Feedback control of transition using reduced-order models

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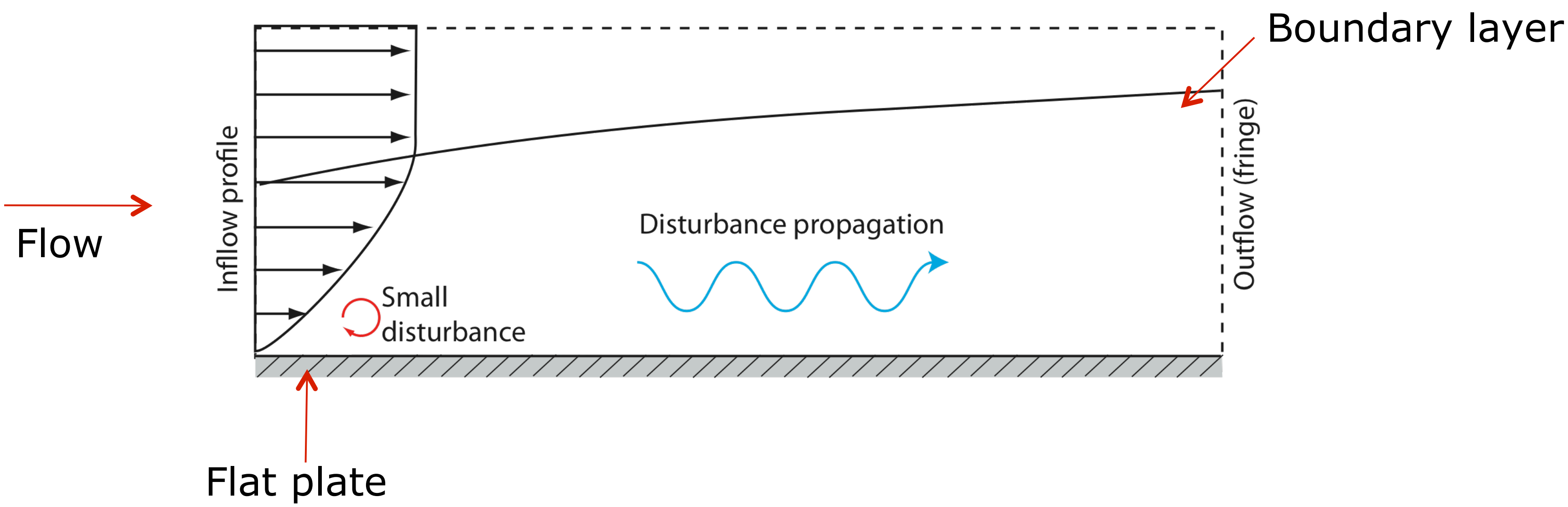
Flow on a Flat Plate

- The flow on a flat-plate



- Disturbances grow, break down, and trigger turbulence
- Using direct numerical simulations and control theory can we delay transition to turbulence?

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Laminar-turbulent transition

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TS-wavepackets

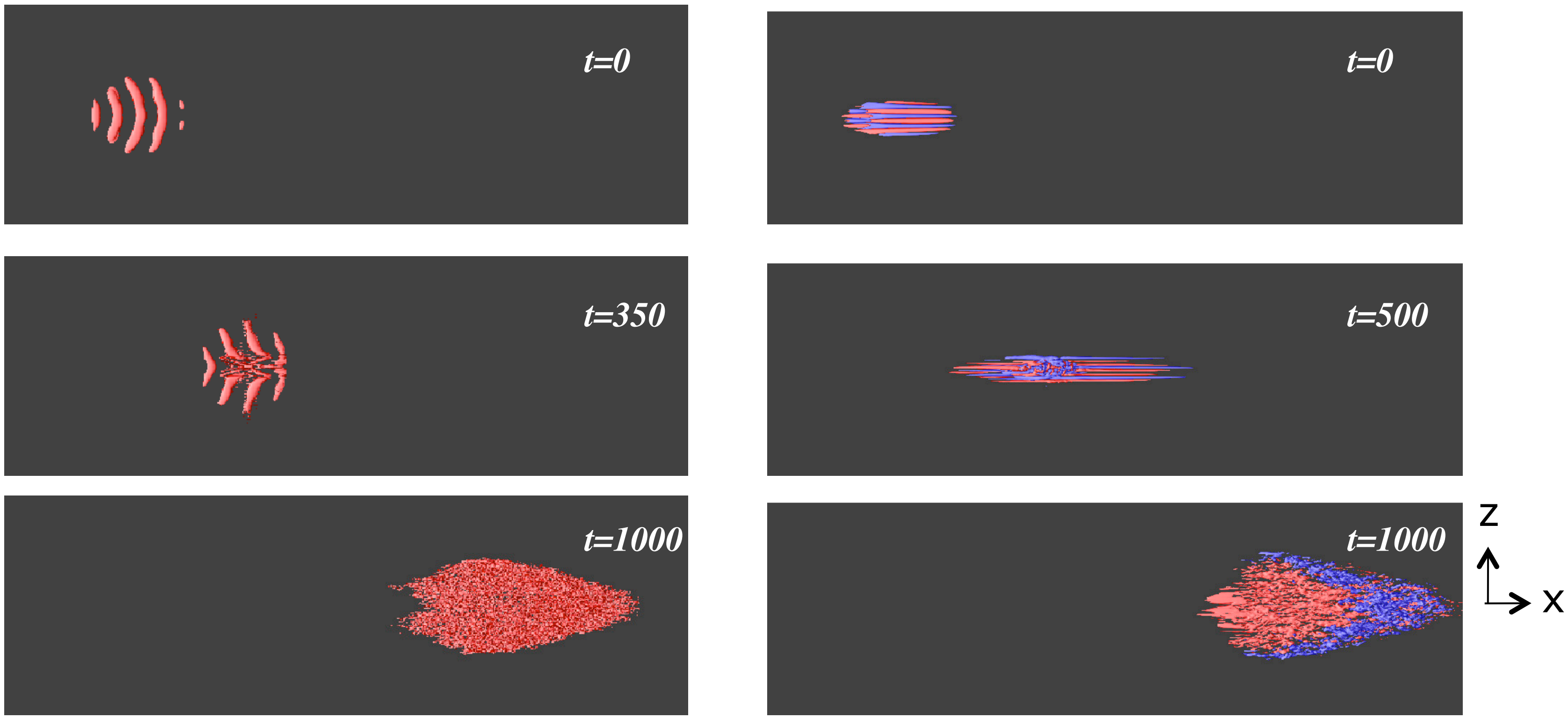
Low levels of FST (<1%)

Dominant streamwise wavelength

Streaks

Moderate levels of FST (>1%)

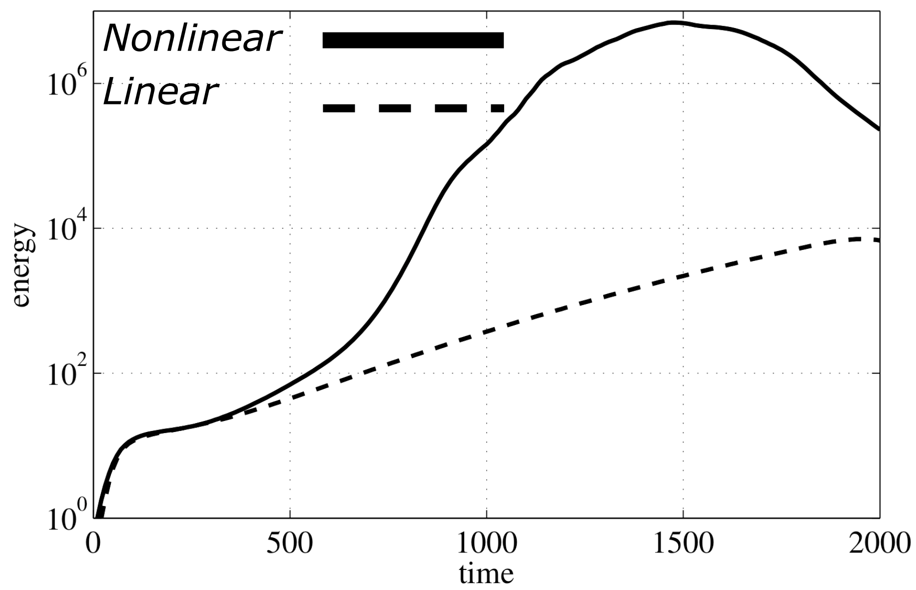
Dominant spanwise wavelength



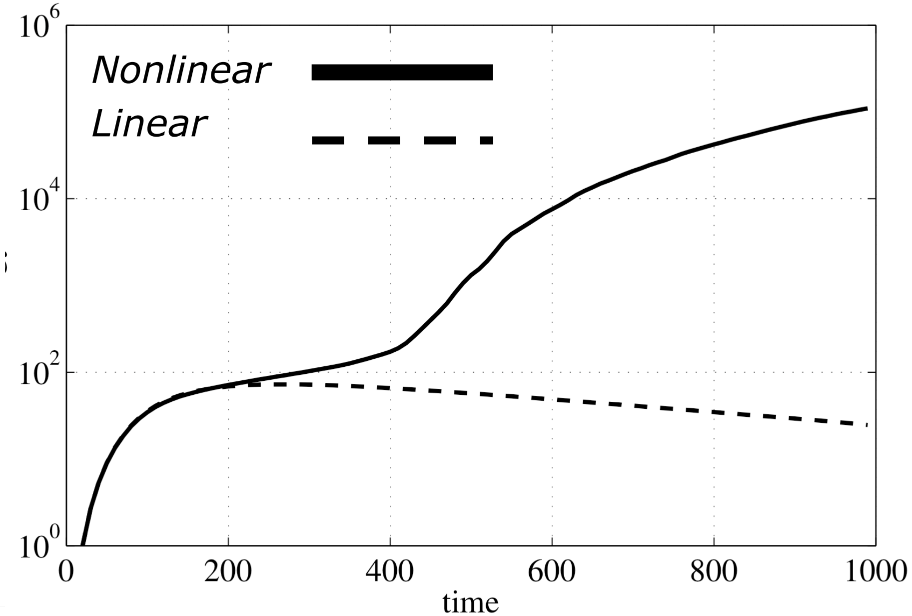
- Spatial disturbance structure sensitive to external environment

Laminar-turbulent transition

TS-wavepackets
Exponential time growth



Streaks
Algebraic time growth



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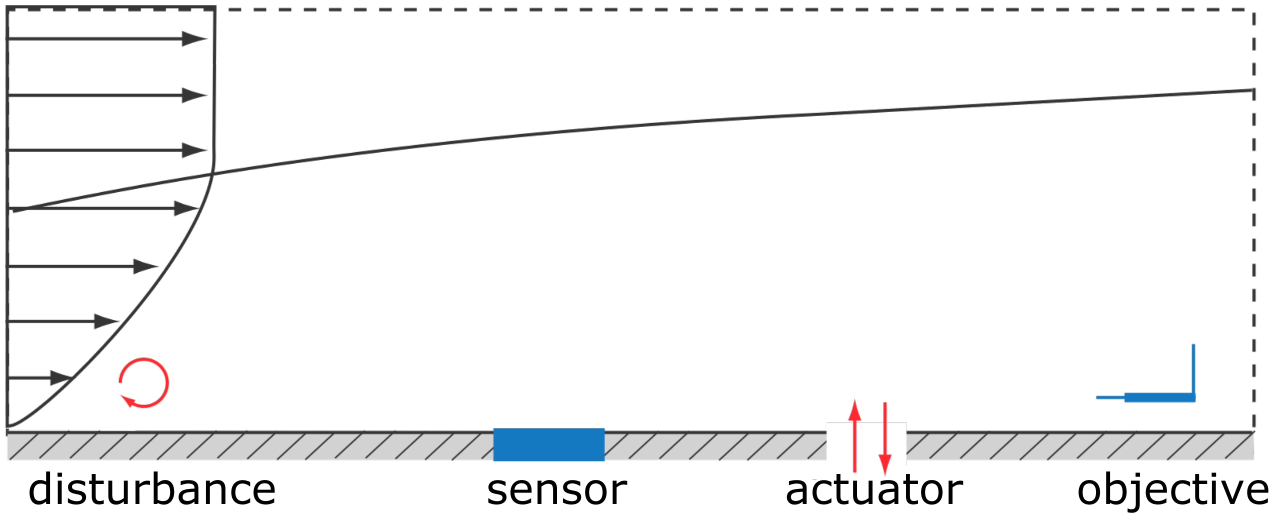
- Temporal behavior also completely different
- Initial phase is linear for both disturbance types
- Use linear feedback control to control both disturbance types

Model of flow system

- Use actuators and sensors to minize disturbance downstream:



- Mathematical linear model of input-output dynamics:

$$\mathbf{G} : \mathbf{w} \rightarrow \mathbf{y}$$

Output signals
(sensor & objective function)

Input signals
(disturbances & actuators)

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Capturing Input-Output Behavior

- **Problem:** The linear mapping between inputs to outputs:

$$\mathbf{G} : \mathbf{w} \to \mathbf{y}$$

has complexity order of millions (due to discretization of N-S)

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- How to construct an approximation

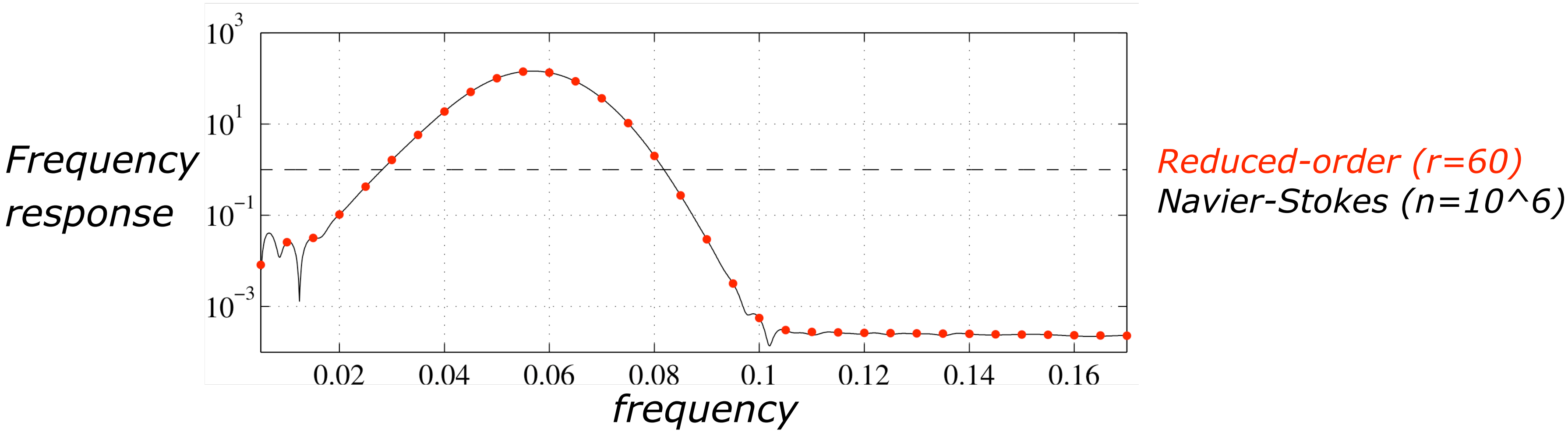
$$\mathbf{G}_r : \mathbf{w} \to \mathbf{y}$$

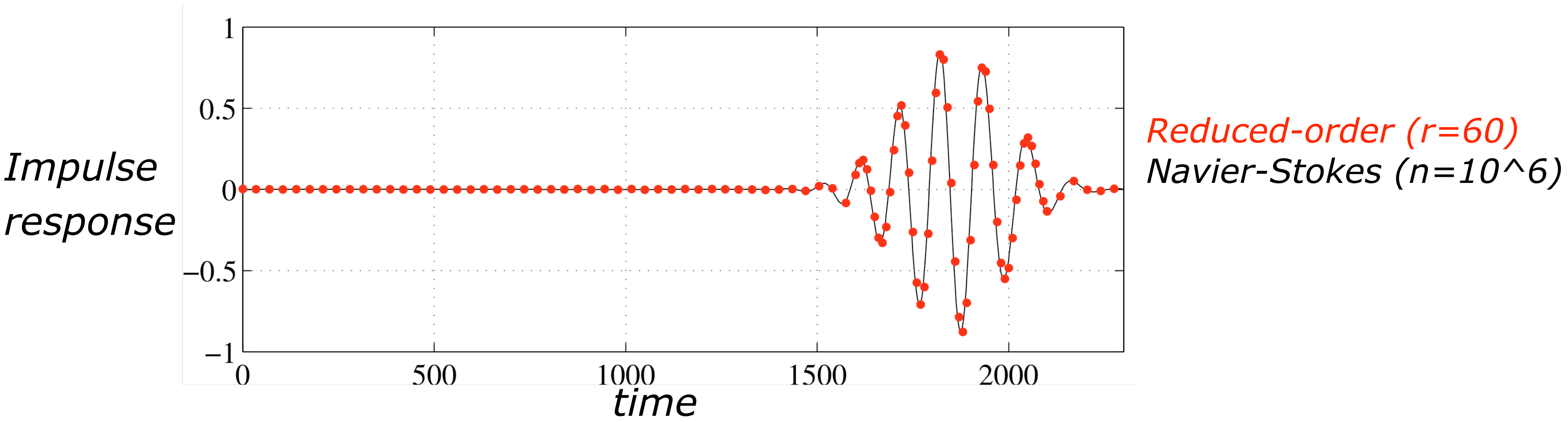
complexity is of order 10-100

such that $||\mathbf{G} - \mathbf{G}_r||$ is small?

→ **Solution:** Balanced truncation *(Moore 1981, Rowley 2005)*

Validation of Reduced-Order Model





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- Reduced-order model & Navier-Stokes show same input-output behavior

Control Design

- Linear quadratic Gaussian (LQG)
  Based on noisy sensor measurements, find control signal that minimize effects of disturbances in a subdomain

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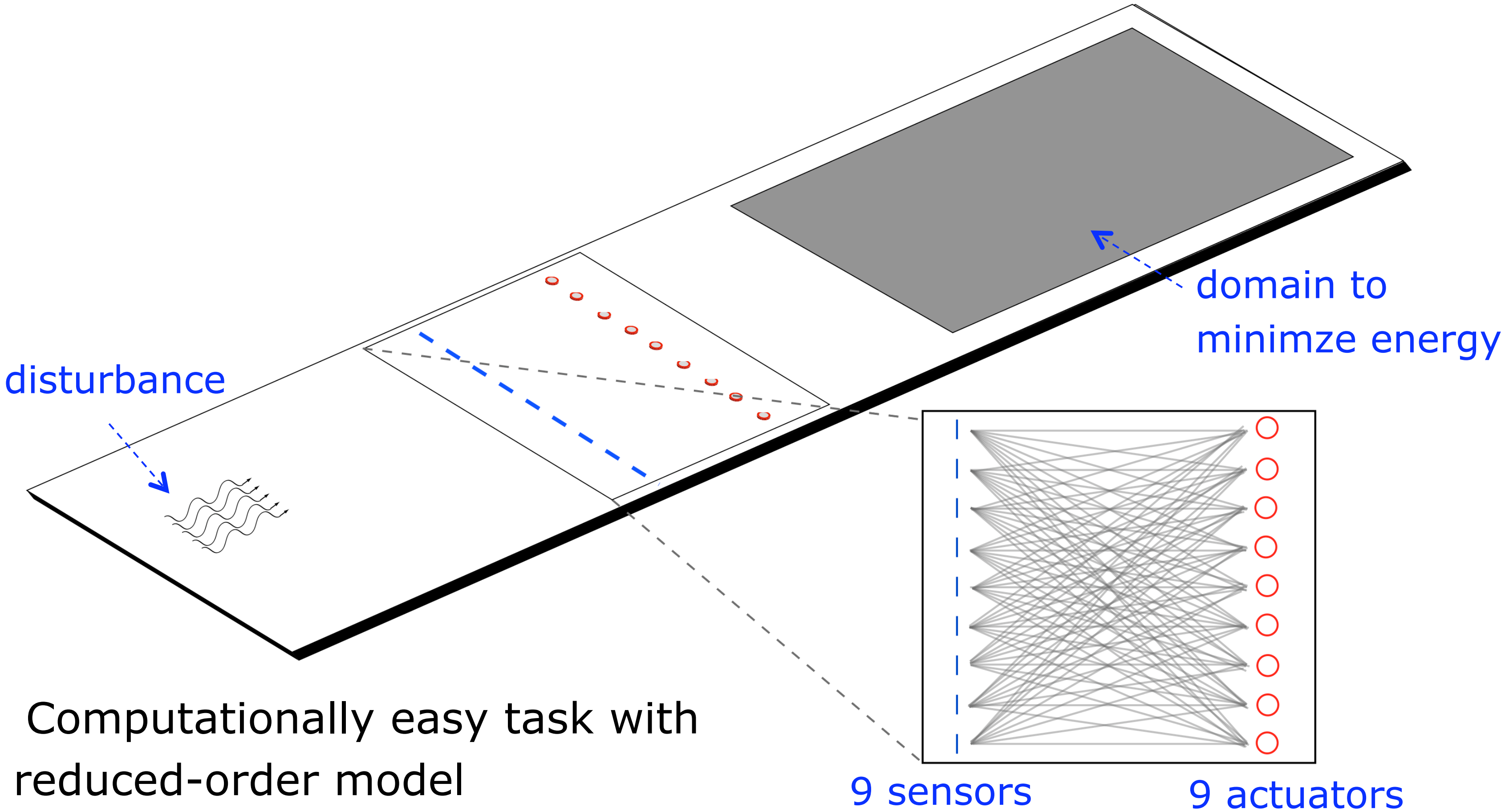
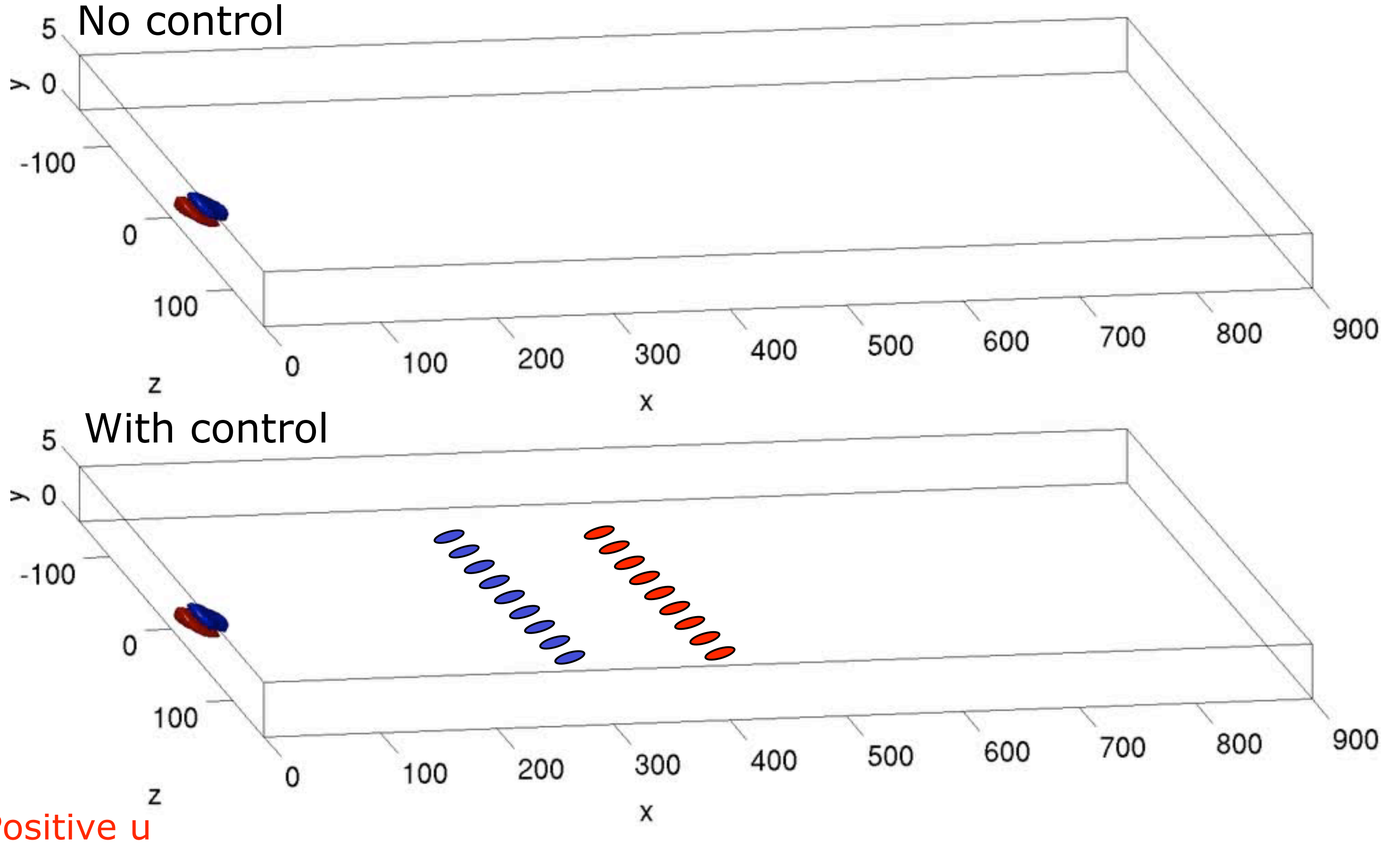


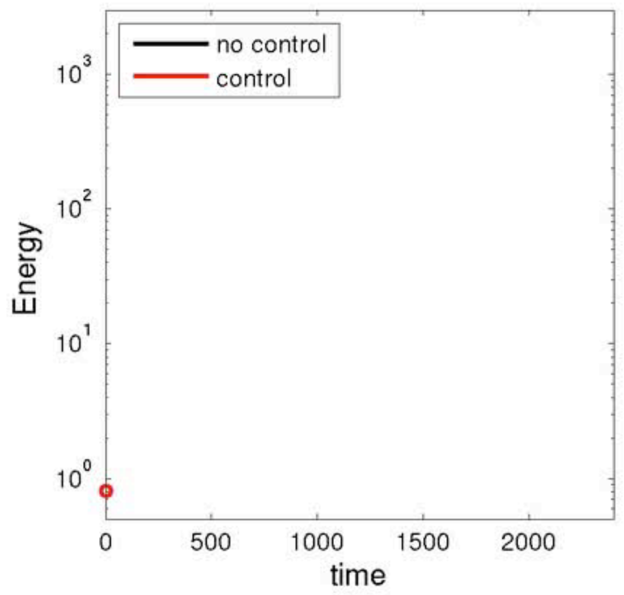
- Computationally easy task with reduced-order model

Controlled Flow

- Disturbance energy reduced orders of magnitude

  Using 9 small sensors & 9 actuators

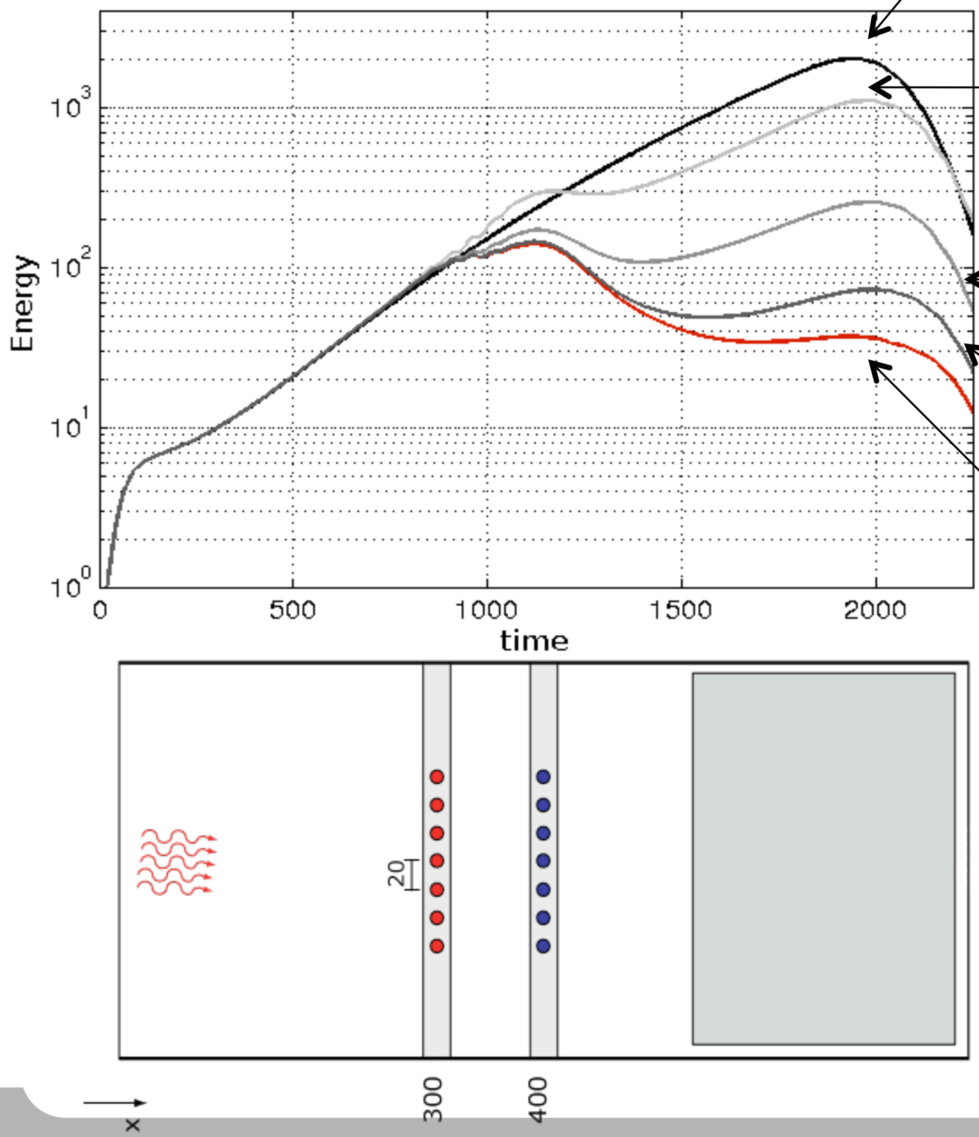




Positive u

Negative u

Parametric analysis - actuators



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Robustness analysis – variation in Re

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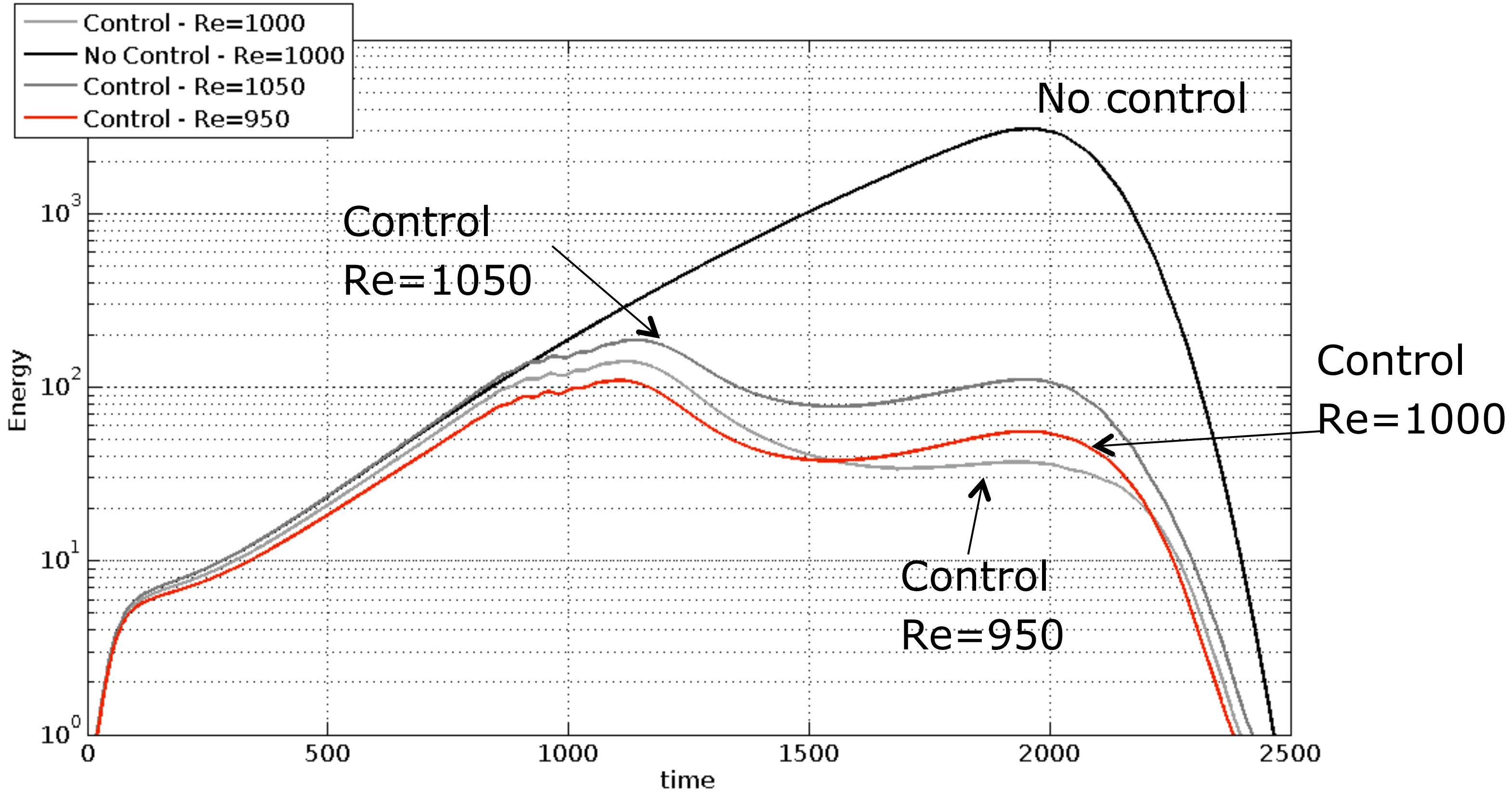

Substantial deviation from the nominal Reynolds number still shows an overall satisfactory behaviour.

Control performance: streaks

- Three cases considered: cheap controller (l=70), intermediate controller (l=100), expensive controller (l=150)

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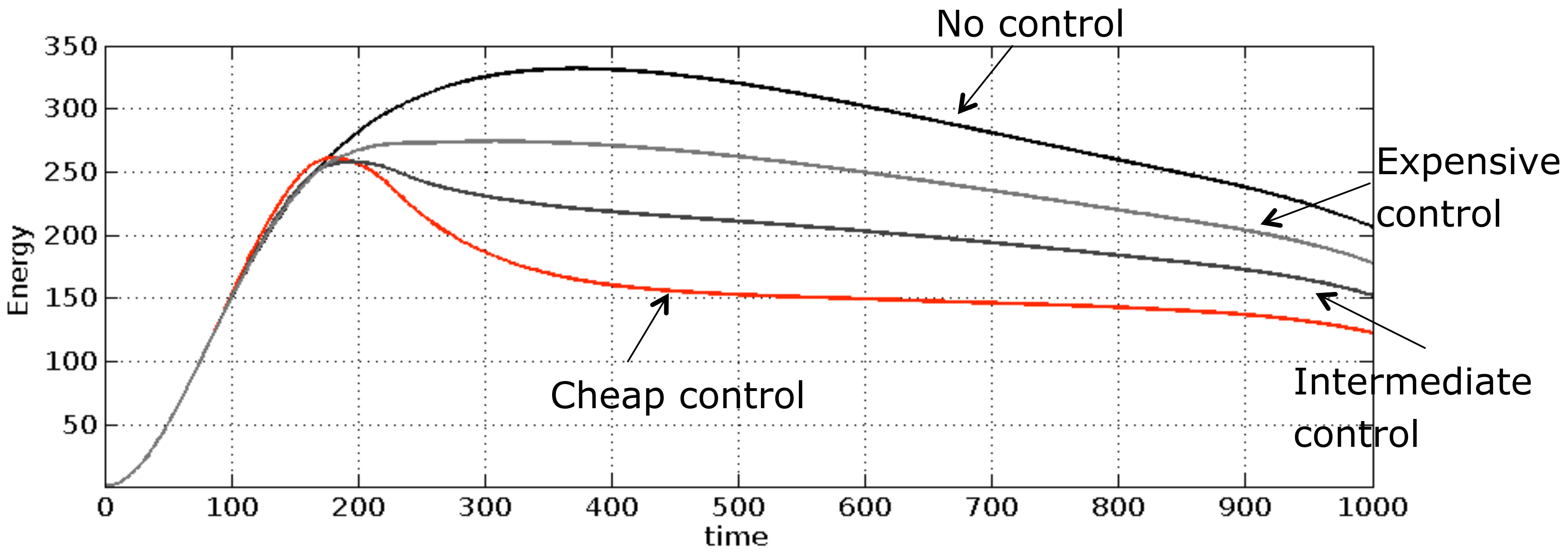

Outlook & Conclusions

- Few small localized sensors and actuators can achieve significant flow manipulation, by exploiting the large sensitivity typical of wall-bounded flows.

- Nonlinear simulations has been performed, transition is delayed using the present controller *(see talk by O. Semeraro on Thursday)*

- Shortcomings of localized sensing and acting:
  (1) Placement of actuators and sensors is related to spatial and temporal scales of the disturbances.

  (2) Computational cost of model algorithms increase rapidly with number of inputs and outputs.

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