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Analysis and Control of Transitional Shear Flows Using Global Modes

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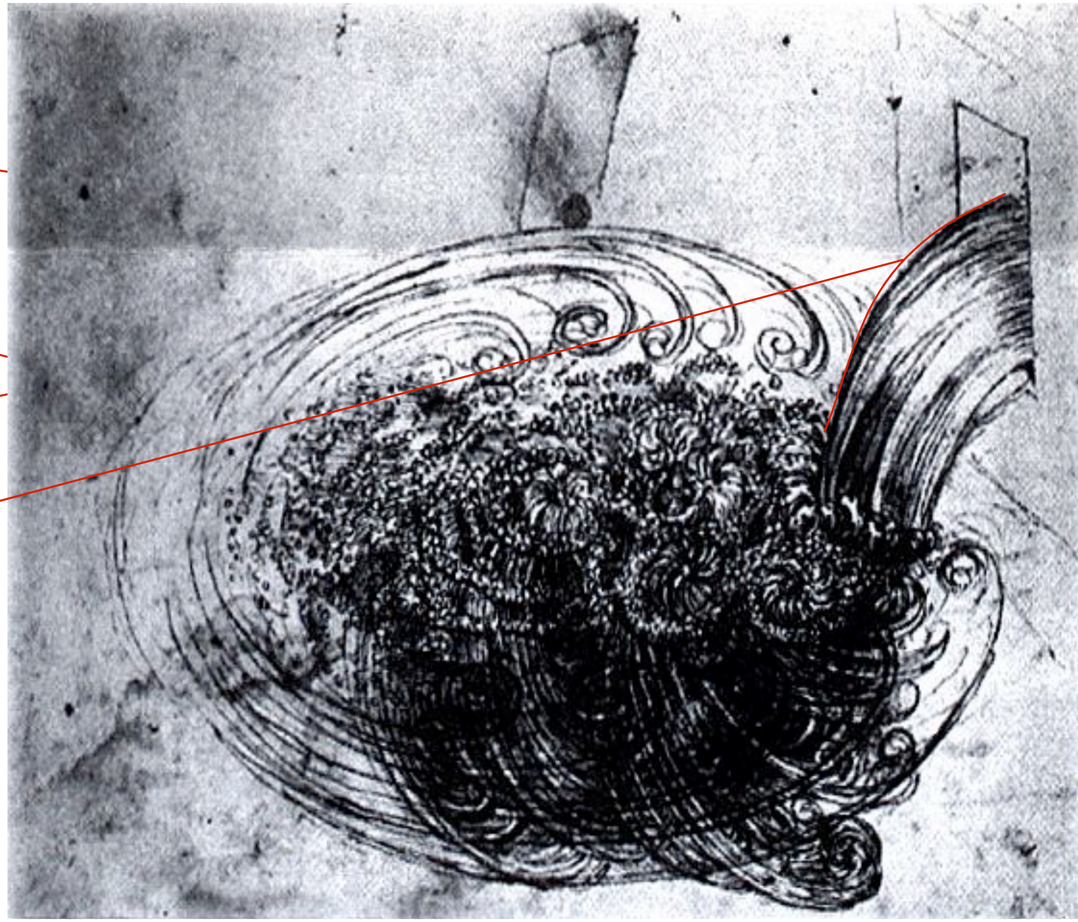
September 28, 2010
KTH Mechanics, Stockholm

A Free Water Jet Into a Pool

- Laminar flow
- Turbulent flow
- Vortical structures
- Shear layers



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Leonardo da Vinci

Cigarette Smoke

- Ordered and predictable smoke becomes chaotic and unpredictable
- Transition of a laminar flow to a turbulent one

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Cloud Structure

- Clouds roll up into Kelvin-Helmholtz vortices
- Two streams of different velocity – shear layer instabilities

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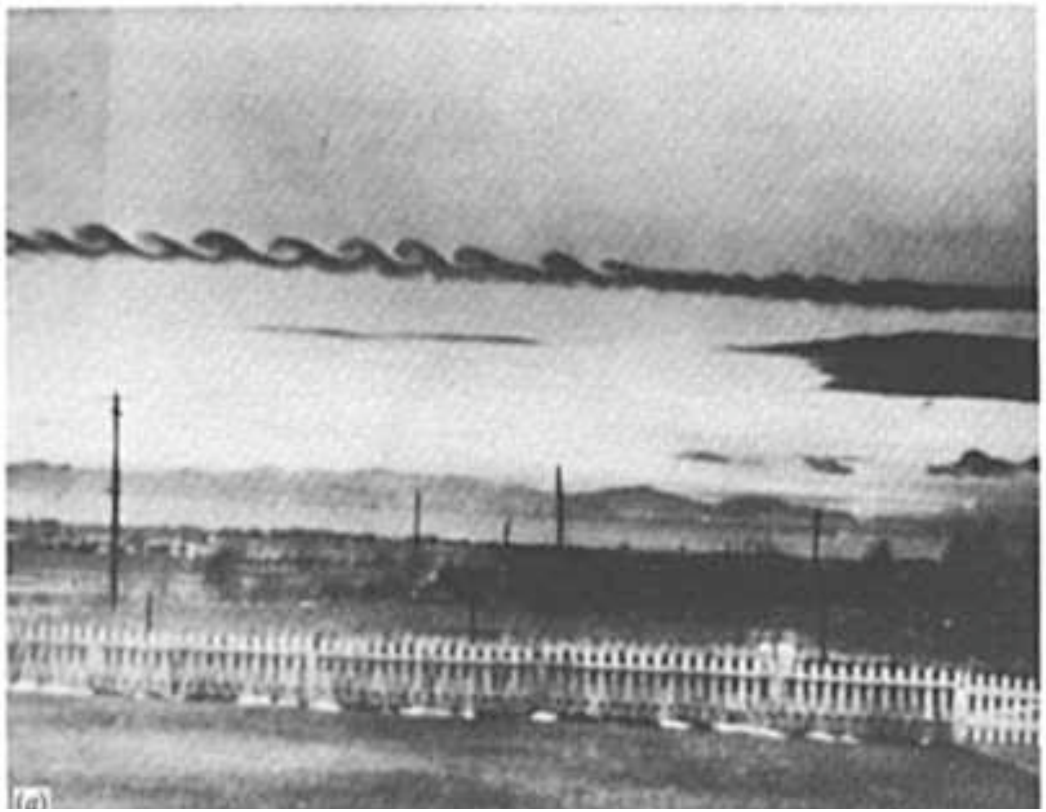

Clould Structure

- von Kármán vortex street developing behind an island
- Periodic vortex shedding

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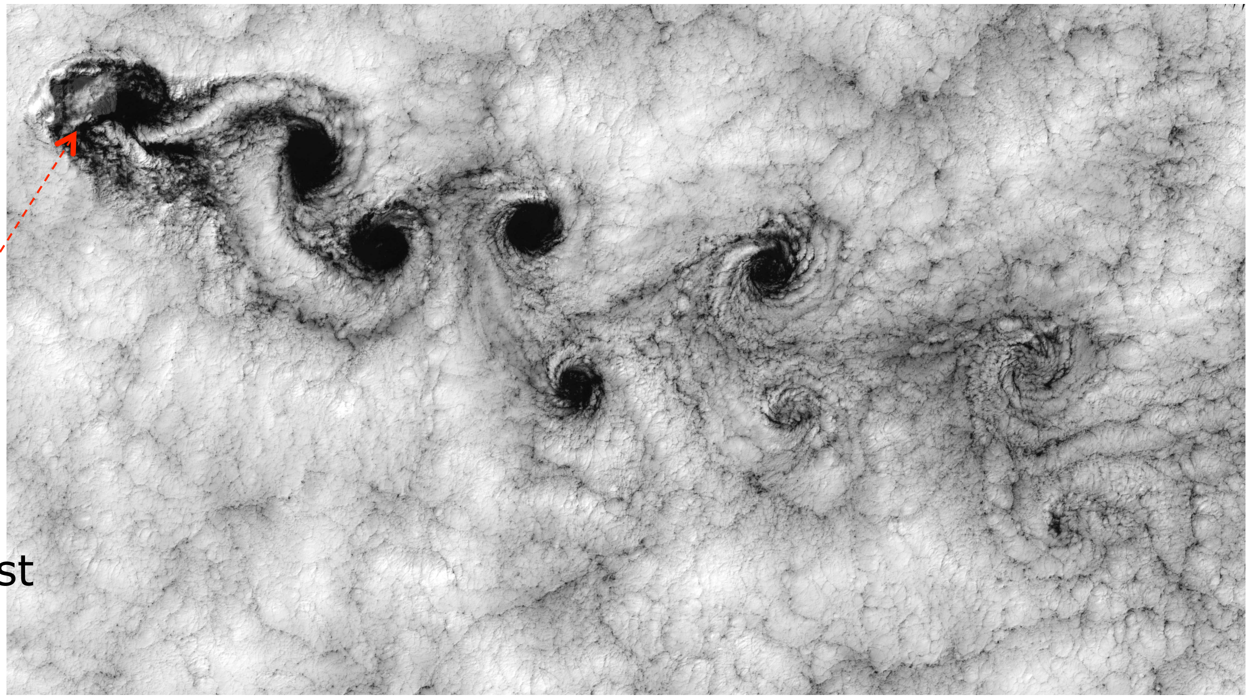

Island near Chilean coast

Open Issues

- Fundamental issues touched:
  - How does a flow transition from laminar to turbulent? Can we control the transition process?
  - Why does unsteadiness in some flows take the form of periodic shedding of vortices?

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- Approaches:
  - Numerical simulations
  - Mathematical tools for analysis and control
  - Both simple & complex flow configurations

Outline

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- Flow phenomena
- Part I: Flow analysis
- Part II: Flow control

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Part I
Flow Analysis

Jet in Crossflow

- Fluid injected through a hole into a crossflow

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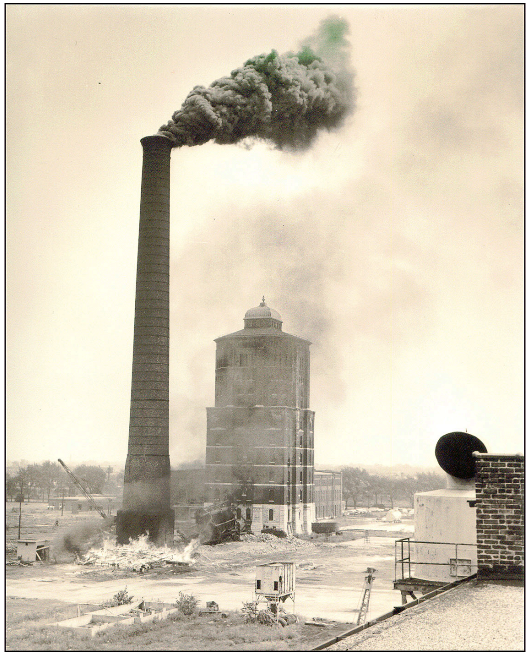
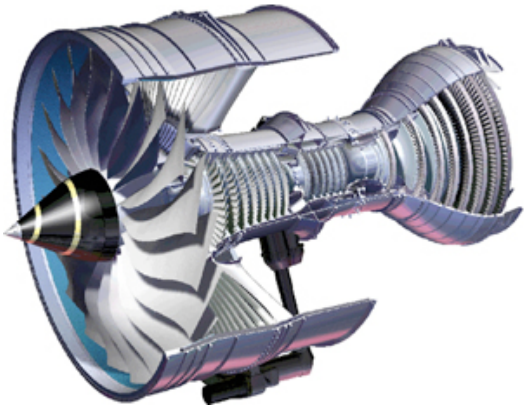
Smoke stacks



Volcano eruptions

Fuel injection/film cooling



The Four Vortical Structures of Jet in Crossflow

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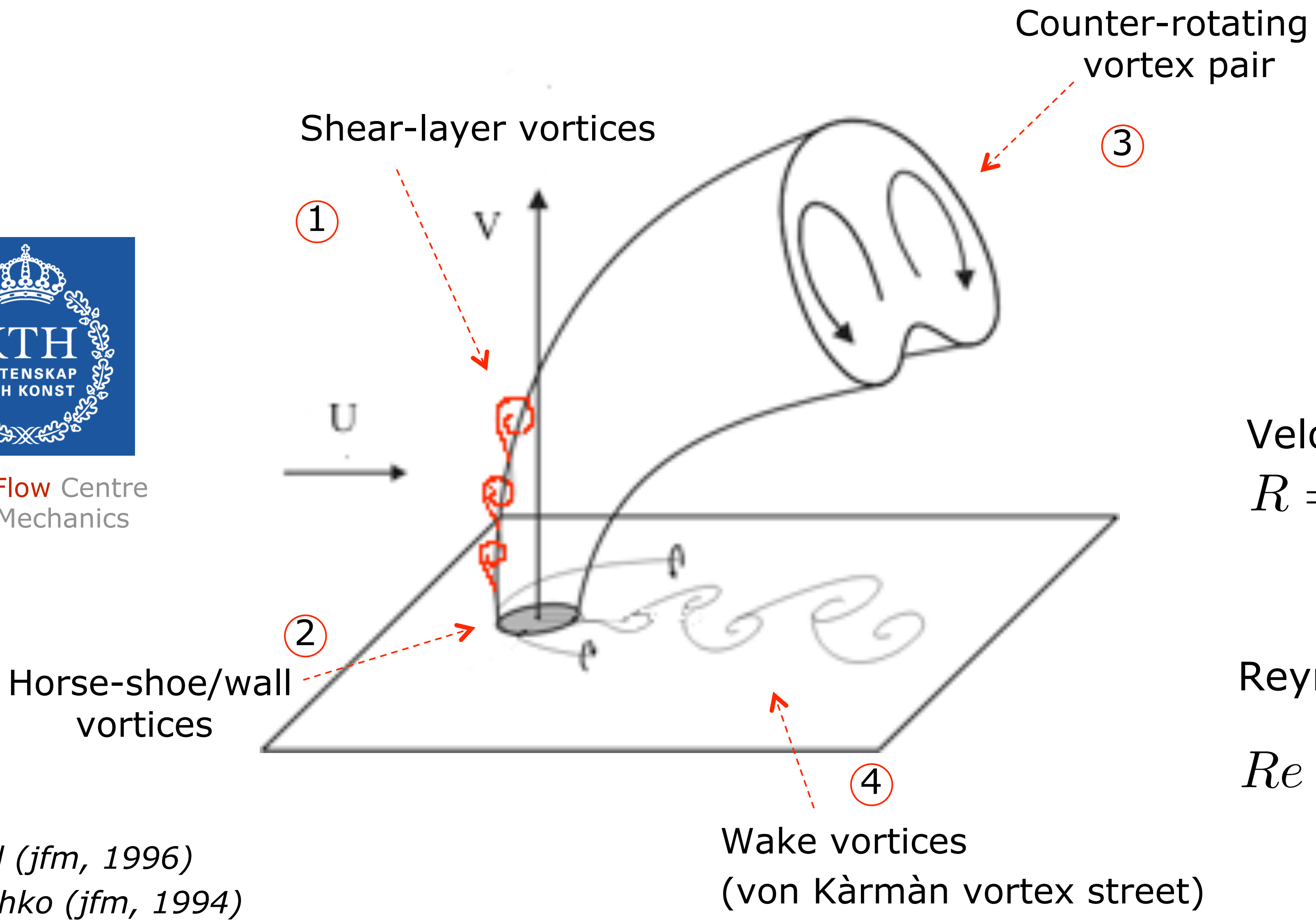

Velocity ratio:

$$R = V/U = 3$$

Reynolds number:

$$Re = \frac{\delta_0^* U}{\nu} = 165$$

Kelso et al (jfm, 1996)
Fric & Roshko (jfm, 1994)

Numerical Simulations

- Direct numerical simulations (DNS)
  - Fully spectral code
  - 10 million gridpoints

- Identified from DNS:
  - all 4 vortical structures
  - 2 events of vortex shedding (oscillation of separated region)

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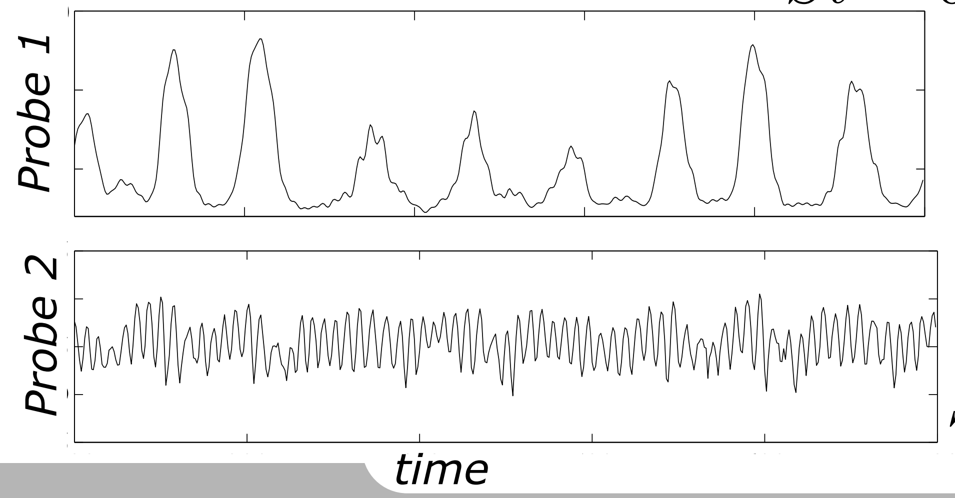
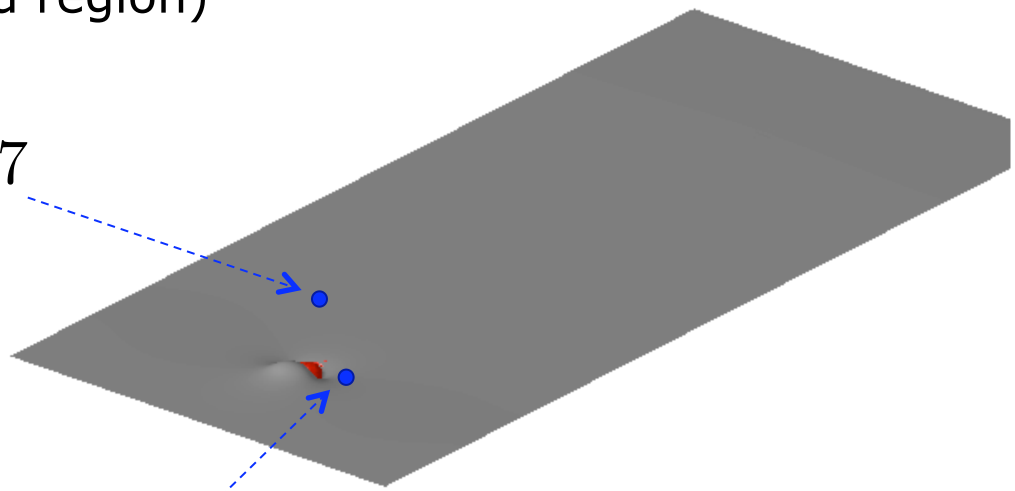
$St = 0.017$





$St = 0.141$

λ_2 - Vortex criterion

Streamwise velocity

Analysis of the Jet in Crossflow

- Observations from DNS:
  - 4 large vortical structures
  - 2 events of periodic vortex shedding

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1. Stability analysis: Perturbation dynamics near steady state
   - Determine which vortical structures are steady or unsteady
   - Determine the physical mechanisms for unsteadiness

2. Nonlinear analysis: Flow dynamics in an attractor region
   - Identify which vortical structures are oscillating periodically

Bagheri *et al. JFM 2009*
Rowley *et al. JFM 2009*
Schlatter *et al. TCFD 2010*

Overview of Stability Analysis

- The Navier-Stokes equations :

$$\dot{\mathbf{u}} = \mathbf{f}(\mathbf{u})$$

- Find a steady solution:

$$0 = \mathbf{f}(\mathbf{u}_s)$$

- Perurb the steady solution:

$$\mathbf{u} = \mathbf{u}_s + \mathbf{u}'$$

Unsteady perturbation

- Linearize around the steady solution:

$$\dot{\mathbf{u}}' = \mathbf{A}\mathbf{u}'$$

Huge matrix $\sim 10^7 \times 10^7$

- Find eigenvectors and eigenvalues:

$$\mathbf{A}\phi_j = \lambda_j \phi_j$$

Linear global eigenmodes

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Schmid & Henningson (Springer, 2001)

Steady Solution of Navier-Stokes Equations

- For fully 3D problem a difficult computational task
  - Selective frequency damping *(Åkervik et al. PoF 2006)*

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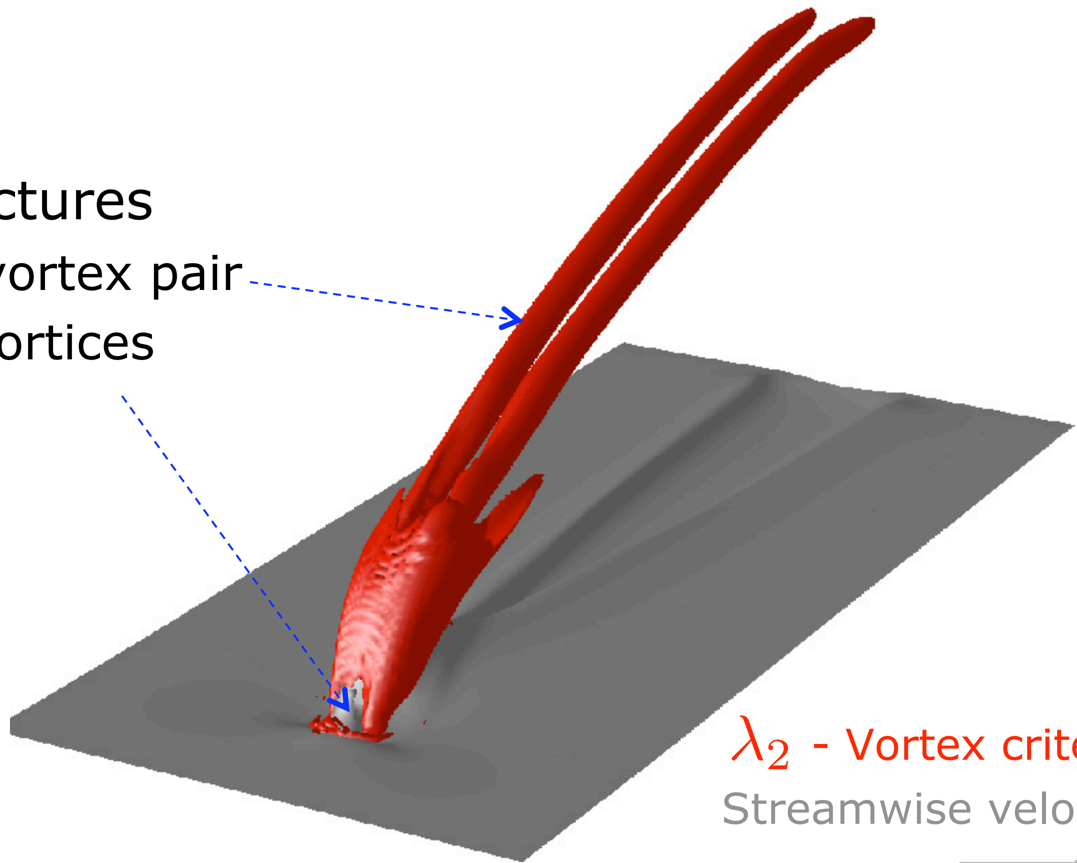
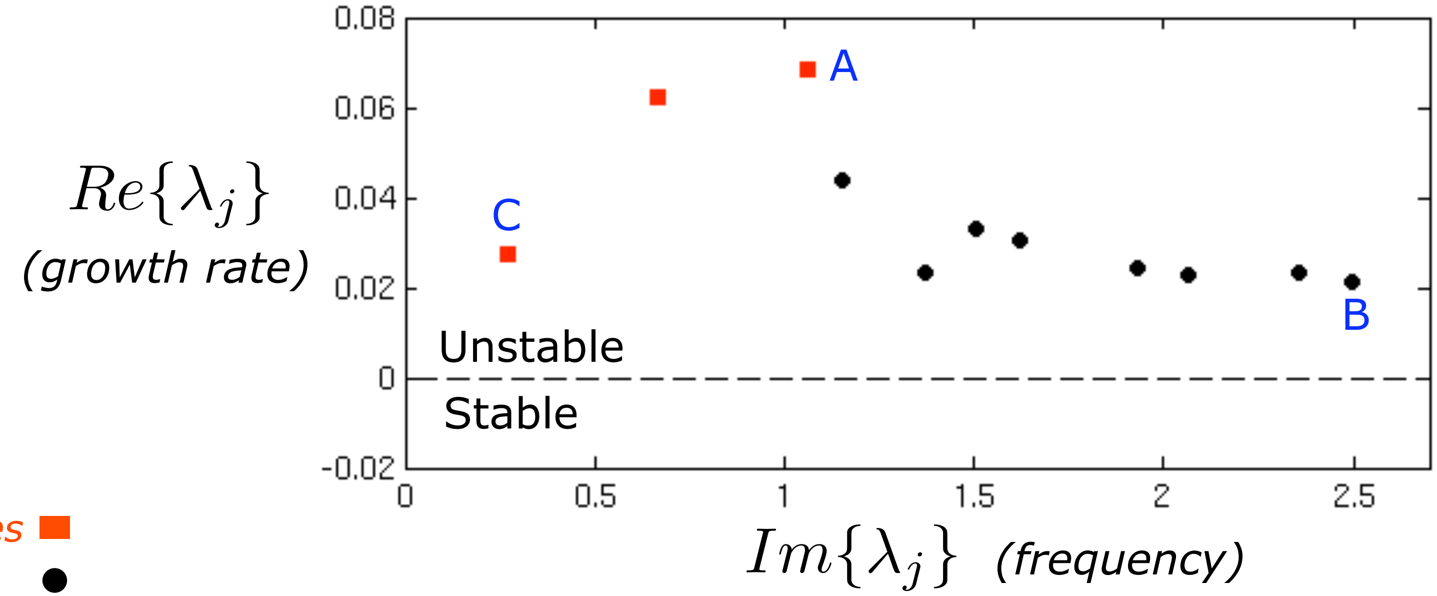
- Steady vortical structures
  - Counter-rotating vortex pair
  - Horse-shoe/wall vortices



Global Stability of Jet in Crossflow

- Global modes computed using iterative techniques (Arnoldi method) in combination with numerical simulations
- 22 unstable global modes found
- Global spectrum:



Anit-Symmetric modes ■

Symmetric modes ●

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Most Unstable Global Eigenmode

- Mode A:
  - Wavepacket on the counter-rotating vortex pair
  - Short-wavelength instability of a vortex pair

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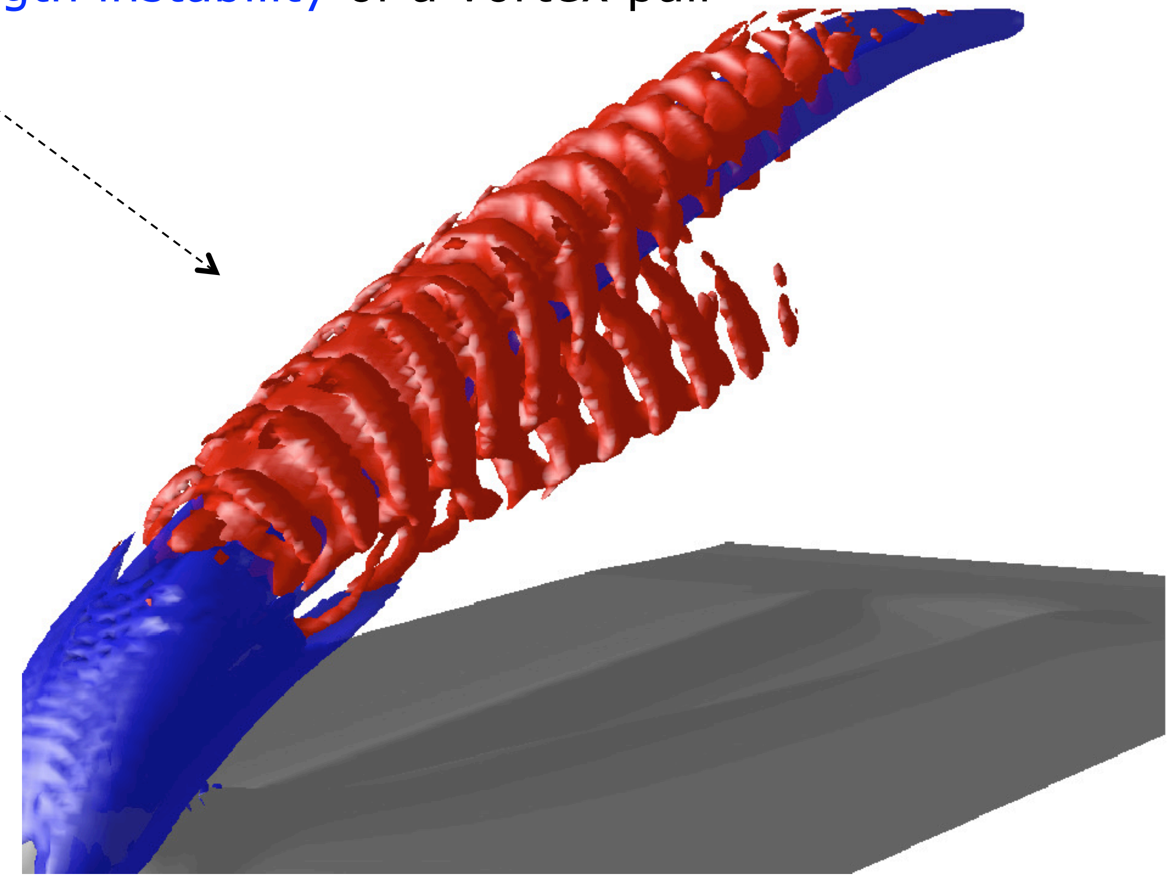


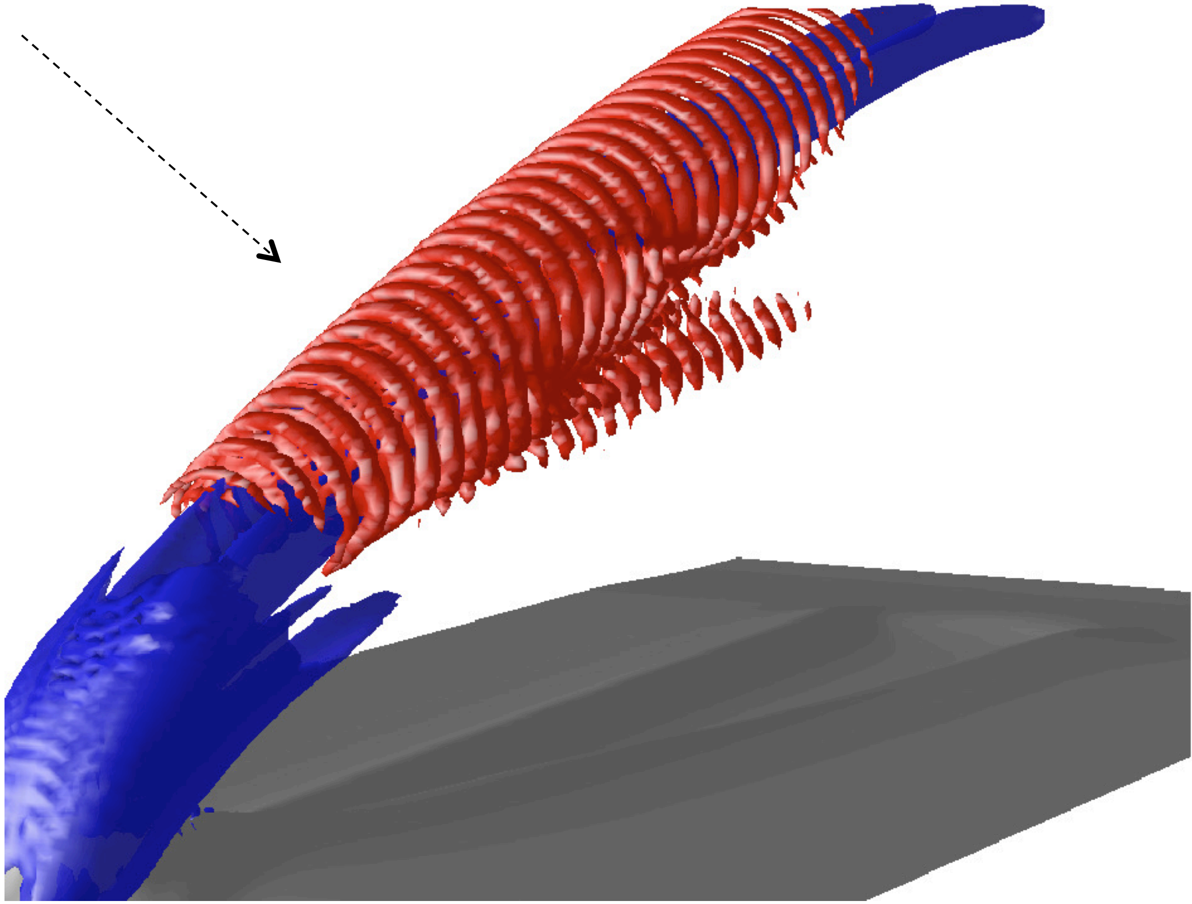
Streamwise velocity (baseflow)

λ_2 Vortex (baseflow)

λ_2 Vortex (global mode)

Symmetric Global Eigenmode

- Mode B:
  - Vortex rings on the counter-rotating vortex pair
  - Kelvin-Helmholtz instability of the shear layer



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Streamwise velocity (baseflow)

λ_2 Vortex (baseflow)

λ_2 Vortex (global mode)

Low-Frequency Global Eigenmode

- Mode C:
  - Mostly located near the wall
  - Karman vortex street in the wake

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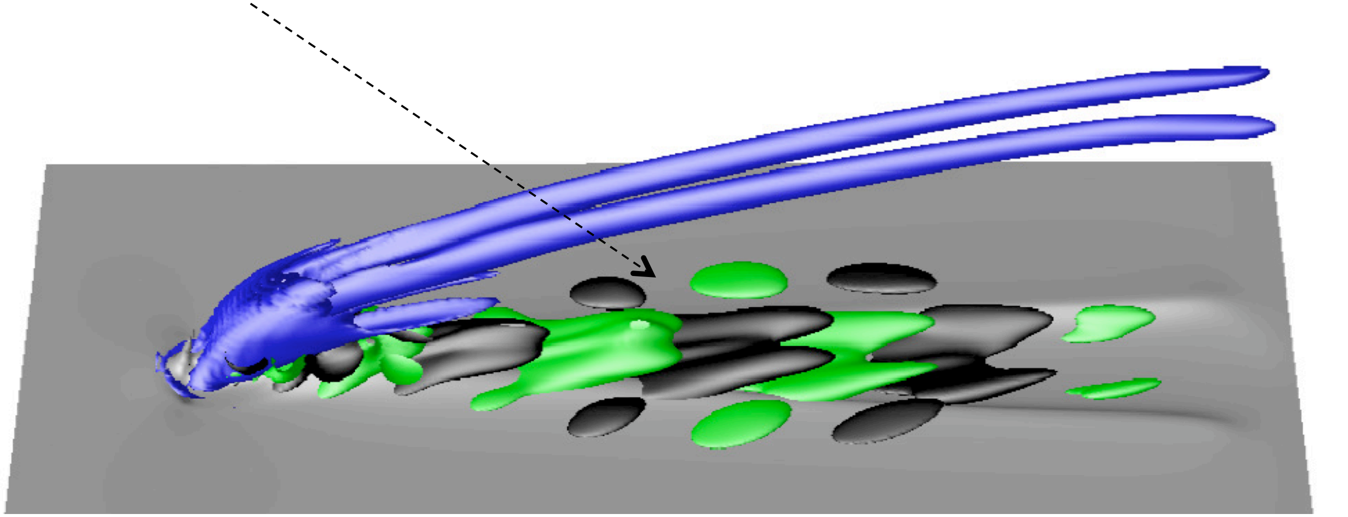

Streamwise velocity (baseflow)

λ_2 *Vortex criterion (baseflow)*

Spanwise velocity (global mode)

Analysis of the Jet in Crossflow

- Observations from DNS:
  - 4 large vortical structures
  - 2 events of periodic vortex shedding

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Koopman Operator

- Define an observable as scalar-valued function

$$a(\mathbf{u}_k) : \mathbb{U} \to \mathbb{R}$$

- Koopman operator U propagates observables in time:

$$Ua(\mathbf{u}_k) = a(\mathbf{u}_{k+1}).$$

Linear, unitary & infinite dimensional operator

- Spectral analysis of U

$$U\varphi_j(\mathbf{u}) = \lambda_j \varphi_j(\mathbf{u})$$

Koopman eigenfunctions

- Expand observables into Koopman eigenfunctions

$$\mathbf{a}(\mathbf{u}_0) = \sum_{j=0}^{\infty} \phi_j \varphi_j(\mathbf{u}_0)$$

Koopman Modes

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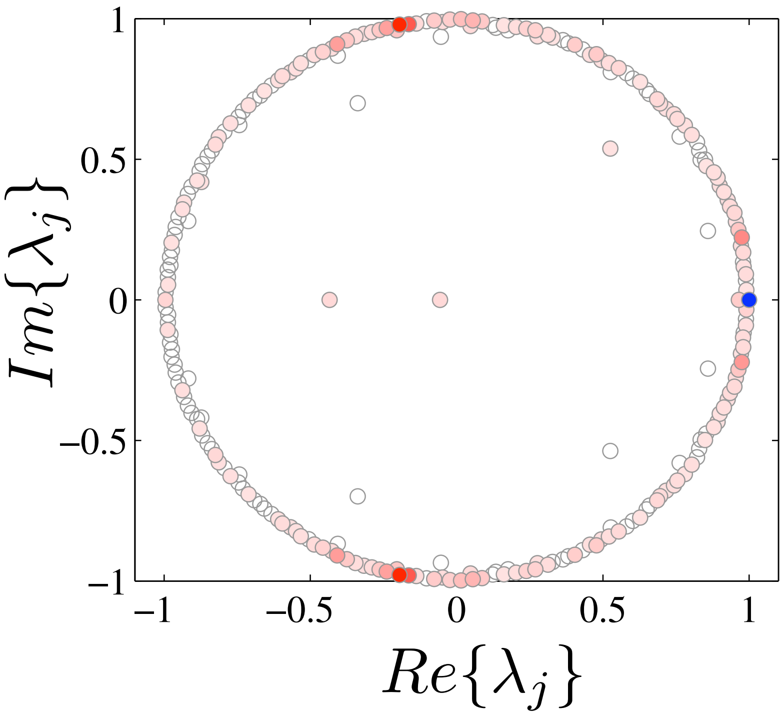
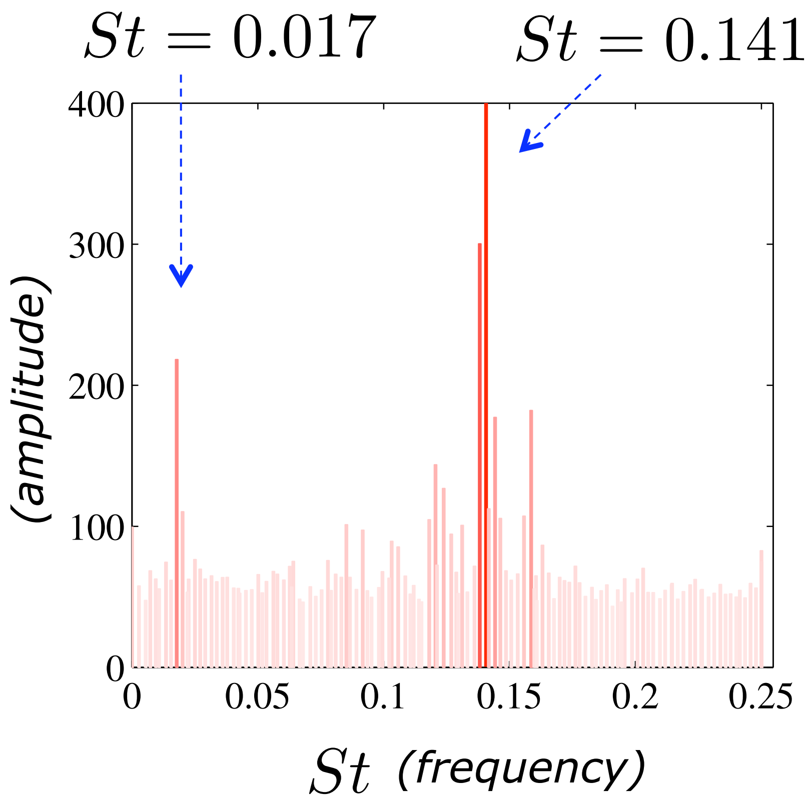
Koopman Spectrum of Jet in Crossflow

- Eigenvalues on the unit circle





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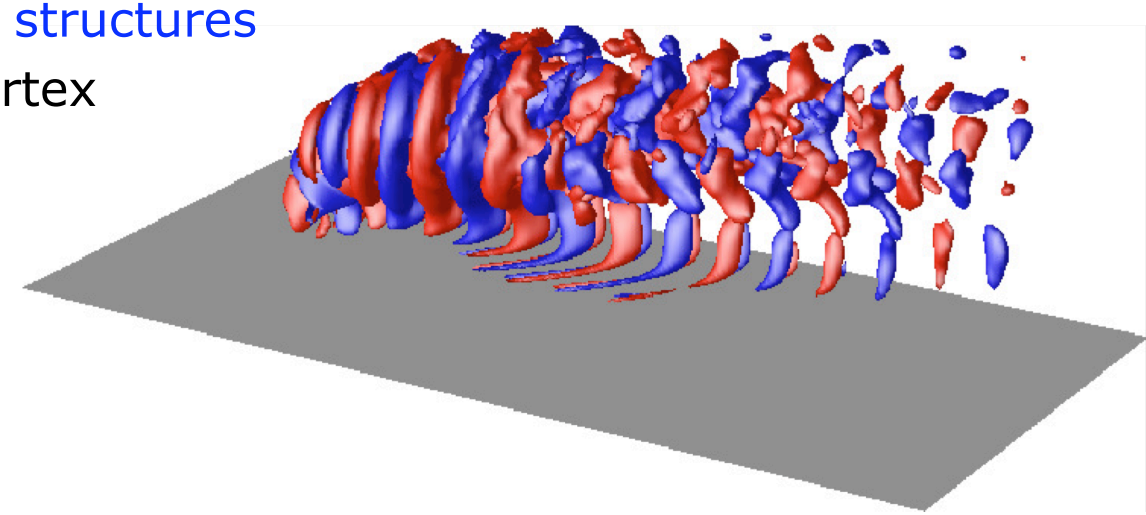
- – Dominant frequencies match vortex shedding frequencies from DNS
- – Computed using DMD (Dynamic Mode Decomposition) *(Schmid 2010)*

Koopman Modes

- High-frequency mode:
  - Captures shear-layer structures
  - Matches first DNS-vortex shedding

$$St = 0.141$$



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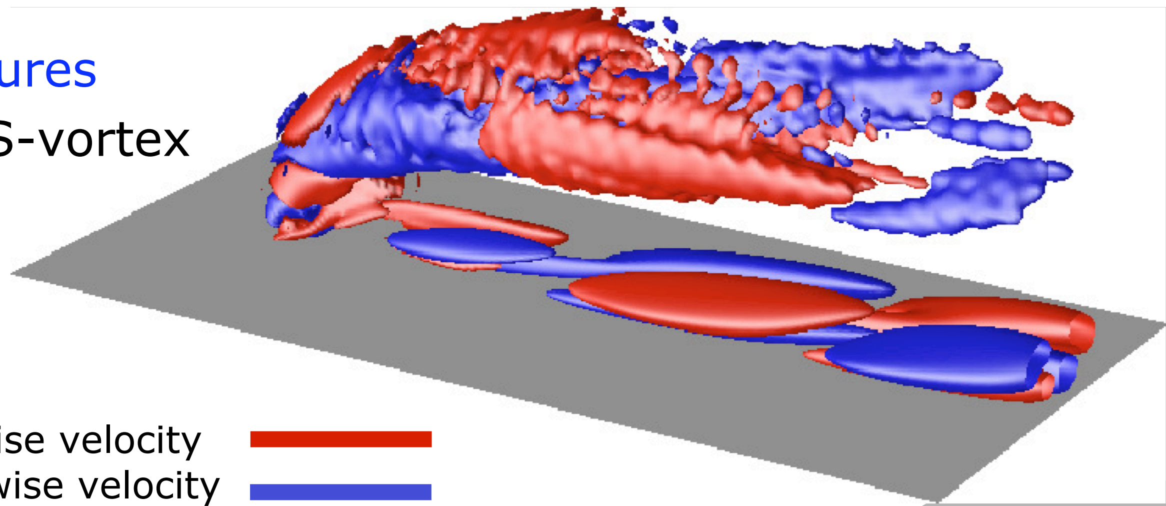
- Low-frequency mode
  - Captures wall structures
  - Matches second DNS-vortex shedding

$$St = 0.017$$



Positive streamwise velocity
Negative streamwise velocity

Summary of Part I

- Decomposition of unsteady flow into global modes
  - Global linear eigenmodes
  - Koopman modes

- Identified three elementary instability mechanisms
  - Kelvin-Helmholtz instability
  - Short-wavelength instability of a vortex pair
  - von Kàrmàn vortex street

- Identified flow structures associated with vortex shedding
  - Wall mode oscillating with low frequency
  - Jet mode oscillating with high frequency

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Part II
Flow Control

Flow on an Airplane Wing

- Friction Drag on surface smaller for laminar than turbulent flows

- Delay the transition to turbulence to save fuel

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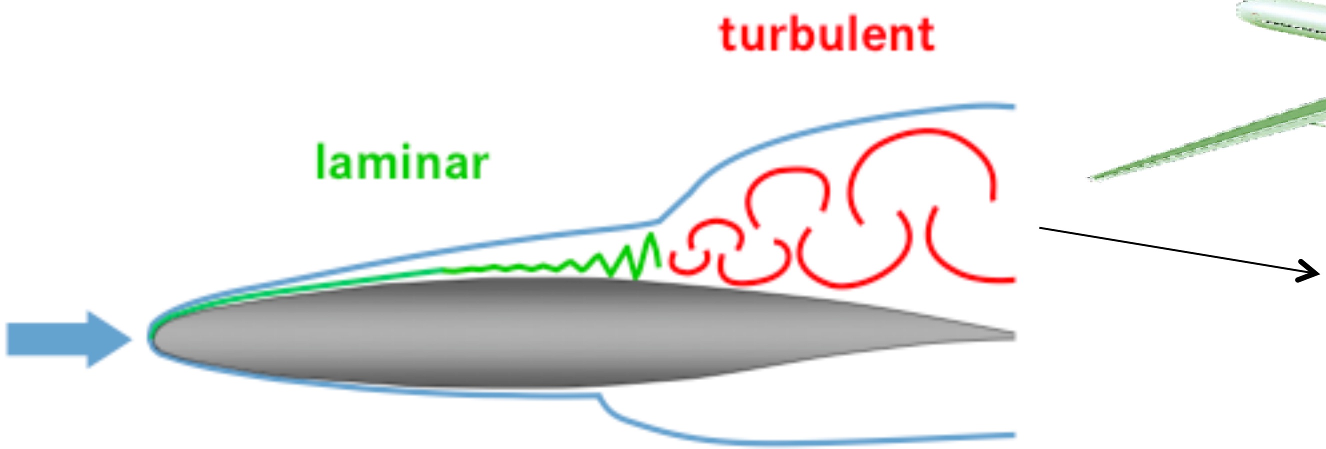

Flow on a Flat Plate

- Simplified geometry
  - flow on a flat plate (downstream of leading edge)

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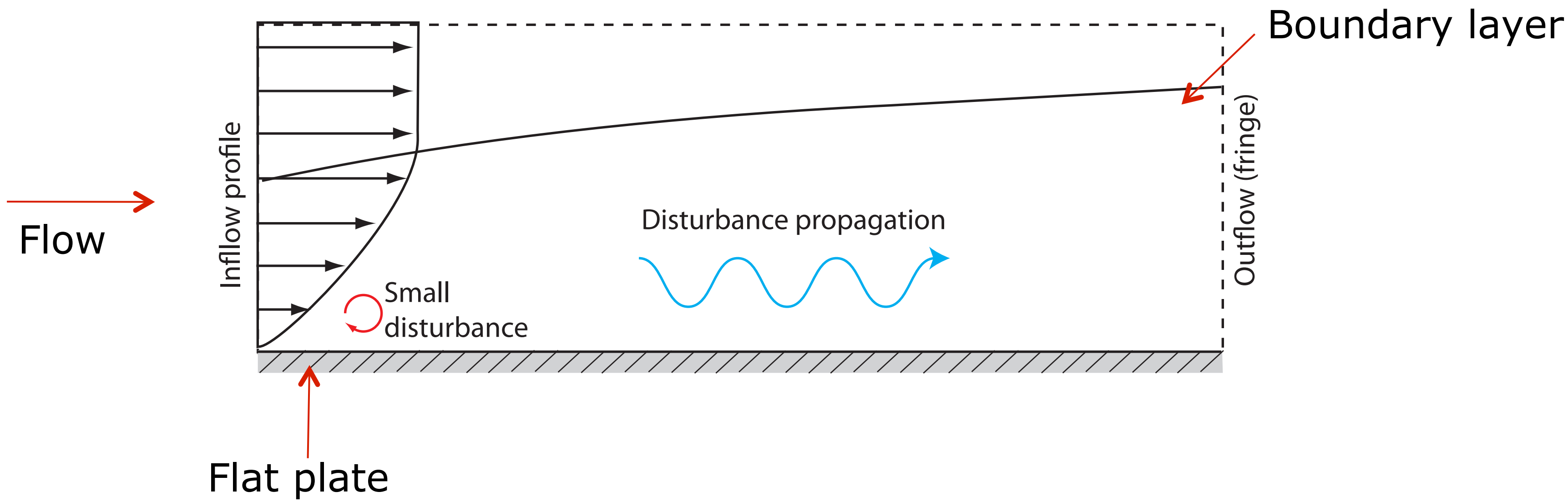

- Direct numerical simulations
  - Re=1000 & 10 million grid points

The 3 Stages of Laminar—Turbulent Transition

- 1st stage is linear growth of perturbations

- 2nd and 3rd stages are nonlinear process'

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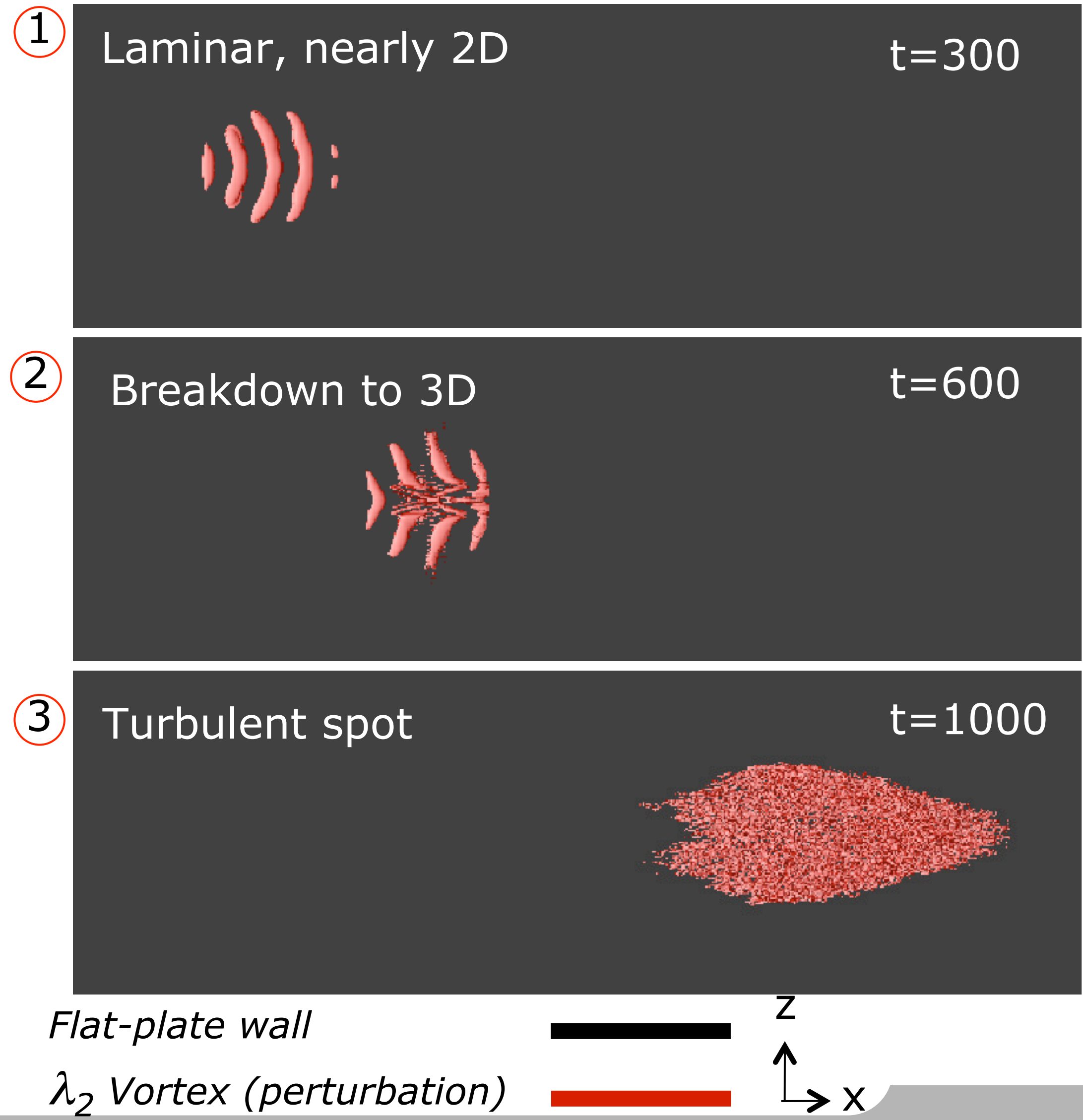

Introduce Actuators & Sensors

- Focus on first stage of transition process
  - can use linearized system
- Control formulation:

$$\dot{\mathbf{u}} = \mathbf{A}\mathbf{u} + \mathbf{B}\mathbf{w}$$
$$\mathbf{y} = \mathbf{C}\mathbf{u}$$

Input signals (disturbances & actuators) → $\mathbf{B}\mathbf{w}$

Output signals (sensor & objective function) → $\mathbf{y}$

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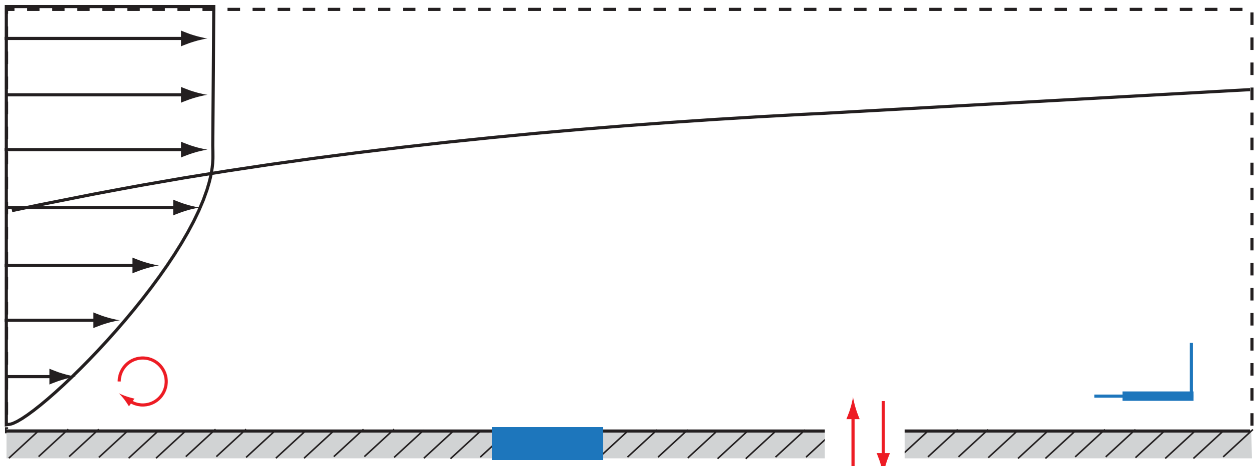

Control Design Issues

- How to:
  - Connect sensors to actuators?
  - What should the actuator do, when we have measurements?
  - Are there guarantees of stability, performance & robustness?
- Answer: Linear control theory
- Problem: tools too expensive for 2D or 3D computational fluid dynamics

  → Model reduction

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Control Design: Two Steps

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1. Develop a low-dimensional model that captures the input-output behavior of high-dimensional Navier-Stokes system
2. Use the low-dimensional model to construct a controller

Bagheri et al. AMR 2009
Bagheri et al. JFM 2009
Bagheri et al. AIAA J. 2009
Semeraro et al. JFM 2010
Bagheri et al. Roy. Proc. A 2010

Capturing Input-Output Behavior

- For a given input signal, what is the output?
- Introduce a mapping between inputs to outputs:

$$\mathbf{G} : \mathbf{w} \rightarrow \mathbf{y}$$

  - Complexity order of millions (due to discretization of N-S)

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- How to construct an approximation

$$\mathbf{G}_r : \mathbf{w} \rightarrow \mathbf{y}$$

such that
  - complexity is of order 10-100
  - norm $\|\mathbf{G} - \mathbf{G}_r\|$ is small

Controllability & Observability

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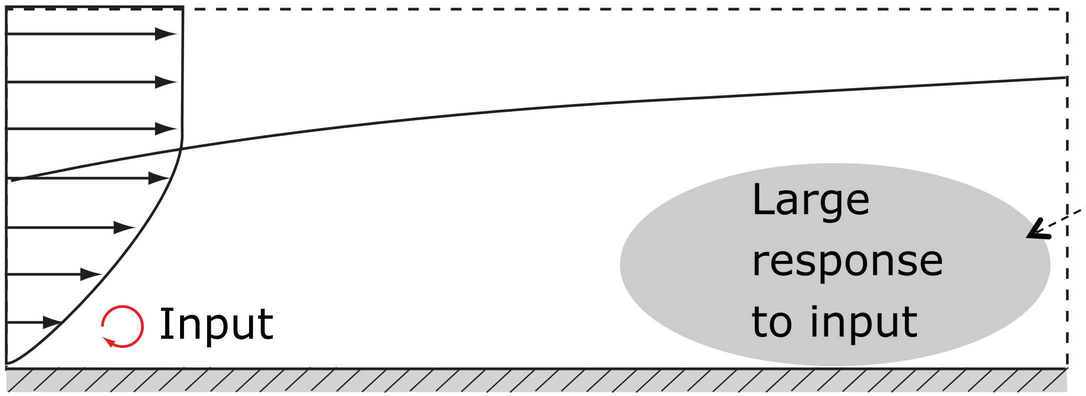
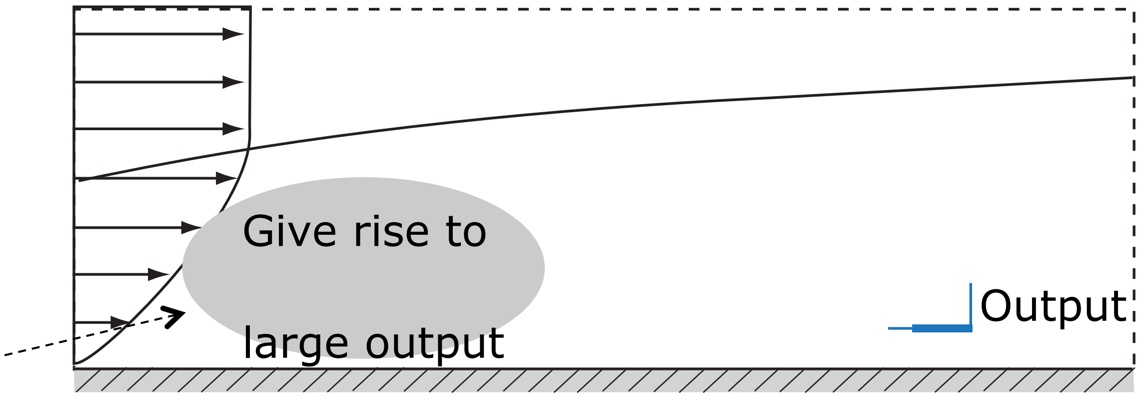
- Which flow structures respond to input forcing?



- Which flow structures generate large output energy?



Balanced Modes

- Controllability Gramian determines controllable structures

$$\mathbf{P} \quad = \int_0^\infty e^{\mathbf{A}t}\mathbf{B}\mathbf{B}^T e^{\mathbf{A}^T t} dt,$$

- Observability Gramian determines observable structures

$$\mathbf{Q} \quad = \int_0^\infty e^{\mathbf{A}^T t}\mathbf{C}^T\mathbf{C}e^{\mathbf{A}t} dt$$

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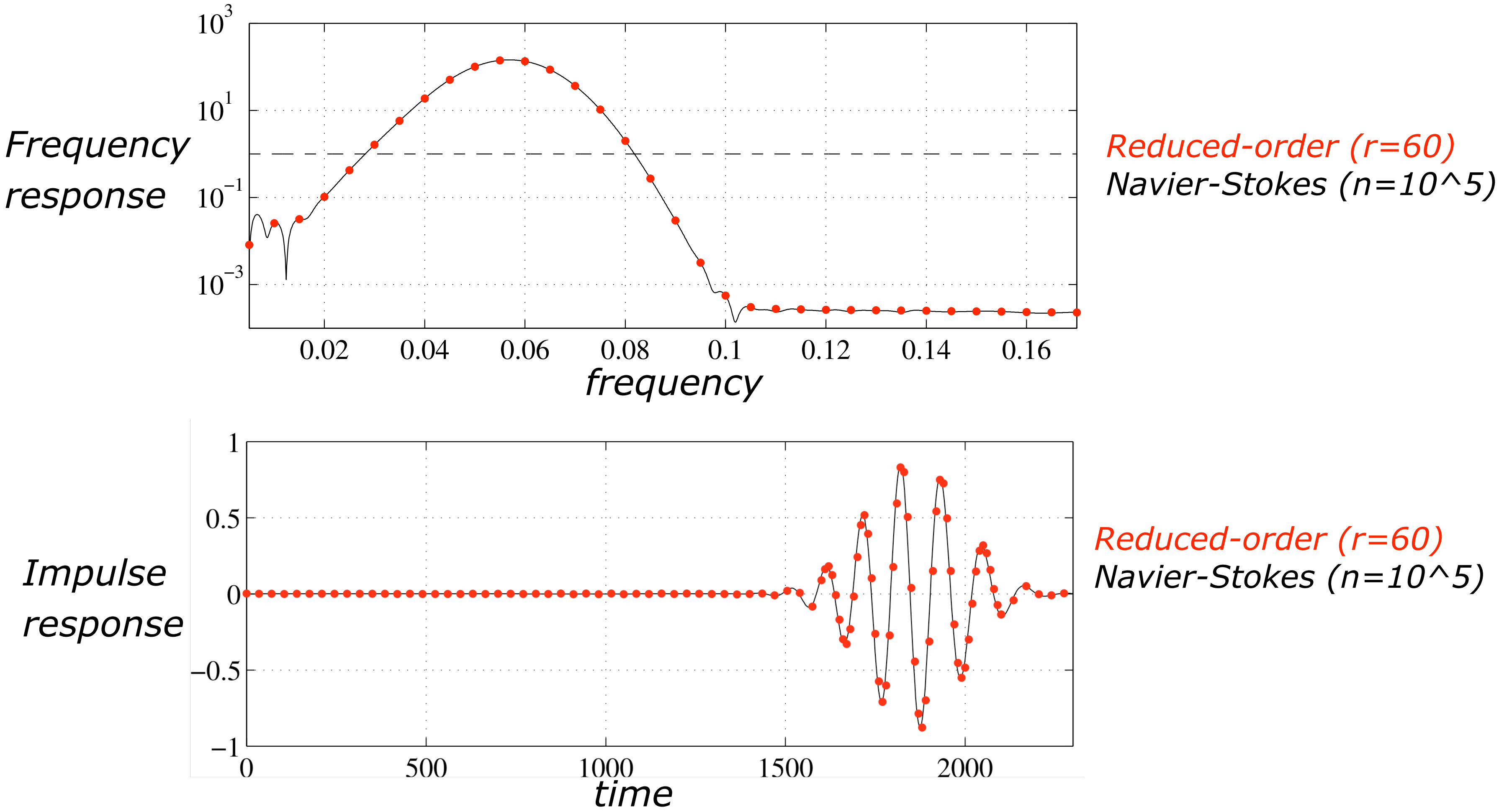
- Balanced modes are eigenmodes of

$$\mathbf{P}\mathbf{Q}\phi_j = \lambda_j \phi_j$$ ← Balanced modes

  - For 2D/3D flows modes computed using the snapshot method *(Rowley, 2005)*

- Reduced model obtained by projection onto balanced modes

Validation of Reduced-Order Model



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- Reduced-order model & Navier-Stokes show same input-output behavior

Two Steps

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1. Develop a low-dimensional model that captures the input-output behavior of high-dimensional Navier-Stokes system

2. Use the low-dimensional model to construct a controller

Bagheri et al. AMR 2009
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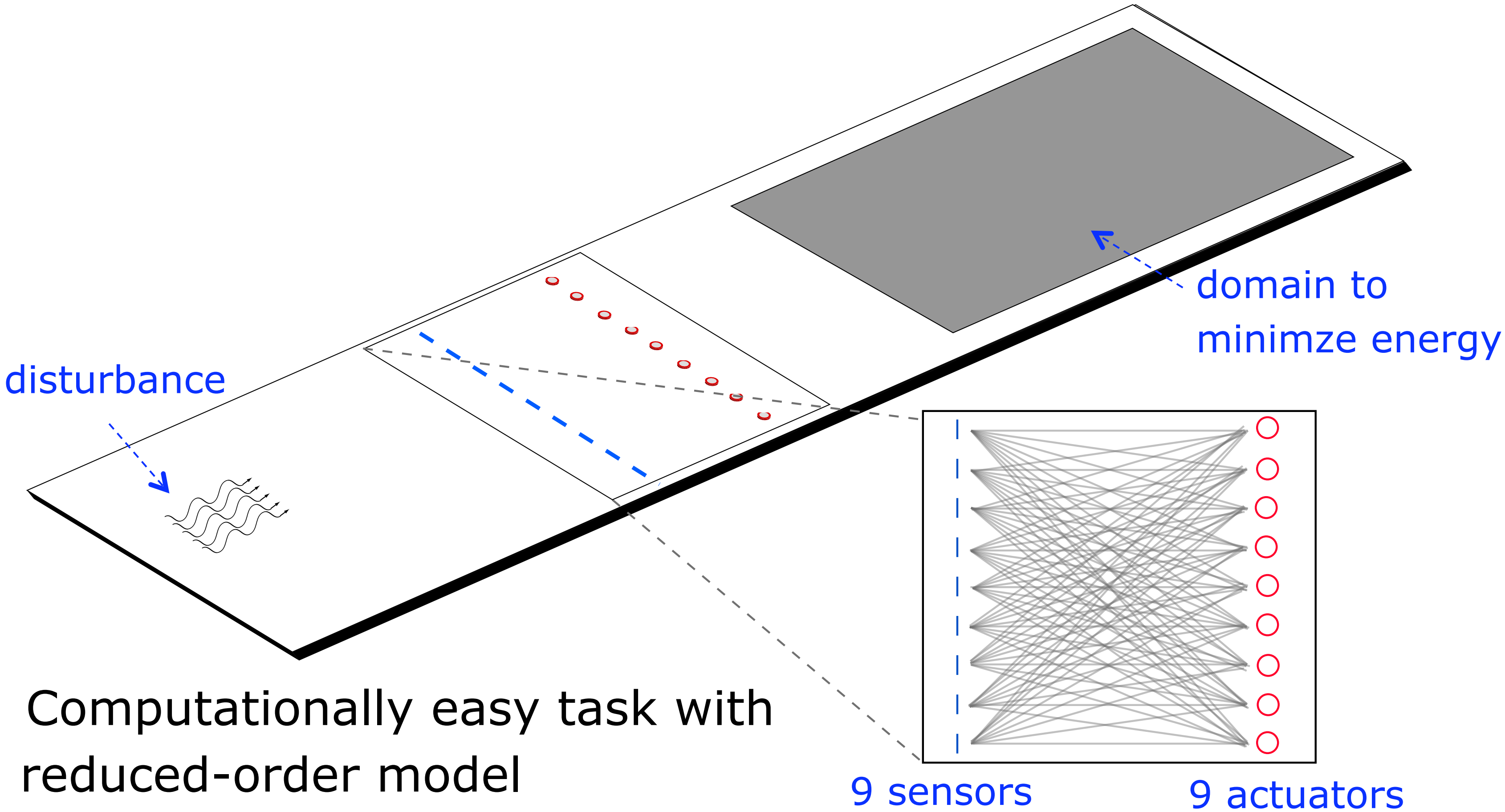
Bagheri et al. Roy. Proc. A 2010

Control Design

- Linear quadratic Gaussian (LQG)
  - Based on noisy sensor measurements, find control signal that minimize effects of disturbances in a subdomain



- Computationally easy task with reduced-order model

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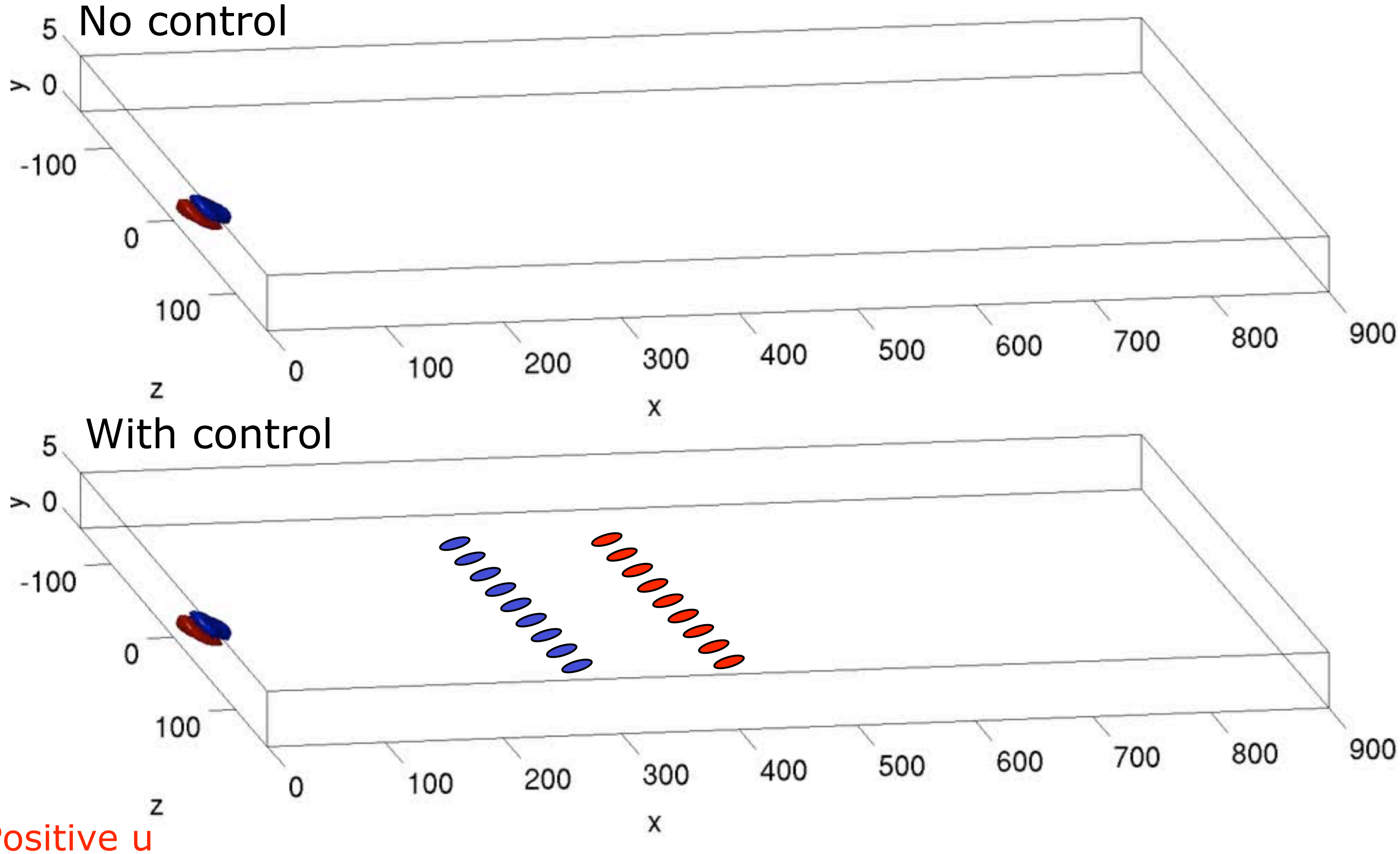
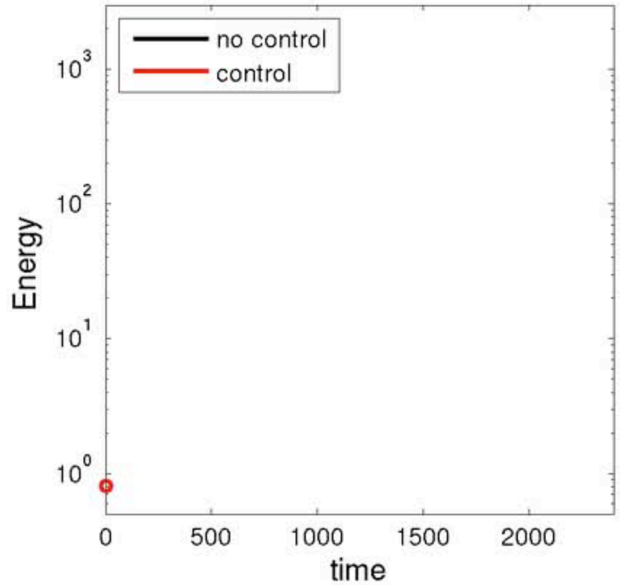
Controlled Flow

- Disturbance energy reduced orders of magnitude

  Using 9 small sensors & 9 actuators



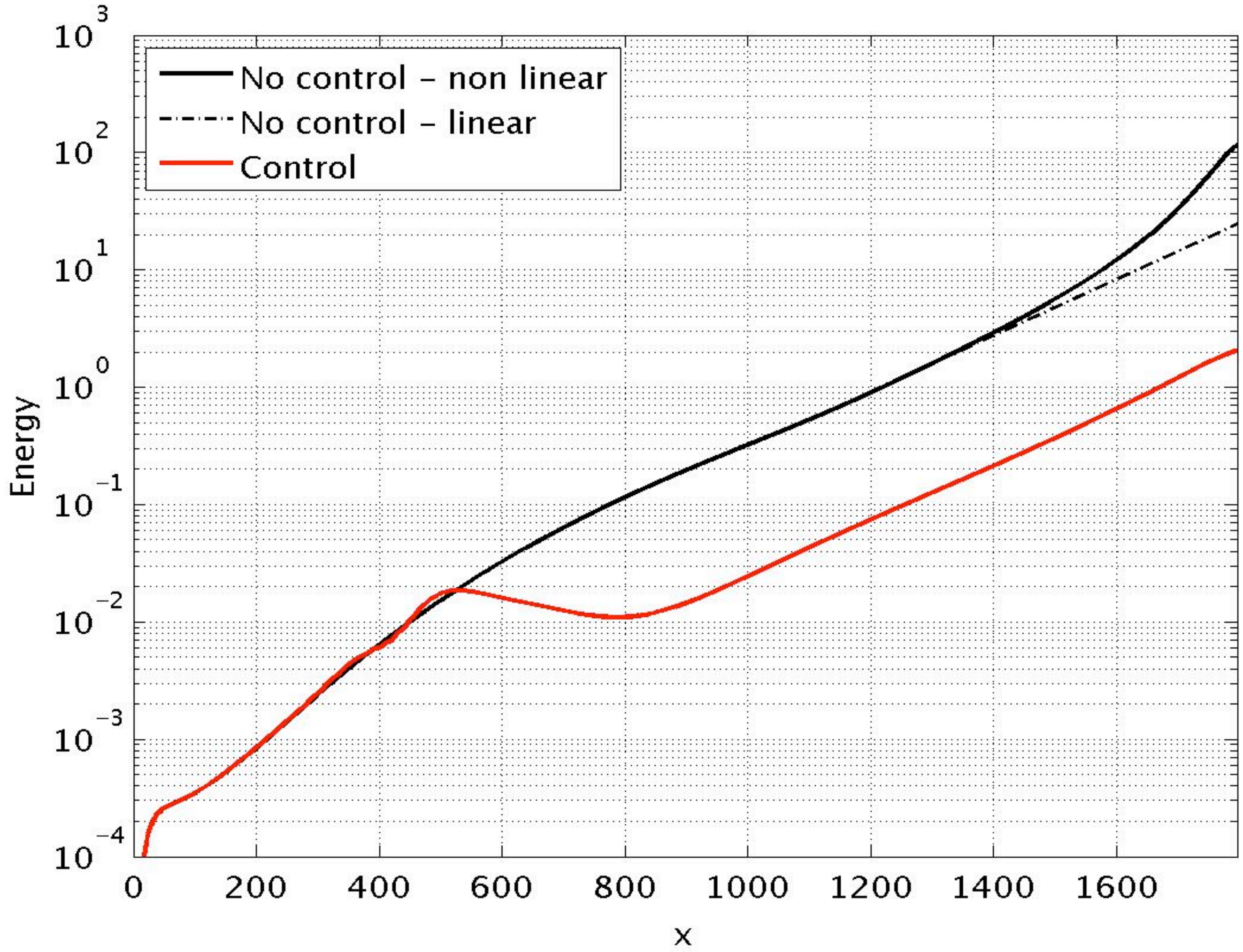


Positive u

Negative u

Transition delay

- Transition delayed using a linear controller



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Summary of Part II

- Using control theory we are able to account for
  - measurement noise
  - control penalty
  - optimality and robustness

- Balanced modes
  - take into account controllability & observability
  - able to capture input-output behavior of Navier-Stokes eqs

- Disturbance energy can be reduced by orders of magnitude using localized sensing/acting

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Future Directions

- Flow analysis of the Jet in Crossflow:
  - *Sensitivity analysis*: identifty locations where steady flow is sensitive to external modifications
  - *Bifurcation analysis:* stability properties depend on the velocity ratio

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- Flow control of the flat-plate flow:
  - *Delay transition:* validate numerically using low-order controller
  - *Wind-tunnel experiments:* Use the low-order controller to delay transition

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Thank you.

Extra Slides

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- Dimension of A
- Dynamic Mode Decomposition
- Arnoldi Method
- Snapshot Method

Dimension of discretized system

	Base Flow	Inhomogeneous direction(s)	Dimension of $\boldsymbol{u}(t)$	Storage of A
Ginzburg-Landau	$U(x)$	1D	10^2	1 MB
Blasius	$U(x,y)$	2D	10^5	25 GB
Jet in crossflow	$U(x,y,z)$	3D	10^7	500 TB

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- Matrix A very large for spatially developing flows
- Use Navier-Stokes solver (DNS) or any CFD code to approximate the action of exponential matrix:

$$\mathbf{u}(\Delta t) = e^{A(\Delta t)}\mathbf{u}_0$$

- Time-stepper technique: Never store matrices and use only velocity fields

Iterative eigenvalue methods

- Eigenvalue problem

$$\mathcal{F}(\Delta t)\mathbf{u}_j = \sigma_j \mathbf{u}_j \qquad (n \times n), \qquad n > 10^5$$

- Construct a small subspace from snapshots

$$\mathcal{K} \;=\; \text{span}\{\mathbf{u}_0, \mathcal{F}(\Delta t)\mathbf{u}_0, \mathcal{F}(2\Delta t)\mathbf{u}_0, \ldots, \mathcal{F}((m-1)\Delta t)\mathbf{u}_0\}$$

- Solve small eigenvalue problem
  - Orthonormalize (e.g. Arnoldi) $\mathbf{V} = [V_1, \ldots, V_m]$
  - Project operator $\mathcal{F}(\Delta t) \approx \mathbf{V}\mathbf{H}\mathbf{V}^T$
  - Solve small eigenvalue problem $\mathbf{H}\mathbf{S} = \mathbf{S}\mathbf{\Sigma} \qquad (m \times m), \quad m < 100$

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Global eigenmodes

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- The eigenvalue problem

$$e^{At}\mathbf{u}_j = \sigma_j \mathbf{u}_j$$

- Subspace

$$\mathcal{K} = \{\mathbf{u}_0, e^{A\Delta t}\mathbf{u}_0, e^{A2\Delta t}\mathbf{u}_0, \ldots, e^{A(m-1)\Delta t}\mathbf{u}_0\}$$

- Basis vector: snapshots of flow fields separated by constant time

$$T_O < \Delta t < T_{\text{Nyquist}}$$

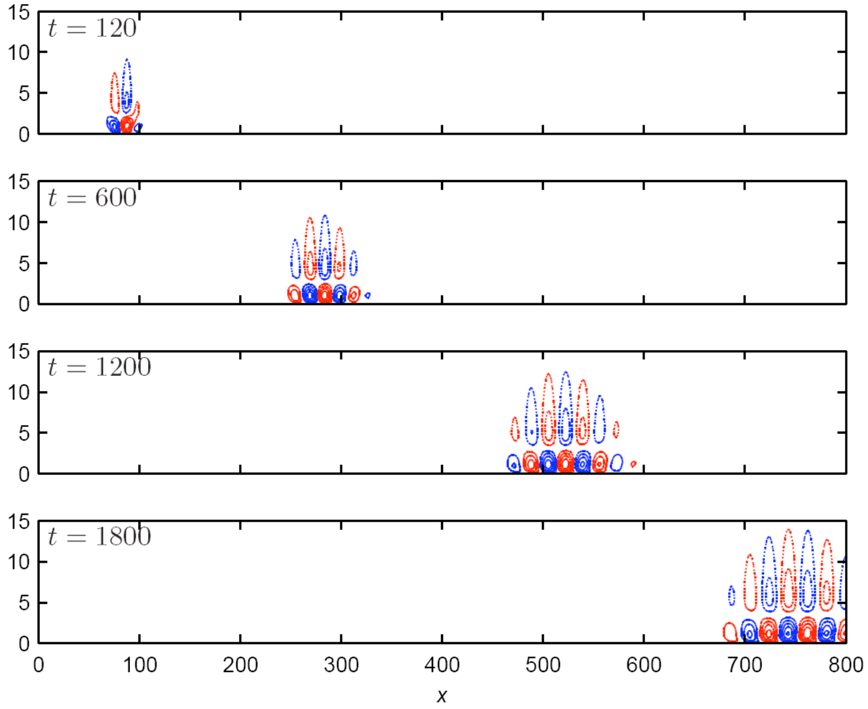
- Eigenvalues of A recovered from

$$\omega = \ln(\sigma)/\Delta t$$

The Snapshot Method

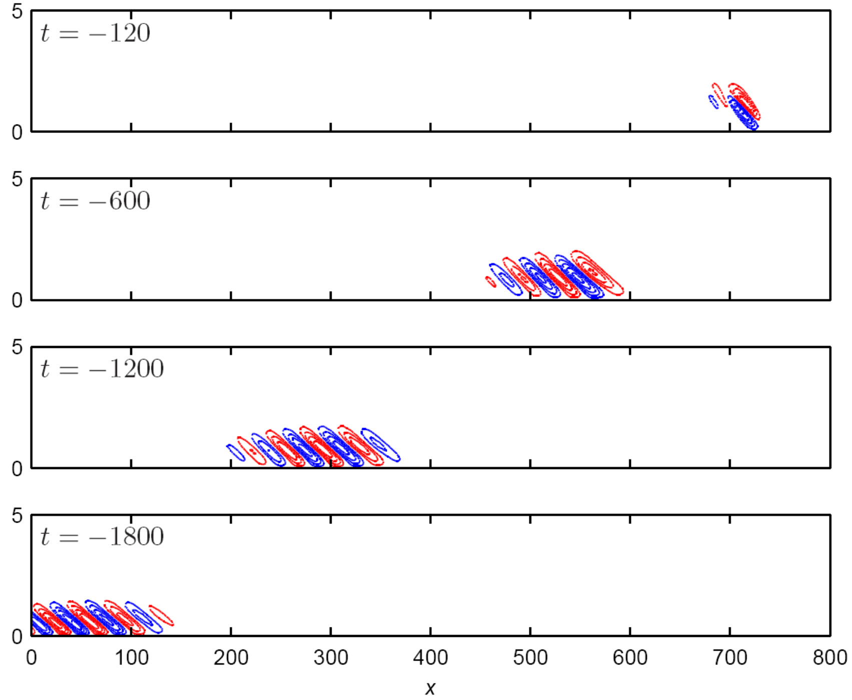
Snaphots of direct simulation simulation

$$\mathbf{X} = \{B_1, \ldots, e^{A\Delta t} B_1, B_2, \ldots, e^{A\Delta t} B_p\}$$



Snapshots of adjoint

$$\mathbf{Y} = \{C_1^*, \ldots, C_1^*, e^{A^*\Delta t} C_2^*, \ldots, e^{A^*\Delta t} C_r^*\}$$



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The snapshot metod

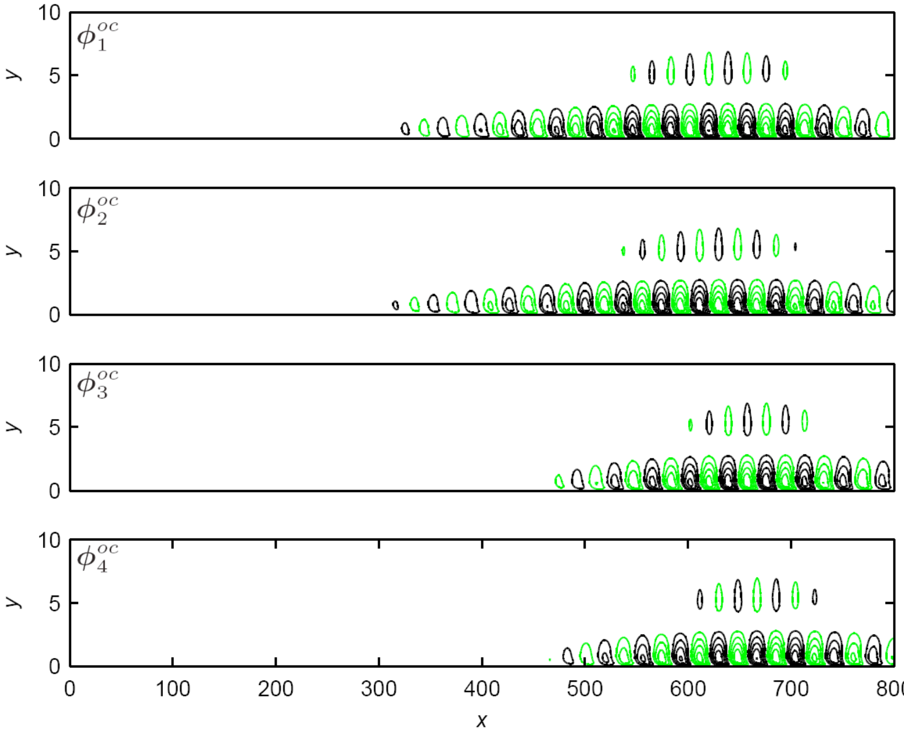
- Singular value decomposition of size: (pm x rm)

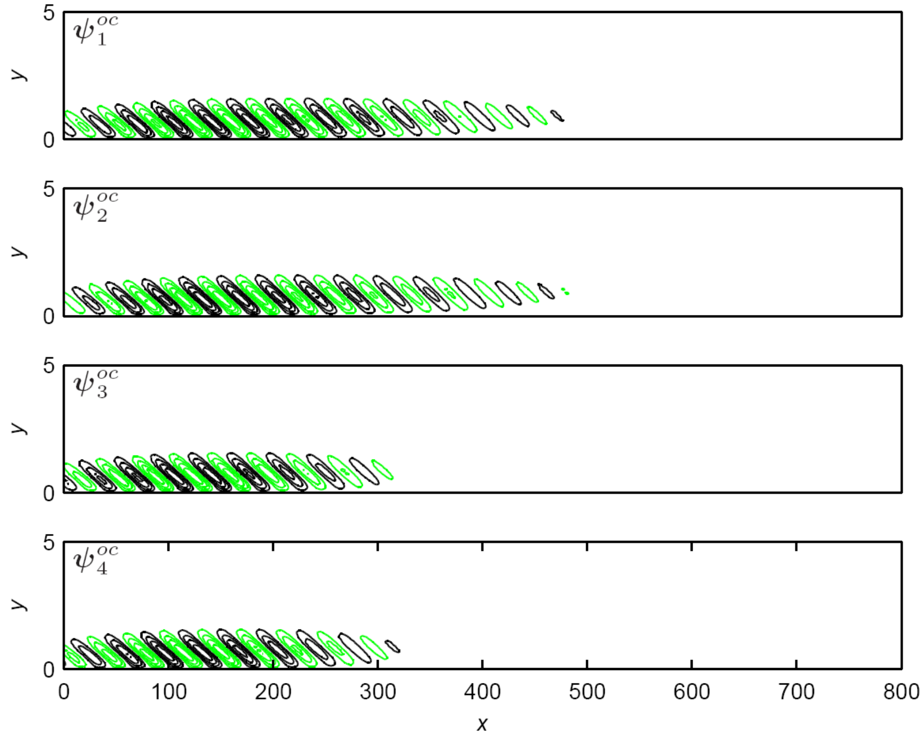
$$\mathbf{Y}^T\mathbf{X} = \mathbf{U}\Sigma\mathbf{V}^T$$

- Balanced modes Adjoint balanced modes

$$\mathbf{T} = \mathbf{X}\mathbf{V}$$

$$\mathbf{S} = \mathbf{Y}\mathbf{U}$$





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