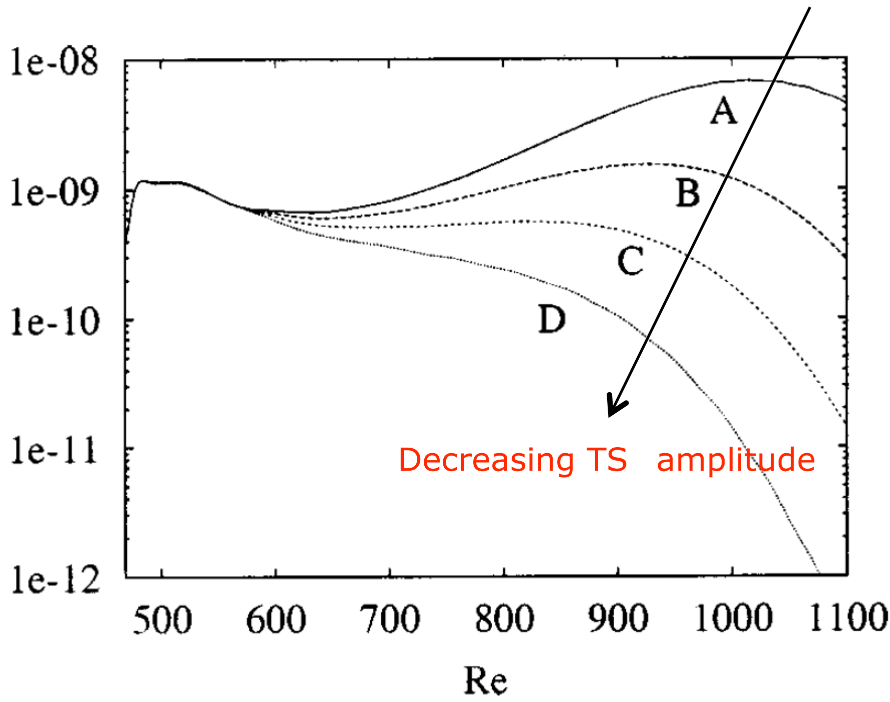
- Evolution of streaks



- Evolution of TS waves

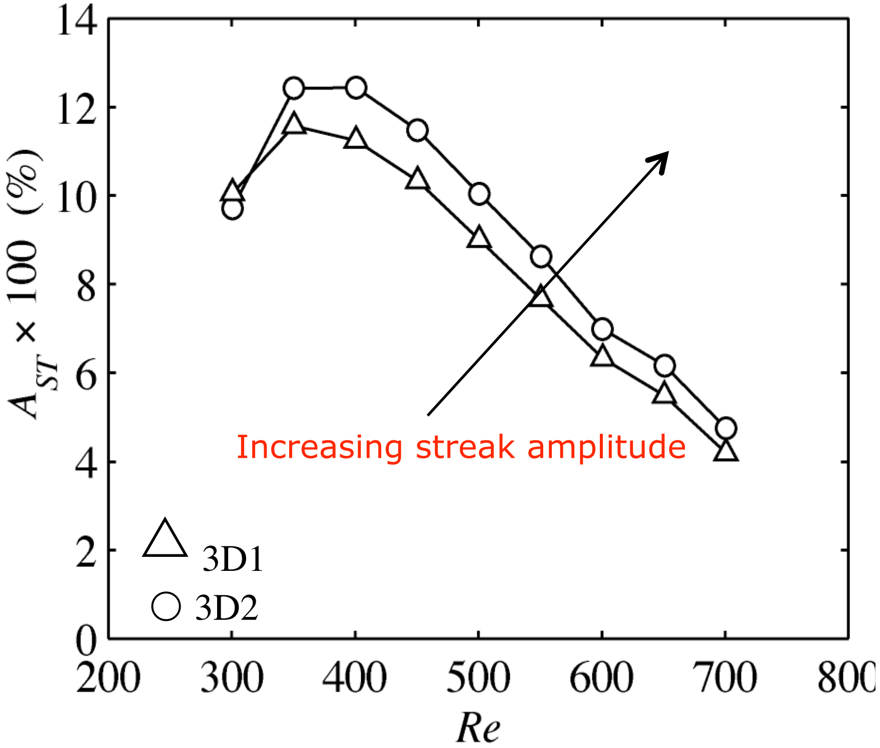


Streamwise streaks of low finite amplitude (<25%U1) are able to stabilize TS waves

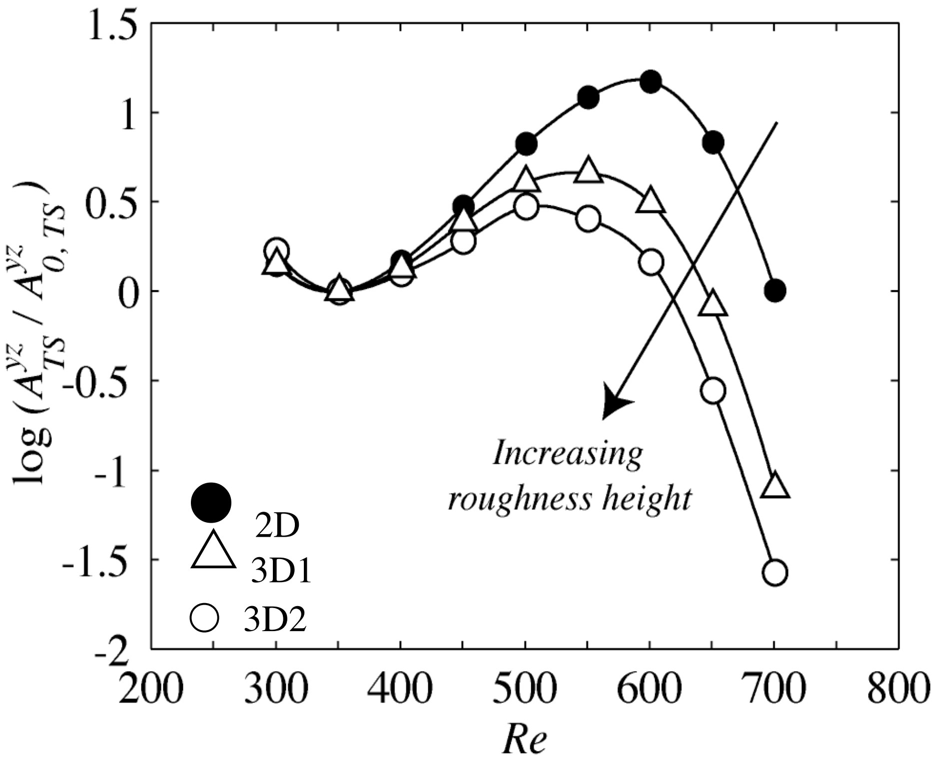
Ref: Cossu & Brandt, Phys. Fluids (2002)

Experiments: Stabilizing Effect

- Streaks generated by roughness



- TS waves generated by blowing and suction



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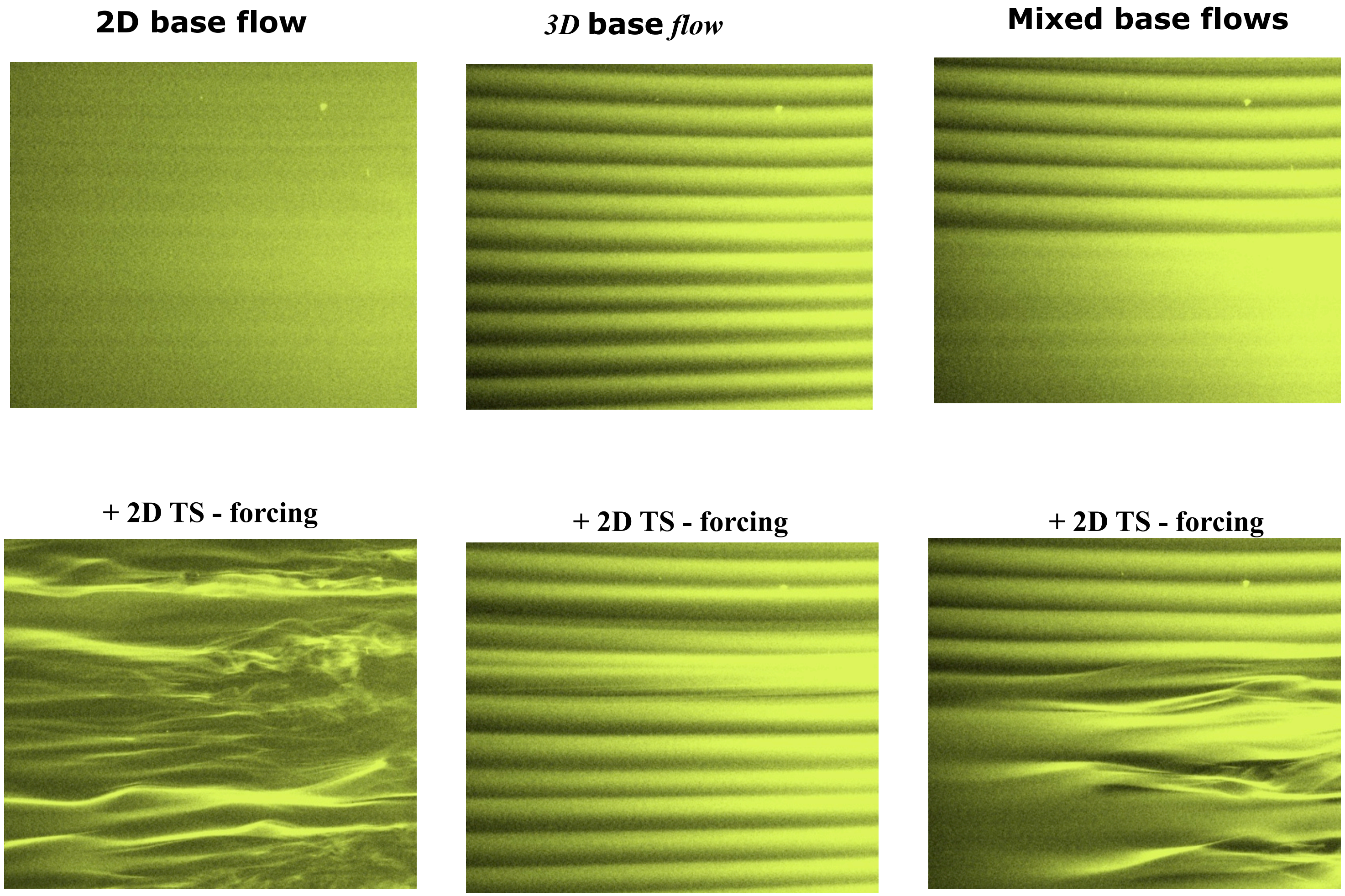
Ref: Fransson, Brandt, Talamelli & Cossu, Phys. Fluids (2005)

Experiments: Transition Delay



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Ref: Fransson, Talamelli, Brandt & Cossu, PRL (2006)

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Parabolized Stability Equations

Aim of Present Study

- Can we capture the nonlinear interaction of streaks and TS waves with nonlinear PSE?
- What are the possibilities of optimizing the stabilization process, e.g. what is the optimal streak spacing?

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Parabolized Stability Equations

- Developed by Herbert & Bertolotti (J. Fluid Mech, 1992)
- Convection dominated flows (e.g. flat-plate boundary-layer): all disturbances are convected downstream
- Effect of disturbance upstream is very small → neglect terms (elliptic) in Navier-Stokes
- Model nonparallel and nonlinear effects at fractional cost of DNS
- PSE reproduces established experimental & numerical results on transition up to the breakdown stage

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Formulation

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- Navier-Stokes equations and continuity (incompressible formulation) governing disturbance behavior

$$
\begin{cases}
\dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathrm{U} + (\mathrm{U}\cdot\nabla)\mathbf{u} + \nabla p - Re^{-1}\nabla^2\mathbf{u} = -(\mathbf{u}\cdot\nabla)\mathbf{u} \\
\nabla\cdot\mathbf{u} = 0
\end{cases}
$$

- Baseflow: $\mathrm{U} = (U(x,y), V(x,y), 0)$

- Perturbation:

$$
\begin{aligned}
\mathbf{q}(x,y,z,t) &= \tilde{\mathbf{q}}(x,y)\exp(i\Theta(x,z,t)) \\
\Theta(x,z,t) &= \int_{x_0}^{x_1}\alpha(x)\;dx + \beta z - \omega t
\end{aligned}
$$

Modal (exponential) Growth

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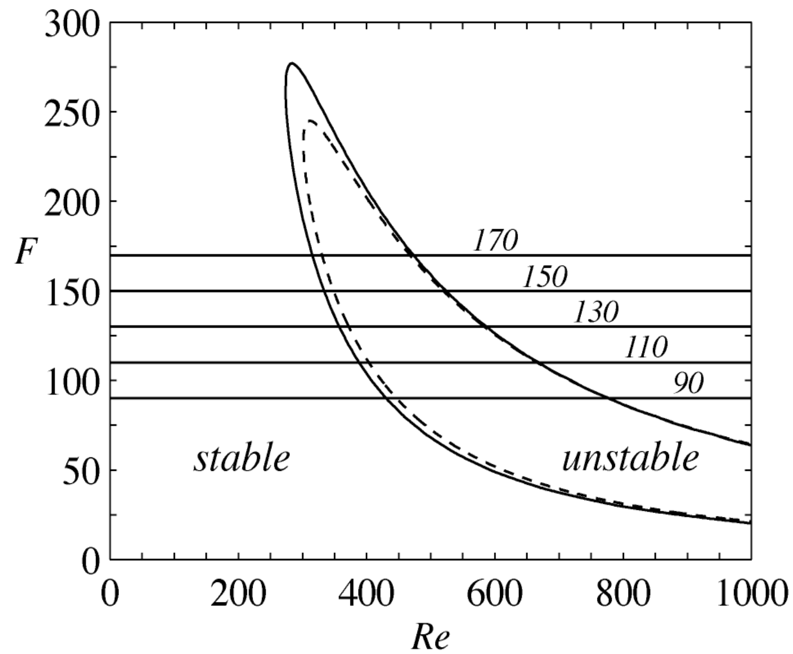
- Classical linear stability analysis for parallel flows
  Orr-Sommerfeld equations, evolution of eigenmodes

- Grow in finite region in streamwise region (between branch I & II)



- Takes into account nonparallel and nonlinear effects

- Described by PSE scaling

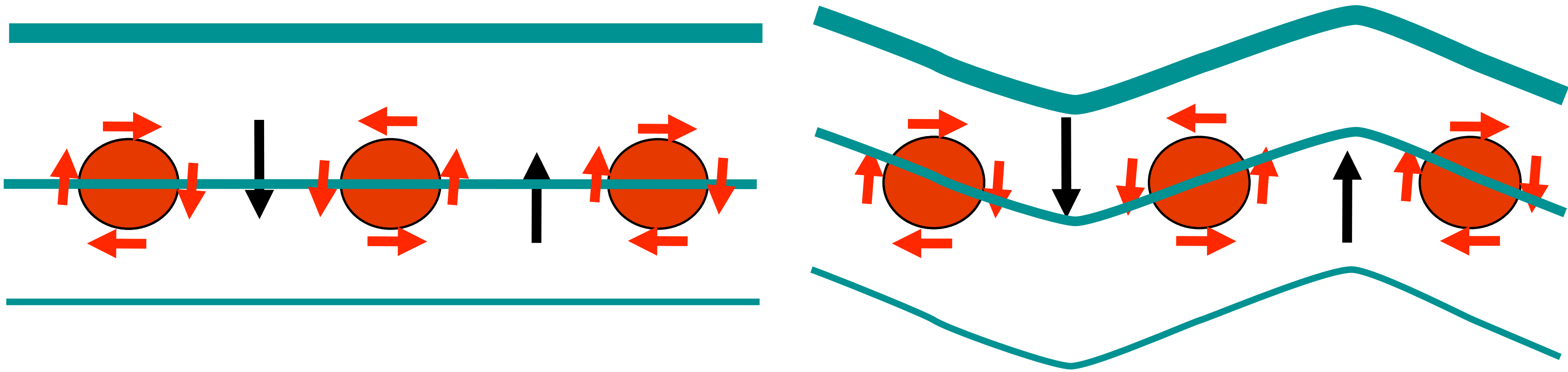
$$U, u, v, w \sim U_\infty$$
$$V \sim U_\infty \delta / L \qquad \delta \ll L$$

- Neglect

$$Re_\delta = \frac{\delta U_\infty}{\nu}$$

Nonmodal Growth

- Streamwise vortices in a shear flow ➔ streamwise streaks
  Taylor 1939,..., Landahl 1980, Ellingsen & Palm 1975...



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- Related to non normality of the global linearised operator
  (Trefethen *et al.* 1993, Schmid & Hennigson 2001)

- Described by boundary layer scalings (neglect) :

$$(U, u, w) \sim U_\infty$$
$$(V, v) \sim U_\infty \delta / L \qquad \delta \ll L$$

Modal vs. Nonmodal Stability

- Range of frequencies and spanwise wavenumbers well separated for streamwise streaks and TS waves
- Modified PSE = PSE scalings + BLE scalings

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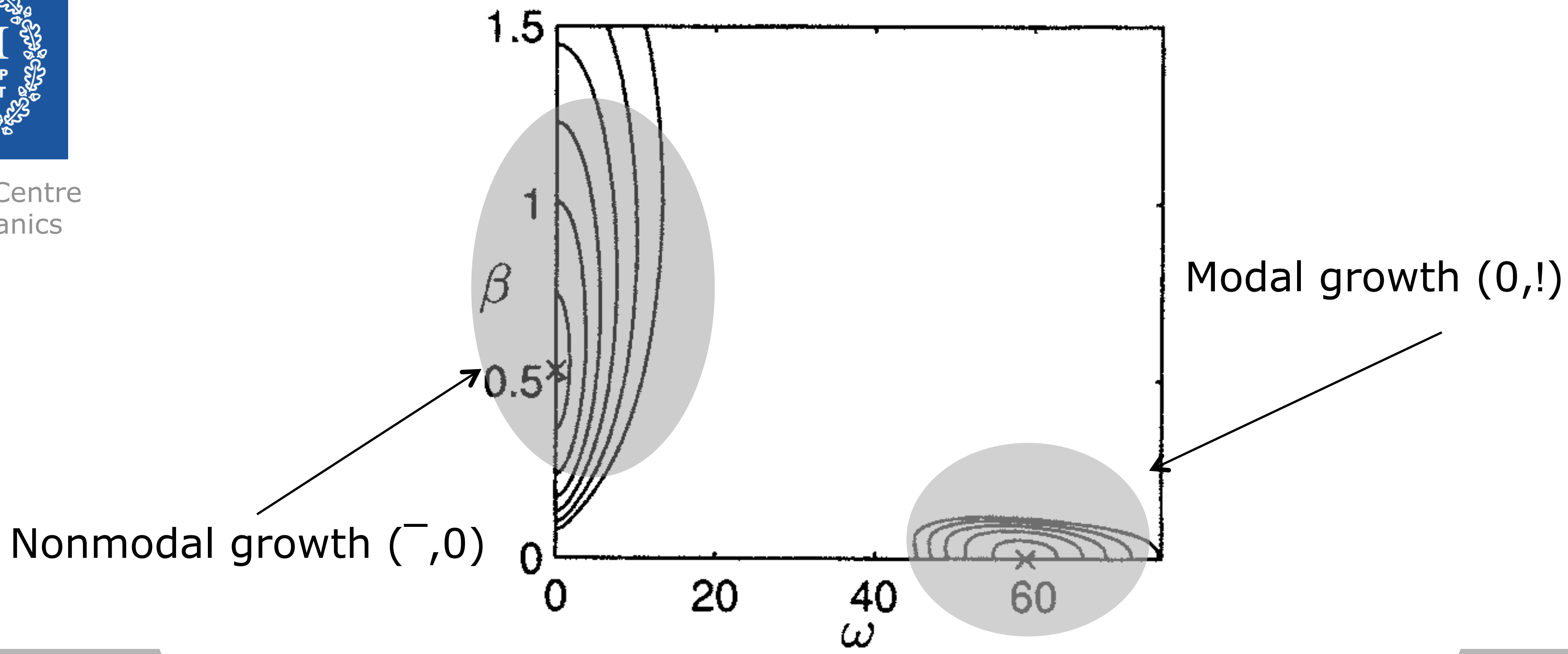

Modified Parabolized Stability Equations

- PSE scalings + BLE scalings + nonlinear terms

$$
\begin{aligned}
0 &= (i\alpha U - i\omega)u + uU_x + vU_y + Uu_x + Vu_y + p_x + i\alpha p + Re^{-1}(k^2 u - u_{yy}) + F_u \\
0 &= (i\alpha U - i\omega)v + uV_x + vV_u + Uv_x + Vv_y + p_y + Re^{-1}(k^2 v - v_{yy}) + F_v \\
0 &= (i\alpha U - i\omega)w + Uw_x + Vw_y + i\beta p_y + Re^{-1}(k^2 w - w_{yy}) + F_w \\
0 &= u_x + i\alpha u + i\beta w + v_y
\end{aligned}
$$

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- Parabolized set of equations (march in space!)

$$
\begin{aligned}
\partial_x \mathbf{q} &= \mathcal{L}\mathbf{q} + \mathcal{F}(\mathbf{q}) \\
\mathbf{q} &= \mathbf{q}_0 \quad \text{at} \quad x = 0
\end{aligned}
$$

- NOLOT code (Hanifi etal,1994)
  4th-order compact finite difference in y
  1st, 2nd-order backward Euler in x
  Small ellipticity can be stabilized (Andersson etal, J. Eng. Math,1998)

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Results

Validation: TS waves

- Linear stability computations

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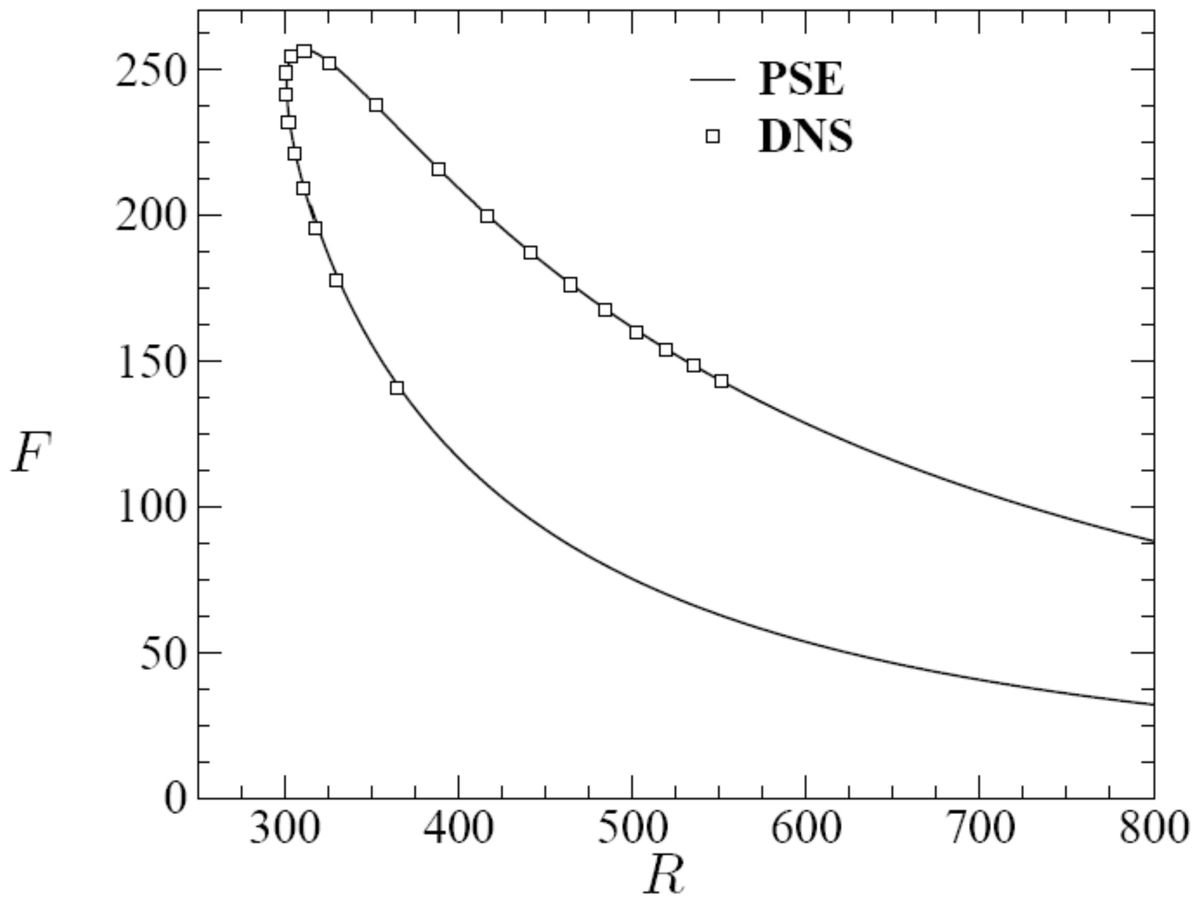

Validation: Streaks

- Initial condition spatial optimal disturbance:
  streamwise vortices (¯=0.45), see e.g. (Andersson etal 1999)

- Nonlinear computations start at downstream position

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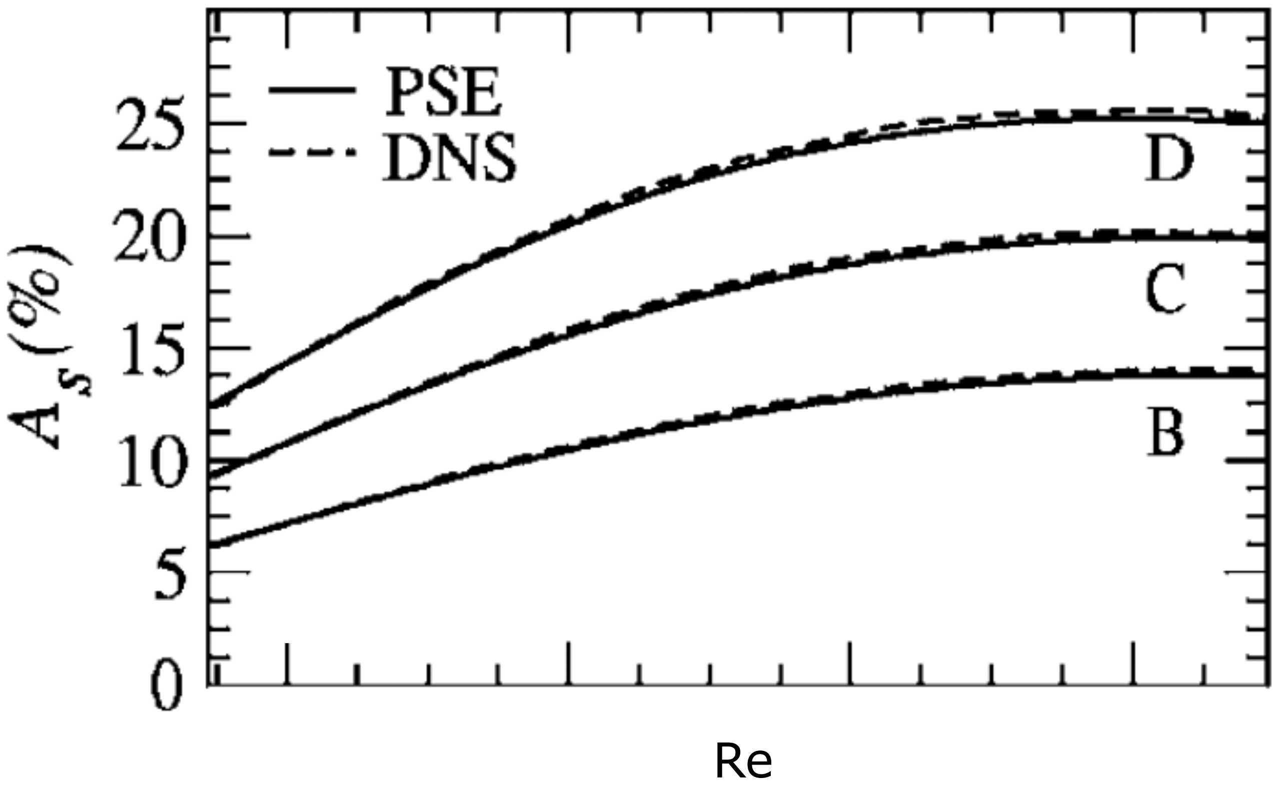
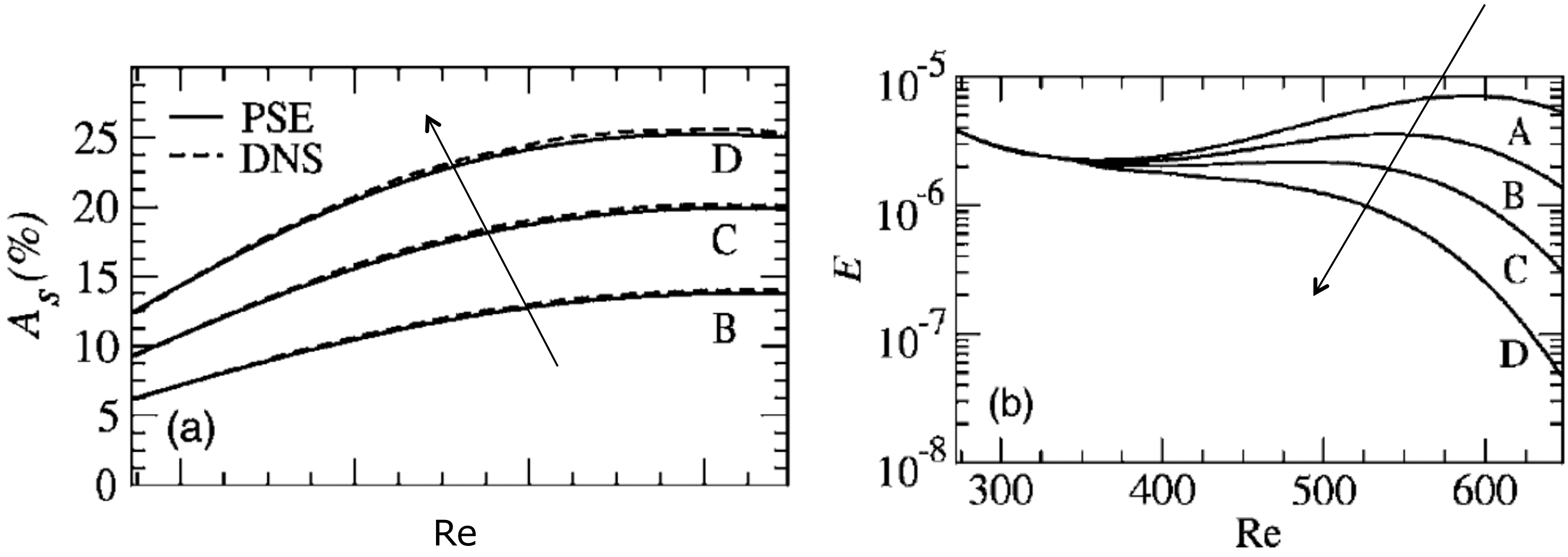


Streak & Small-Amplitude TS waves

- Stabilizing effect of streaks increase with streak amplitude



- PSE can reproduce DNS results with fraction the DNS costs

Ref: Bagheri & Hanifi (Phys. Fluids, 2007)

Streak & Finite-Amplitude TS waves

No Streak:

- Classical H-type scenario
- TS-wave (2,0), 0.46%U_1
- Oblique wave (1,1), 0.0035% U_1
- Transition!

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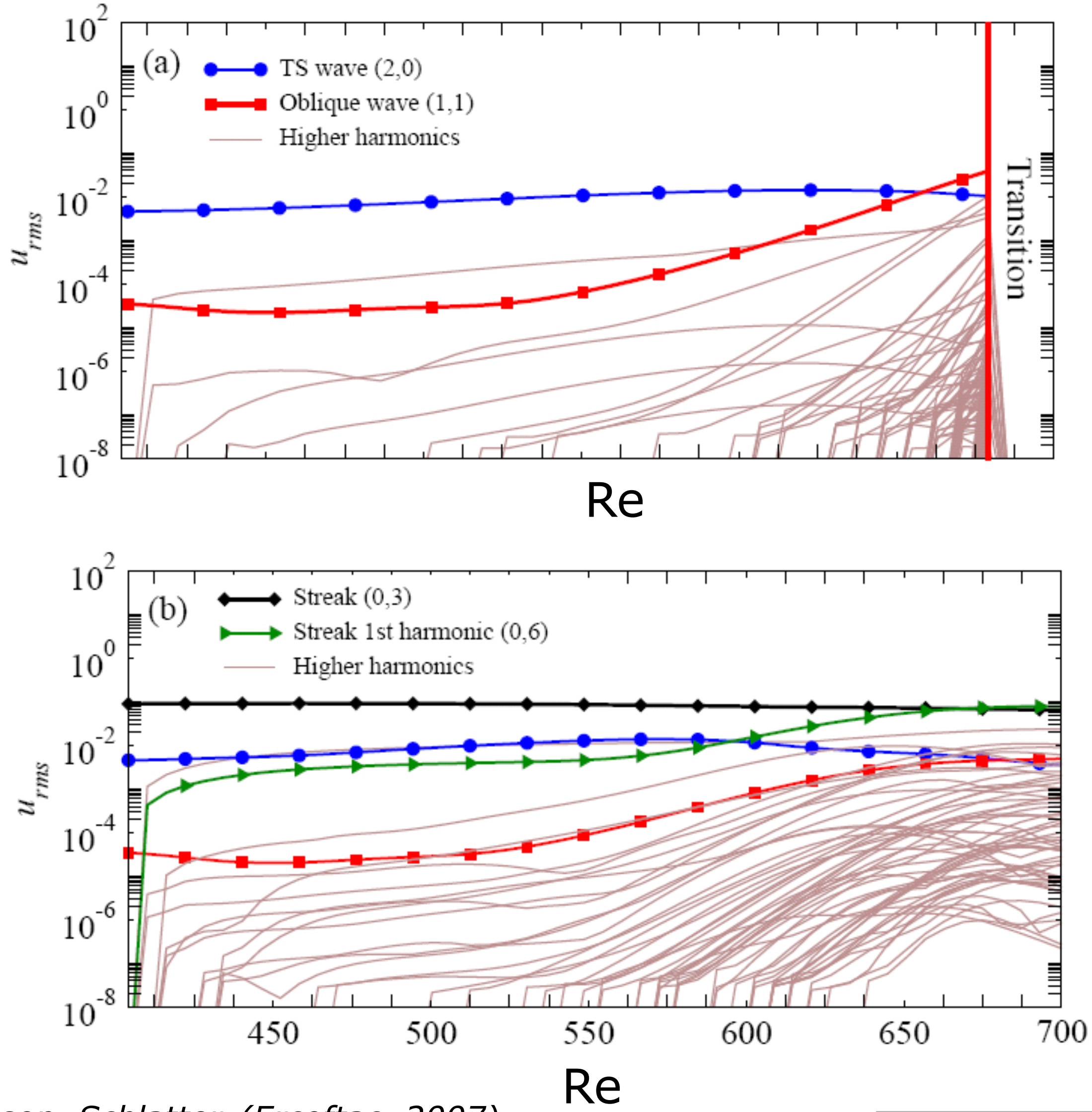
With Streak:

- Streak mode (0,3) 9%U_1
- Streak doubling
- Transiton delayed!



Ref: Bagheri, Fransson, Schlatter (Ercoftac, 2007)

Effect of Streak Spacing

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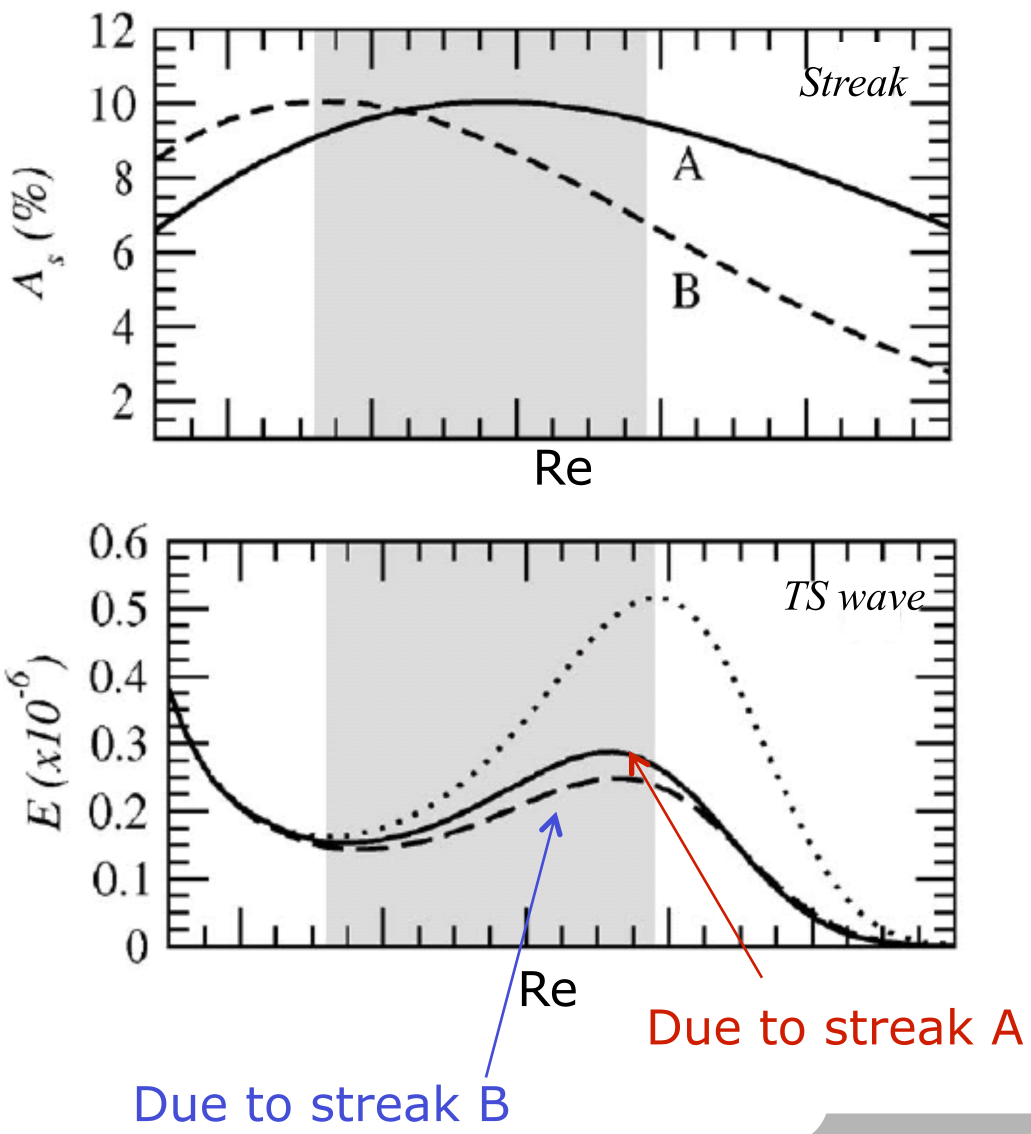
- Fix maximum streak amplitude (10%)
- Streak A: Optimal growing (linear) streak (¯=0.45)
- Streak B: (¯=0.65)
- Streak A has larger amplitude in the unstable region of the TS wave
- Streak B has larger stabilizing effect!



Effect of Streak Spacing

- Streak B induces larger mean flow distortion
- Fuller velocity profile
- Larger stabilzing effect

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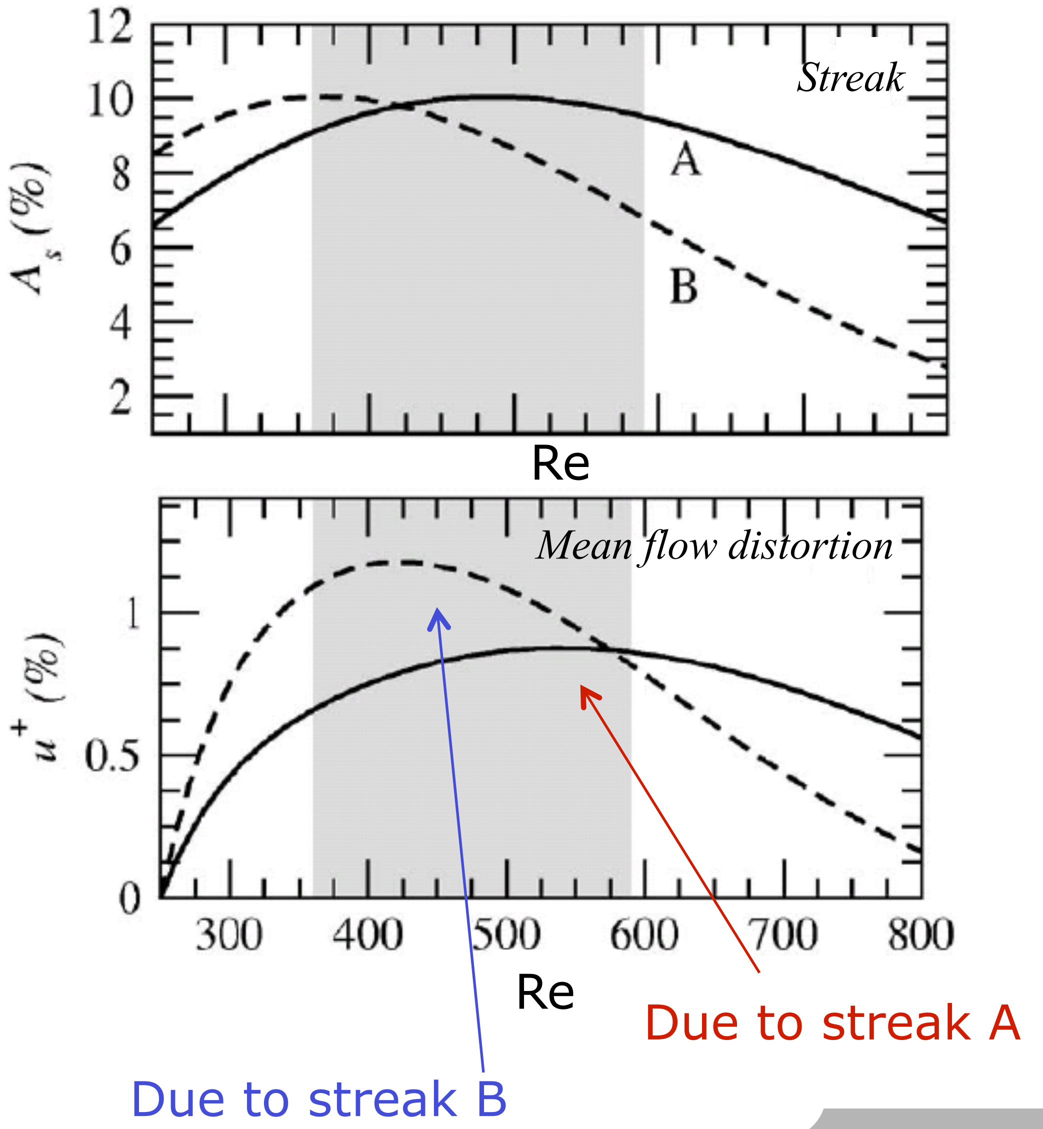

Optimal Growing vs. Optimal Stabilizing

- Choosing the streak spacing correctly the necessary Streak amplitude to completely stabilize TS wave can be reduced by a factor of two!

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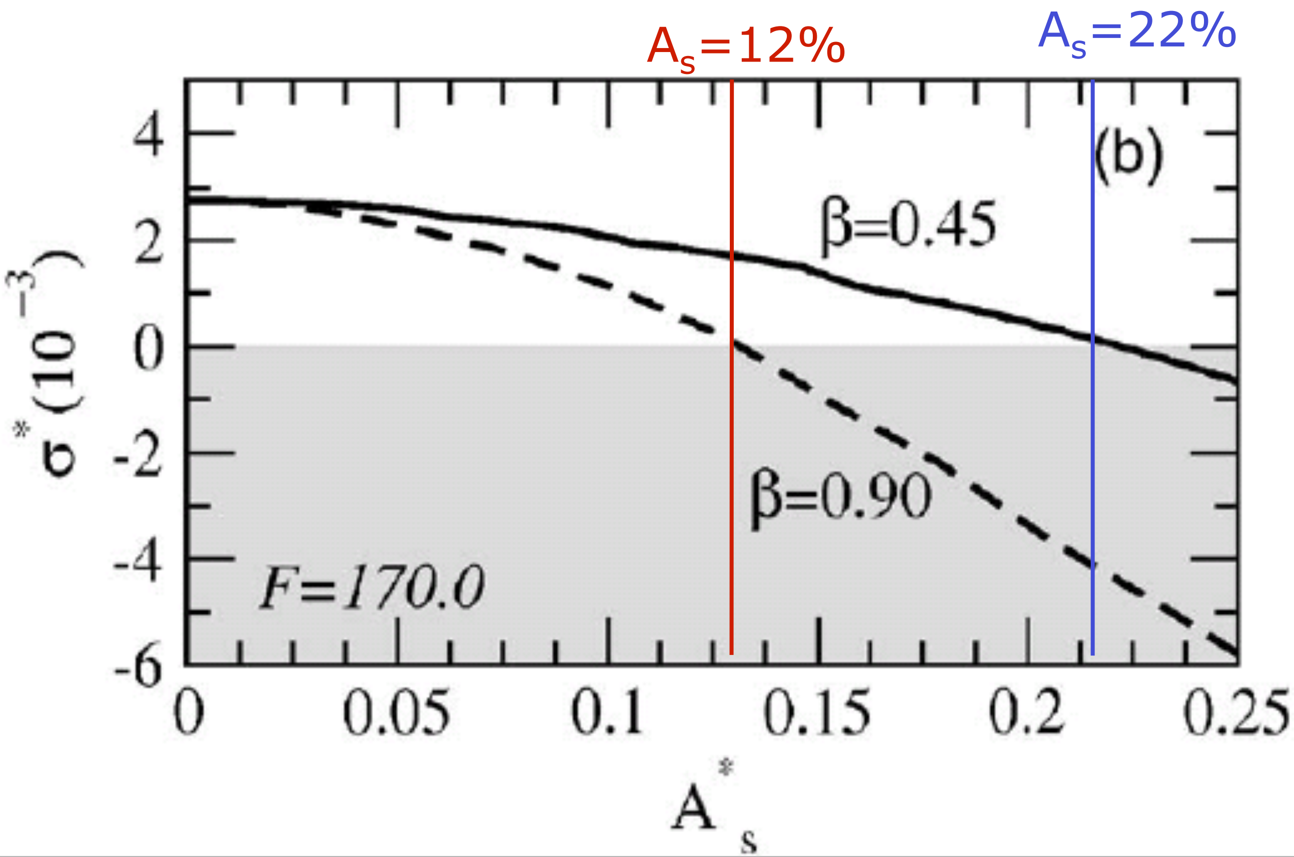

Conclusions

- Using modified PSE the nonlinear interaction of streaks and TS-waves can be captured
  1. Stabilization effect
  2. Transition delay

- Optimal stabilization effect:
  1. is not obtained for the streak with the largest amplitude
  2. is obtained for the streak which induces largest mean flow distortion in the unstable region of the TS wave

- At KTH (experimental & numerical):
  1. Vortex generators to generate streaks
  2. Robustness to noise

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