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Analysis and Control of Transitional Shear Flows Using Global Modes

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Doctoral Thesis Seminar
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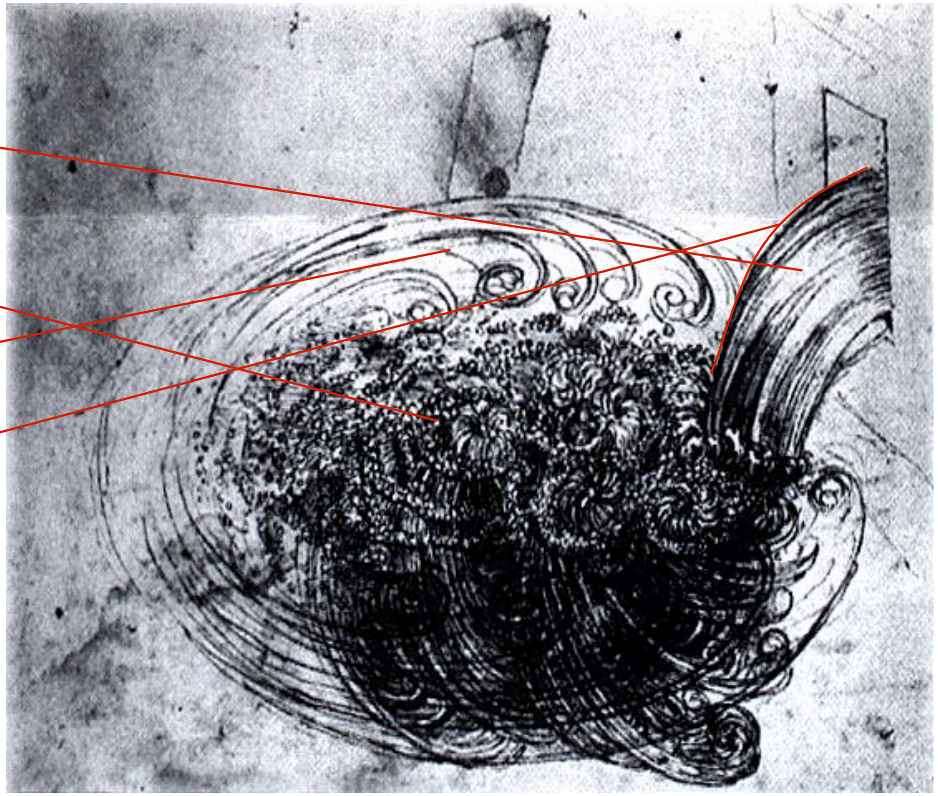
KTH Mechanics, Stockholm

A Free Water Jet Into a Pool

- Laminar flow
- Turbulent flow
- Vortical structures
- Shear layers



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Leonardo da Vinci

Cigarette Smoke

- Ordered and predictable smoke becomes chaotic and unpredictable
- Transition of a laminar flow to a turbulent one

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Cloud Structure

- Clouds roll up into Kelvin-Helmholtz vortices
- Two streams of different velocity – shear layer instabilities

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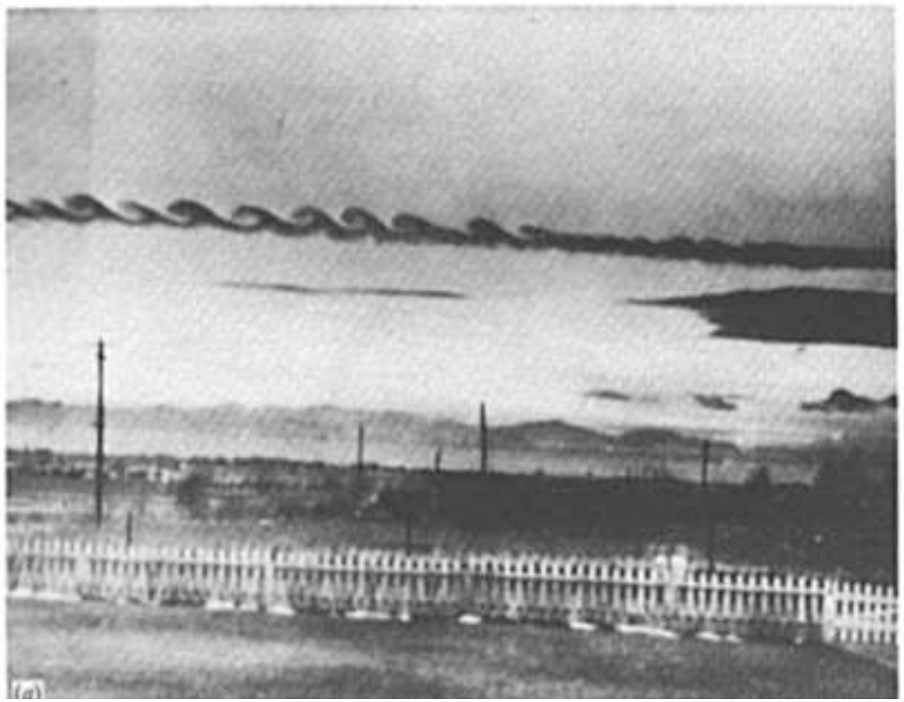

Clould Structure

- von Kármán vortex street developing behind an island
- Periodic vortex shedding

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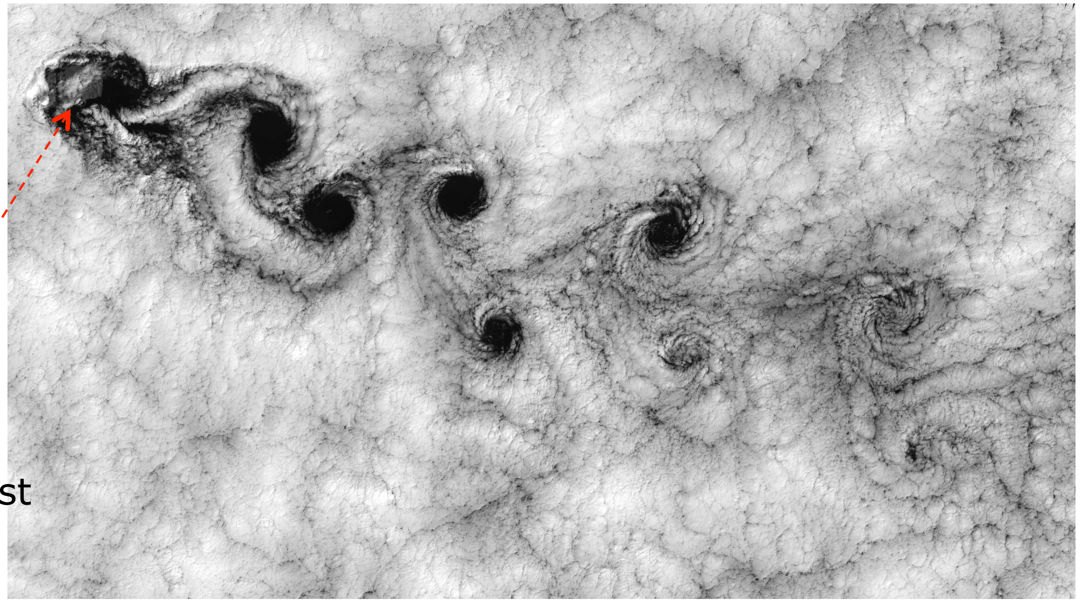

Island near Chilean coast

Open Issues

- Fundamental issues touched upon in this thesis:
  - How does a flow transition from laminar to turbulent? Can we control the transition process?
  - Why does unsteadiness in some flows take the form of periodic shedding of vortices?

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- Approaches:
  - Numerical simulations
  - Mathematical tools for analysis and control
  - Both simple & complex flow configurations

Outline

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- Flow phenomena
- Part I: Flow analysis
- Part II: Flow control

Part I
Flow Analysis

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Jet in Crossflow

- Fluid injected through a hole into a crossflow

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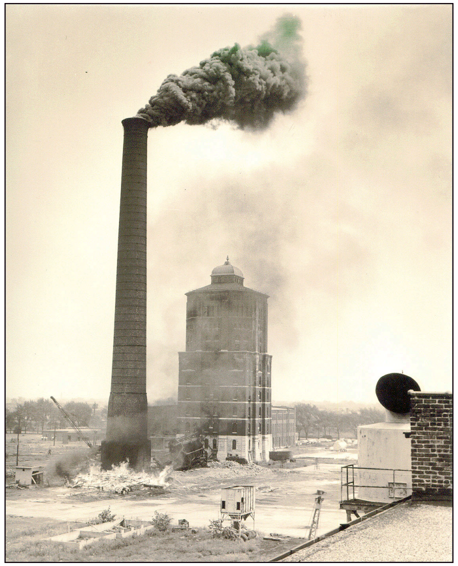
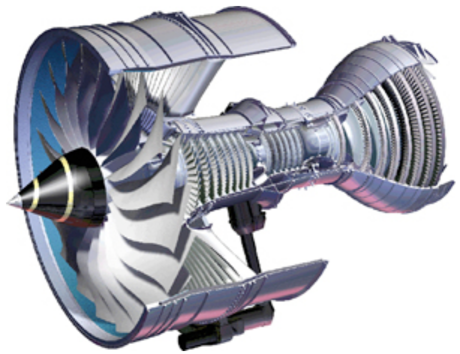
Smoke stacks

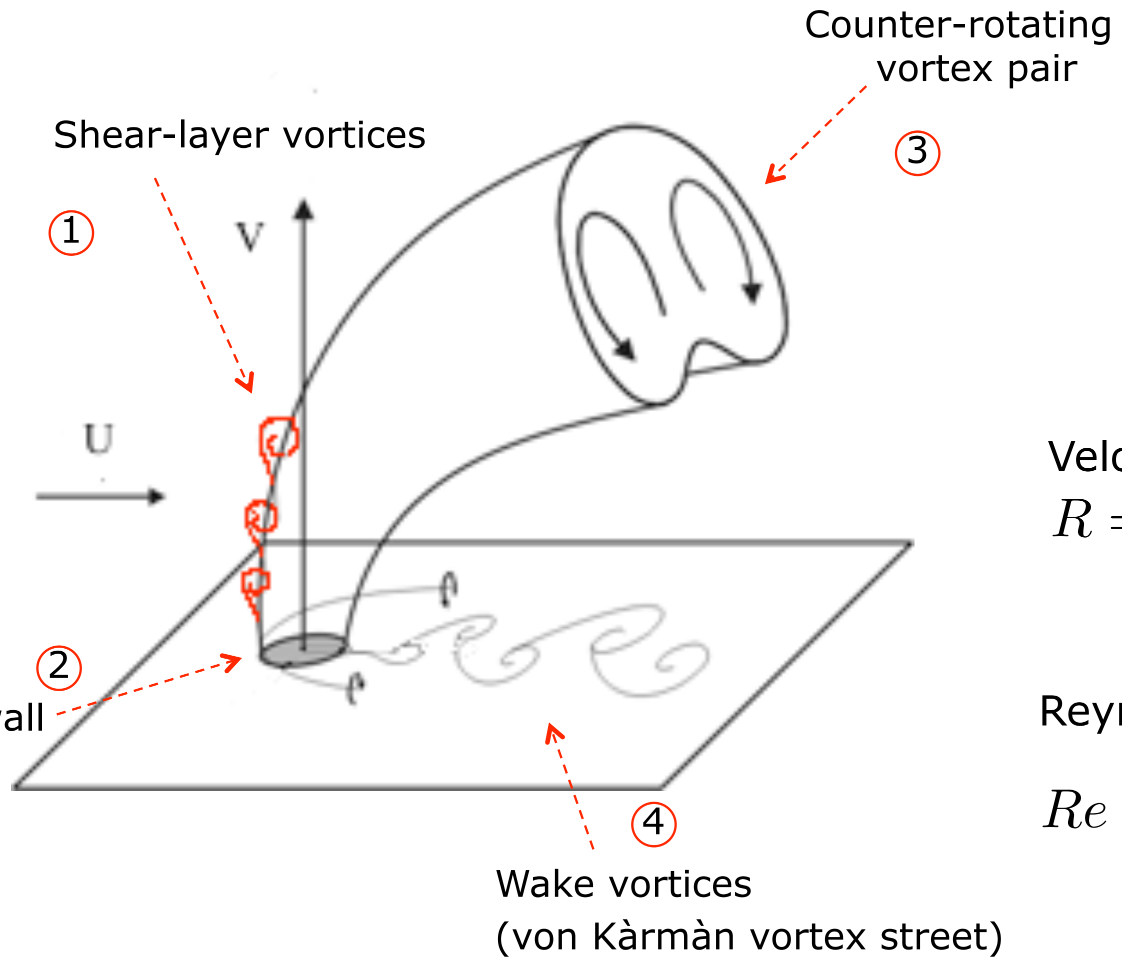


Volcano eruptions

Fuel injection/film cooling



The Four Vortical Structures of Jet in Crossflow



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Velocity ratio:

$$R = V/U = 3$$

Reynolds number:

$$Re = \frac{\delta_0^* U}{\nu} = 165$$

Kelso et al (jfm, 1996)
Fric & Roshko (jfm, 1994)

Numerical Simulations

- Direct numerical simulations (DNS)
  - Fully spectral code
  - 10 million gridpoints

- Identified from DNS:
  - all 4 vortical structures
  - 2 events of vortex shedding (oscillation of separated region)

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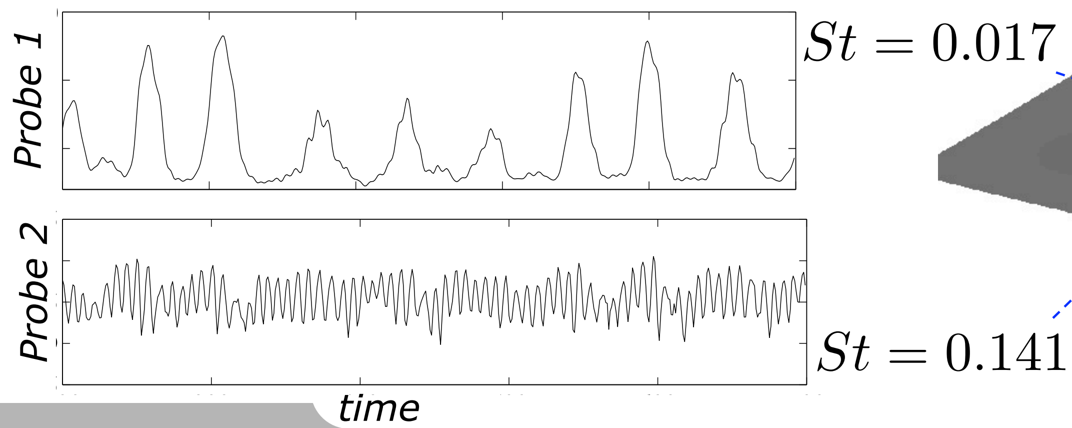

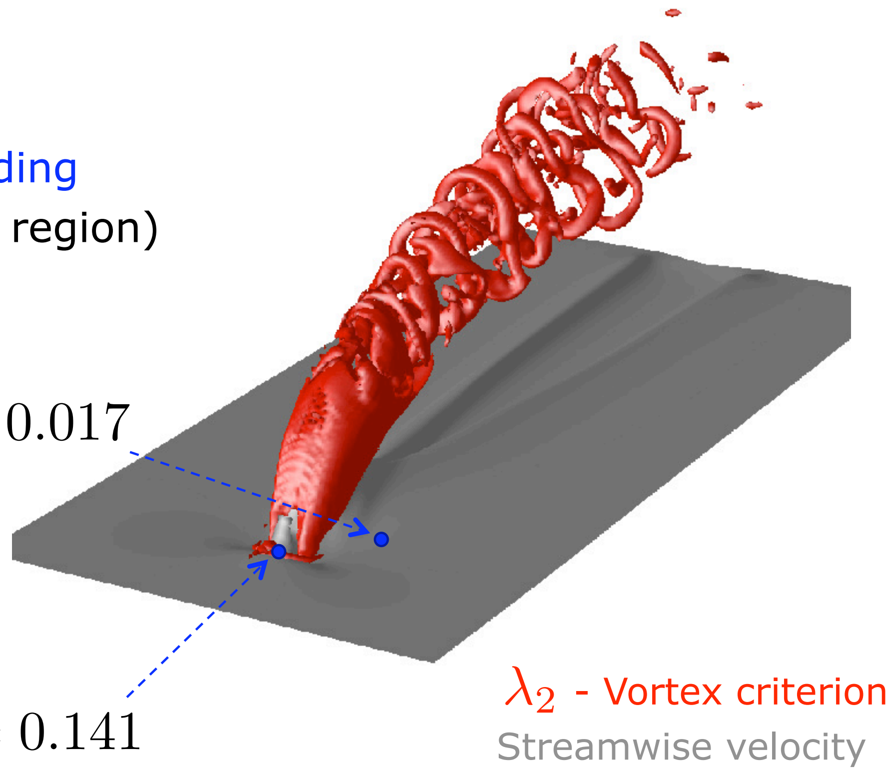

Analysis of the Jet in Crossflow

- Observations from DNS:
  - 4 large vortical structures
  - 2 events of periodic vortex shedding

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1. Stability analysis: Perturbation dynamics near steady state
   - Determine which vortical structures are steady or unsteady
   - Determine the physical mechanisms for unsteadiness

2. Nonlinear analysis: Flow dynamics in an attractor region
   - Identify which vortical structures are oscillating periodically

Papers: 5, 6 & 7

Overview of Stability Analysis

- The Navier-Stokes equations :

$$\dot{\mathbf{u}} = \mathbf{f}(\mathbf{u})$$

- Find a steady solution:

$$0 = \mathbf{f}(\mathbf{u}_s)$$

- Perurb the steady solution:

$$\mathbf{u} = \mathbf{u}_s + \mathbf{u}'$$ ← Unsteady perturbation

- Linearize around the steady solution:

$$\dot{\mathbf{u}}' = \mathbf{A}\mathbf{u}'$$

Huge matrix $\sim 10^7 \times 10^7$

- Find eigenvectors and eigenvalues:

$$\mathbf{A}\phi_j = \lambda_j \phi_j$$

Linear global eigenmodes

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Schmid & Henningson (Springer, 2001)

Steady Solution of Navier-Stokes Equations

- For fully 3D problem a difficult computational task
  - Selective frequency damping *(Åkervik et al. PoF 2006)*

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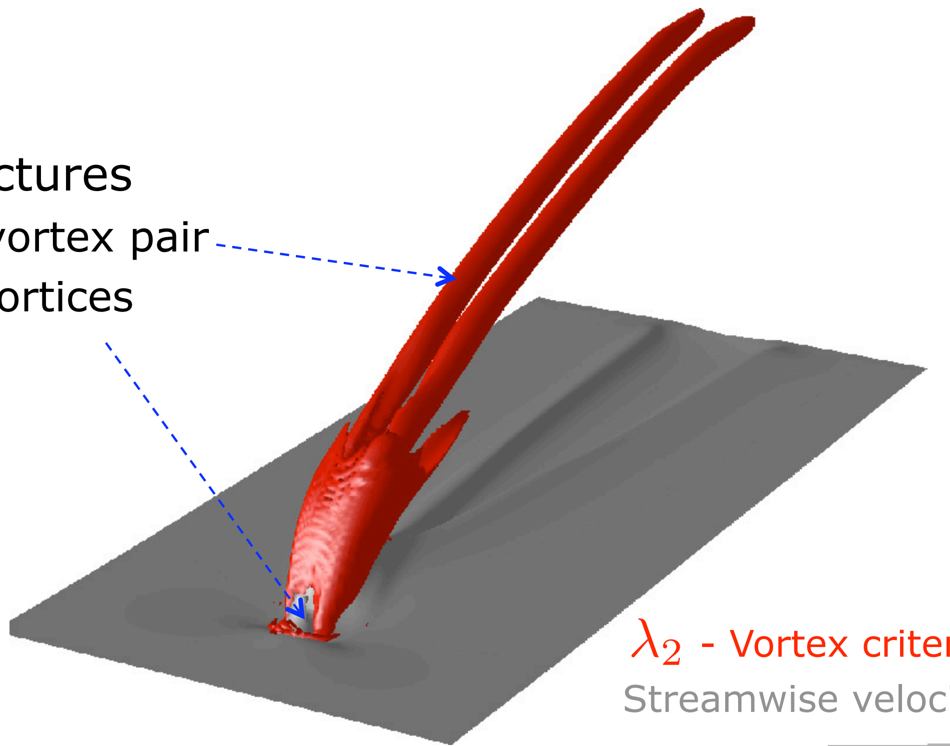
- Steady vortical structures
  - Counter-rotating vortex pair
  - Horse-shoe/wall vortices



Global Stability of Jet in Crossflow

- Global modes computed using iterative techniques (Arnoldi method) in combination with numerical simulations
- 22 unstable global modes found
- Global spectrum:

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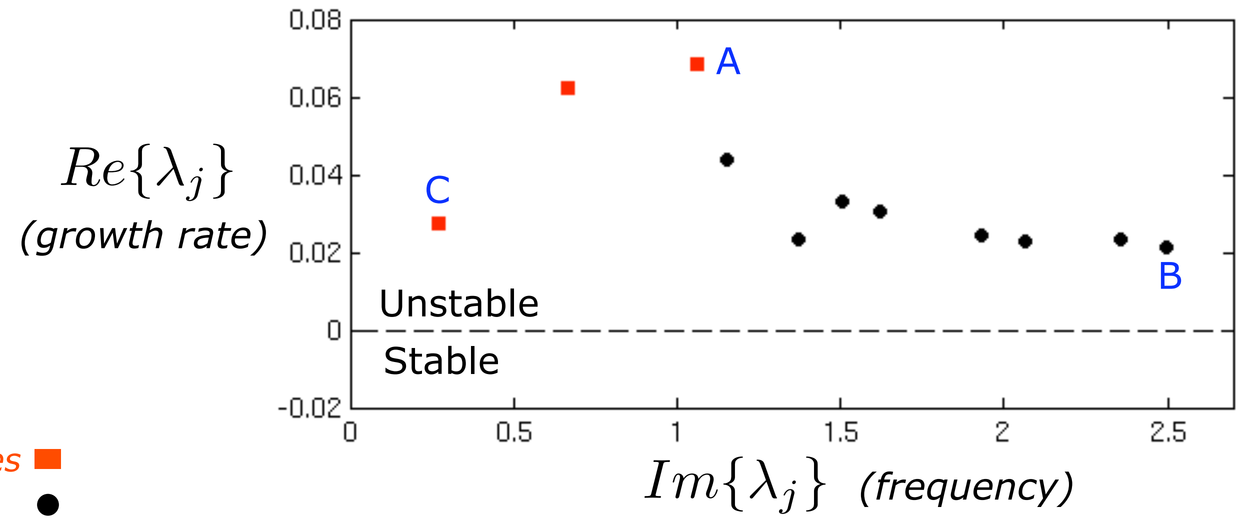
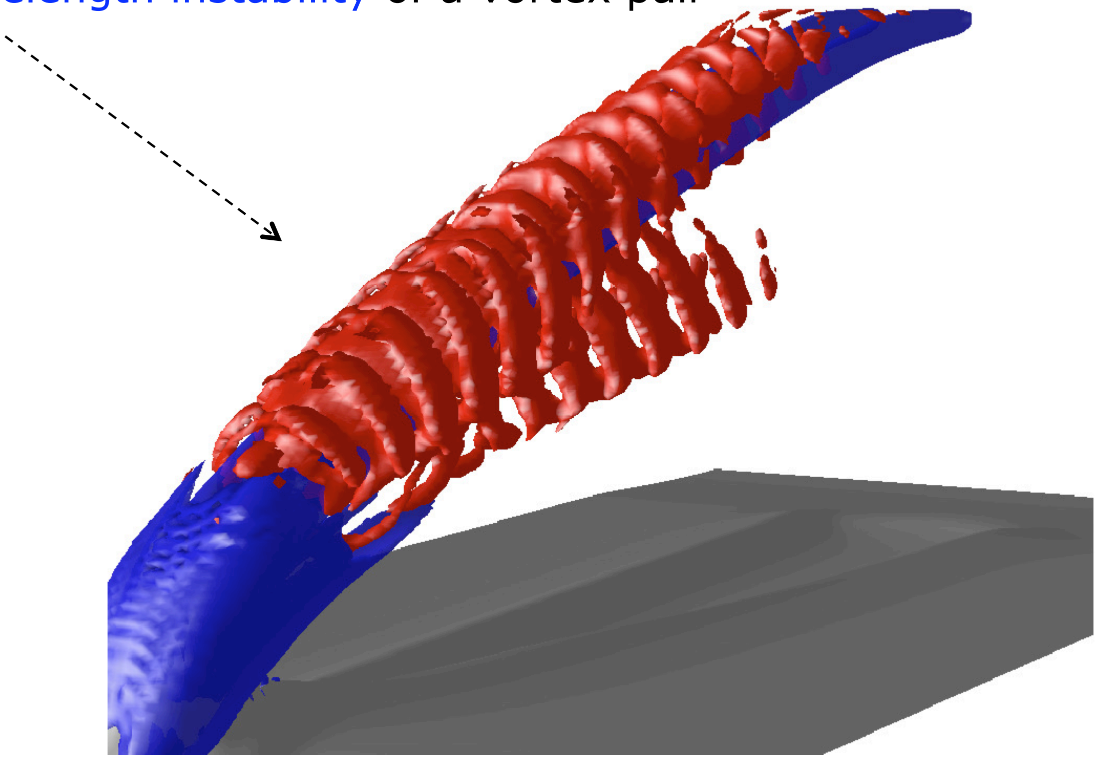


Anit-Symmetric modes ■
Symmetric modes ●

Most Unstable Global Eigenmode

- Mode A:
  - Wavepacket on the counter-rotating vortex pair
  - Short-wavelength instability of a vortex pair



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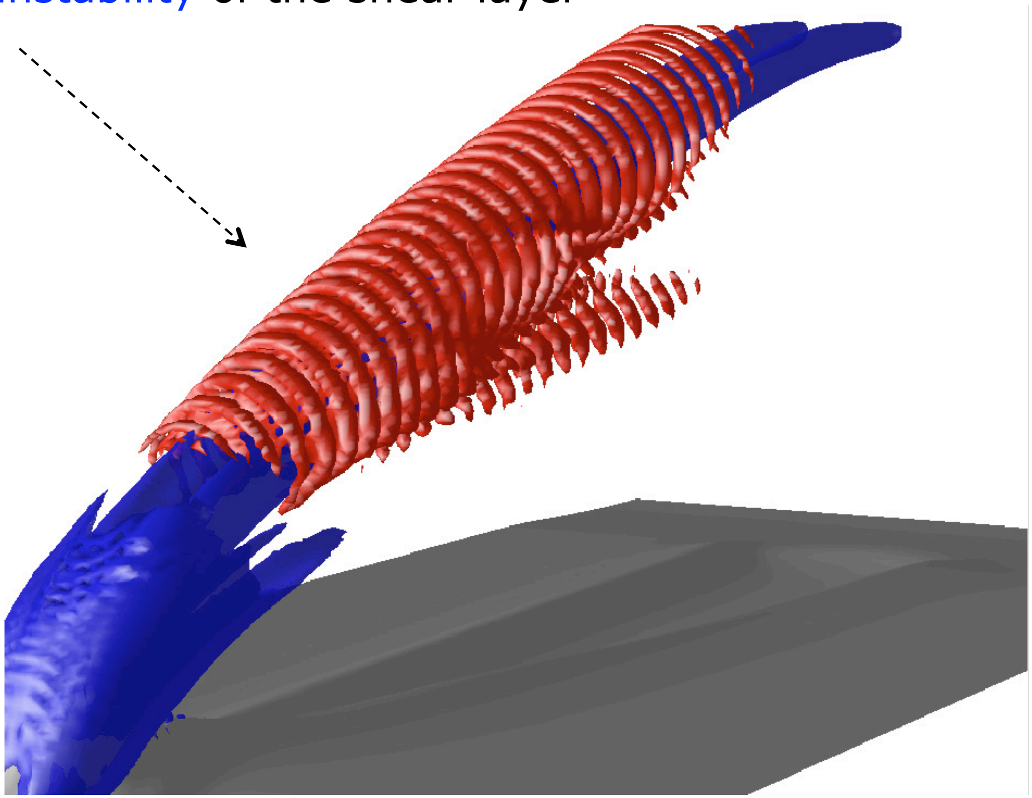
Streamwise velocity (baseflow)
λ_2 Vortex (baseflow)
λ_2 Vortex (global mode)

Symmetric Global Eigenmode

- Mode B:
  - Vortex rings on the counter-rotating vortex pair
  - Kelvin-Helmholtz instability of the shear layer



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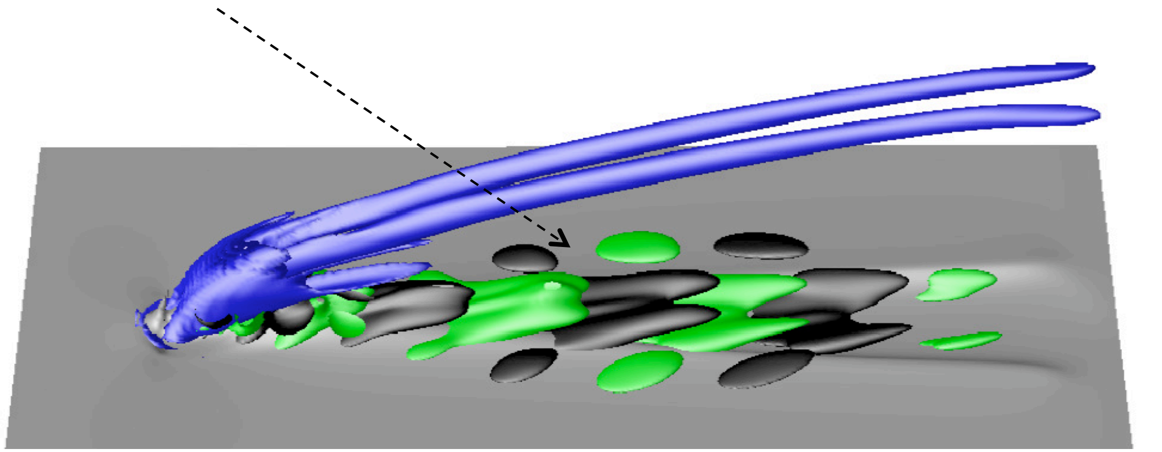
Streamwise velocity (baseflow)

λ_2 Vortex (baseflow)

λ_2 Vortex (global mode)

Low-Frequency Global Eigenmode

- Mode C:
  - Mostly located near the wall
  - Karman vortex street in the wake



Streamwise velocity (baseflow)

λ_2 Vortex criterion (baseflow)

Spanwise velocity (global mode)

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Analysis of the Jet in Crossflow

- Observations from DNS:
  - 4 large vortical structures
  - 2 events of periodic vortex shedding

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(1) Stability analysis: Perturbation dynamics near steady state
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Paper: 7

Koopman Operator

- Define an observable as scalar-valued function

$$a(\mathbf{u}_k) : \mathbb{U} \rightarrow \mathbb{R}$$

- Koopman operator U propagates observables in time:

$$Ua(\mathbf{u}_k) = a(\mathbf{u}_{k+1}).$$

Linear, unitary & infinite dimensional operator

- Spectral analysis of U

$$U\varphi_j(\mathbf{u}) = \lambda_j \varphi_j(\mathbf{u})$$

Koopman eigenfunctions

- Expand observables into Koopman eigenfunctions

$$\mathbf{a}(\mathbf{u}_0) = \sum_{j=0}^{\infty} \phi_j \varphi_j(\mathbf{u}_0)$$

Koopman Modes

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Koopman Spectrum of Jet in Crossflow

- Eigenvalues on the unit circle

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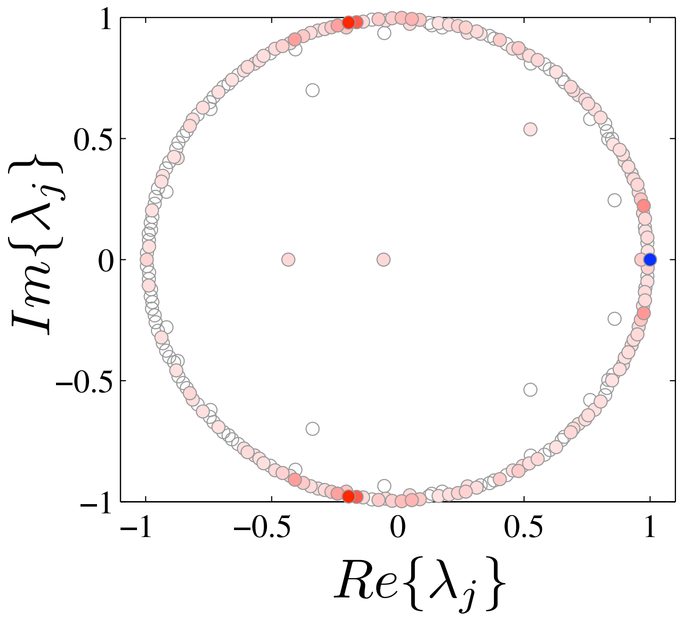

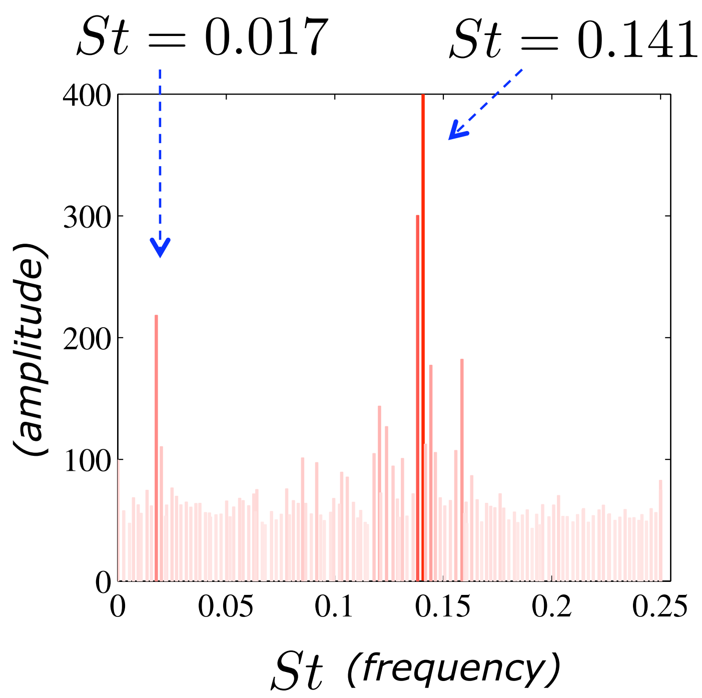

  - Dominant frequencies match vortex shedding frequencies from DNS
  - Computed using DMD (Dynamic Mode Decomposition) *(Schmid 2010)*

Koopman Modes

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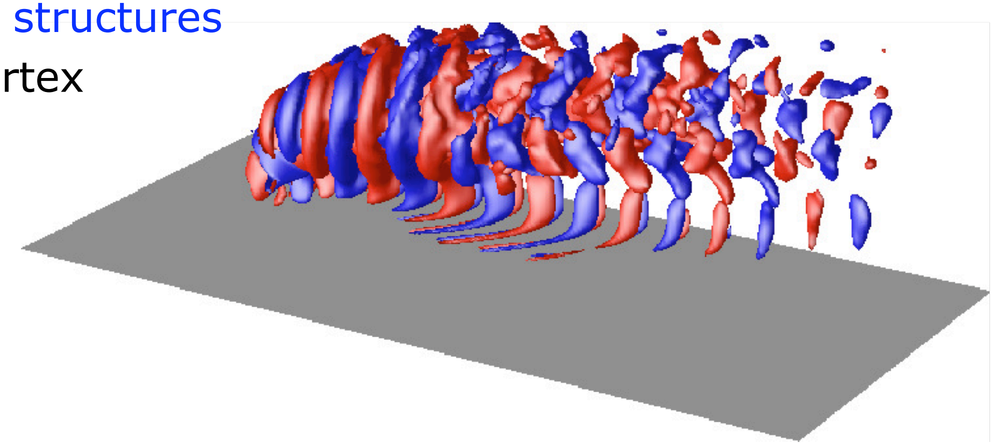
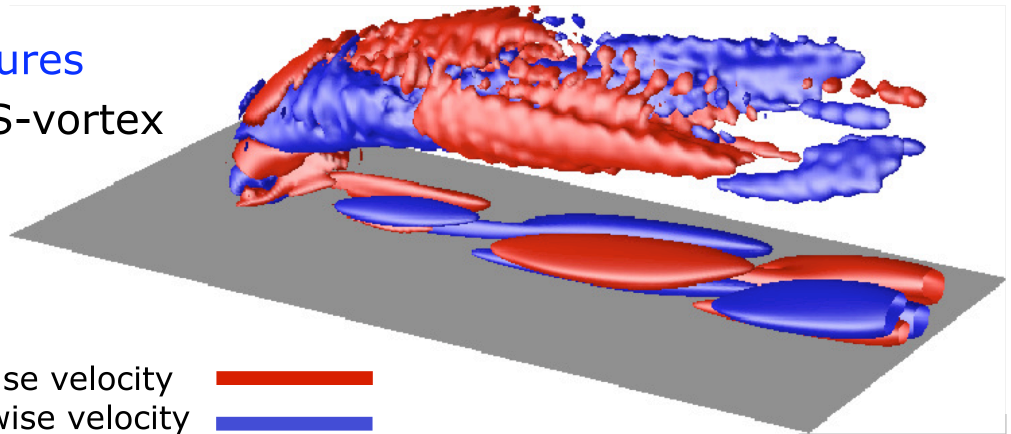
- High-frequency mode:
  - Captures shear-layer structures
  - Matches first DNS-vortex shedding

$$St = 0.141$$



- Low-frequency mode
  - Captures wall structures
  - Matches second DNS-vortex shedding

$$St = 0.017$$



Positive streamwise velocity
Negative streamwise velocity

Summary of Part I

- Decomposition of unsteady flow into global modes
  - Global linear eigenmodes
  - Koopman modes

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- Identified three elementary instability mechanisms
  - Kelvin-Helmholtz instability
  - Short-wavelength instability of a vortex pair
  - von Kàrmàn vortex street

- Identified flow structures associated with vortex shedding
  - Wall mode oscillating with low frequency
  - Jet mode oscillating with high frequency

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Part II
Flow Control

Flow on an Airplane Wing

- Friction Drag on surface smaller for laminar than turbulent flows
- Delay the transition to turbulence to save fuel

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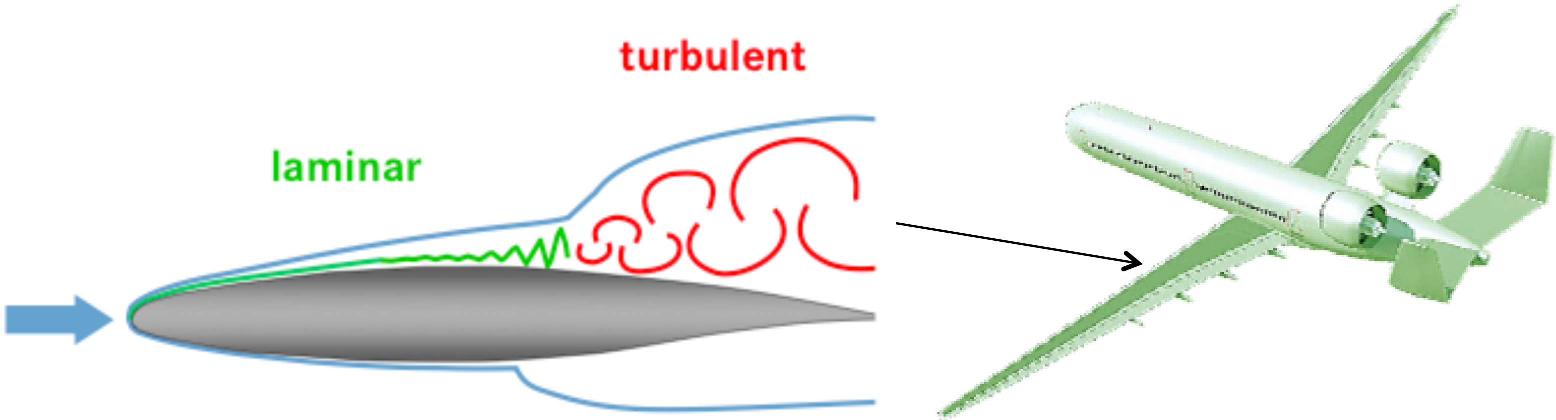
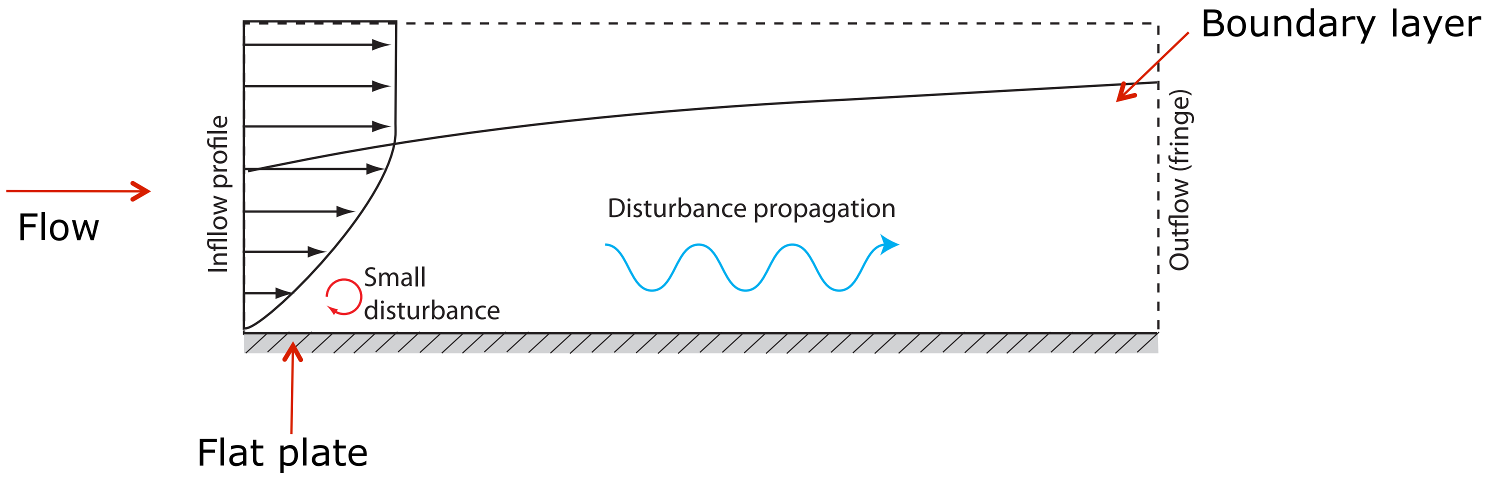


Flow on a Flat Plate

- Simplified geometry
  - flow on a flat plate (downstream of leading edge)



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- Direct numerical simulations
  - Re=1000 & 10 million grid points

The 3 Stages of Laminar—Turbulent Transition

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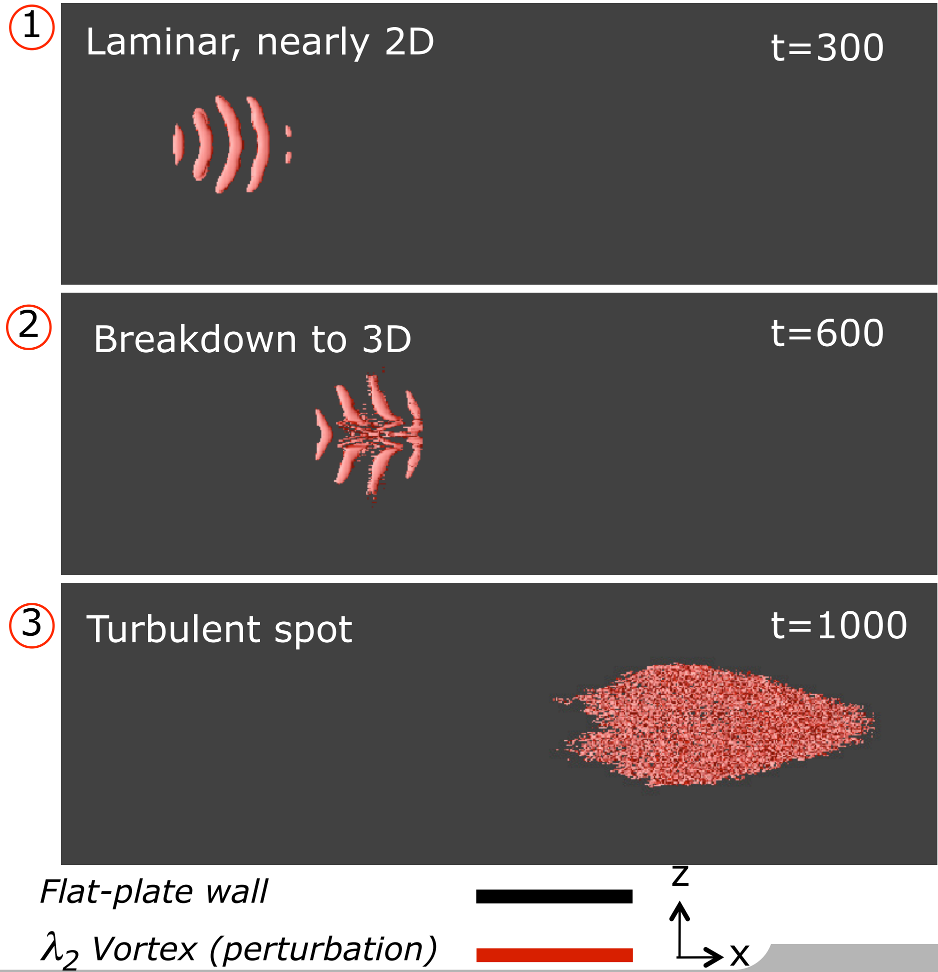
- 1st stage is linear growth of perturbations

- 2nd and 3rd stages are nonlinear process’



Introduce Actuators & Sensors

- Focus on first stage of transition process
  - can use linearized system

- Control formulation:

$$\dot{\mathbf{u}} = \mathbf{A}\mathbf{u} + \mathbf{B}\mathbf{w}$$
$$\mathbf{y} = \mathbf{C}\mathbf{u}$$

Input signals (disturbances & actuators) → $\mathbf{B}\mathbf{w}$

Output signals (sensor & objective function) → $\mathbf{y}$

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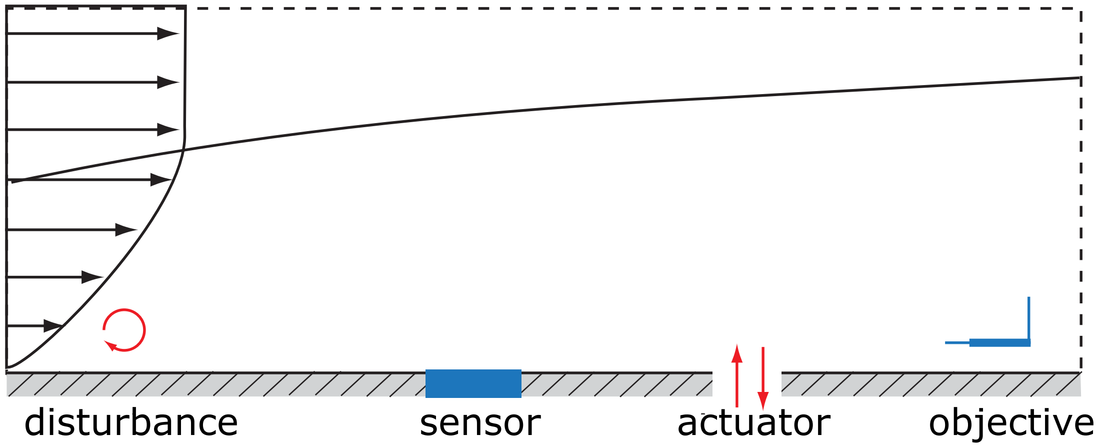

Control Design Issues

- How to:
  - Connect sensors to actuators?
  - What should the actuator do, when we have measurements?
  - Are there guarantees of stability, performance & robustness?
- Answer: Linear control theory
- Problem: tools too expensive for 2D or 3D computational fluid dynamics

  → Model reduction

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Control Design: Two Steps

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1. Develop a low-dimensional model that captures the input-output behavior of high-dimensional Navier-Stokes system
2. Use the low-dimensional model to construct a controller

Papers: 1, 2, 3 & 4

Capturing Input-Output Behavior

- For a given input signal, what is the output?
- Introduce a mapping between inputs to outputs:

$$\mathbf{G} : \mathbf{w} \rightarrow \mathbf{y}$$

  – Complexity order of millions (due to discretization of N-S)

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- How to construct an approximation

$$\mathbf{G}_r : \mathbf{w} \rightarrow \mathbf{y}$$

such that

  – complexity is of order 10-100
  – norm $\|\mathbf{G} - \mathbf{G}_r\|$ is small

Controllability & Observability

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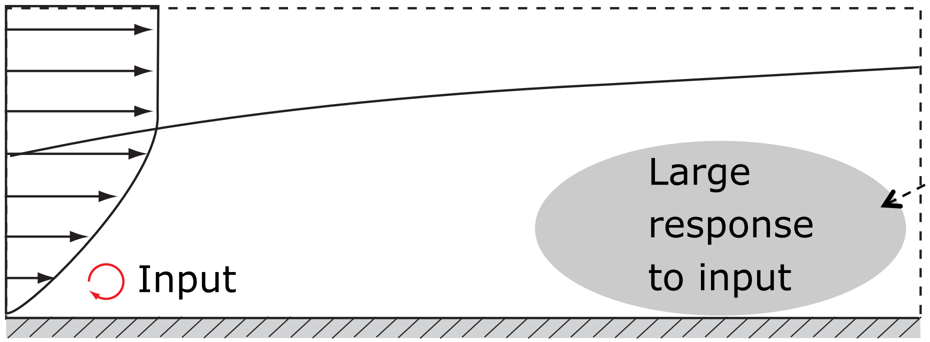
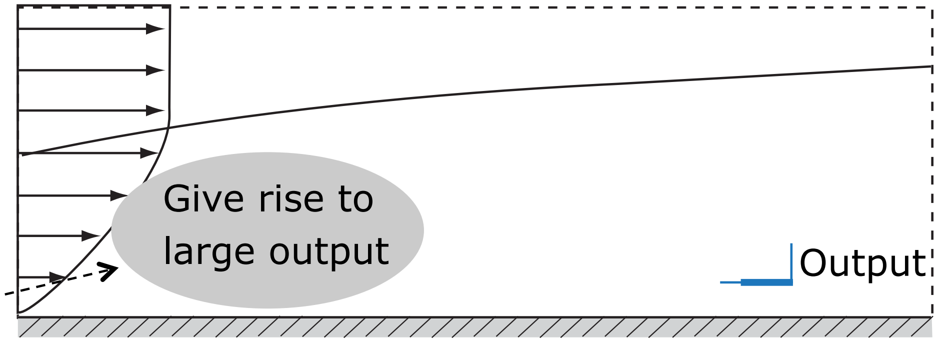
- Which flow structures respond to input forcing?



Strong controllability

- Which flow structures generate large output energy?



Strong observability

Balanced Modes

- Controllability Gramian determines controllable structures

$$\mathbf{P} = \int_0^\infty e^{\mathbf{A}t}\mathbf{B}\mathbf{B}^T e^{\mathbf{A}^T t} dt,$$

- Observability Gramian determines observable structures

$$\mathbf{Q} = \int_0^\infty e^{\mathbf{A}^T t}\mathbf{C}^T\mathbf{C}e^{\mathbf{A}t} dt$$

- Balanced modes are eigenmodes of

$$\mathbf{P}\mathbf{Q}\phi_j = \lambda_j \phi_j$$ <------ Balanced modes

  - For 2D/3D flows modes computed using the snapshot method *(Rowley, 2005)*

- Reduced model obtained by projection onto balanced modes

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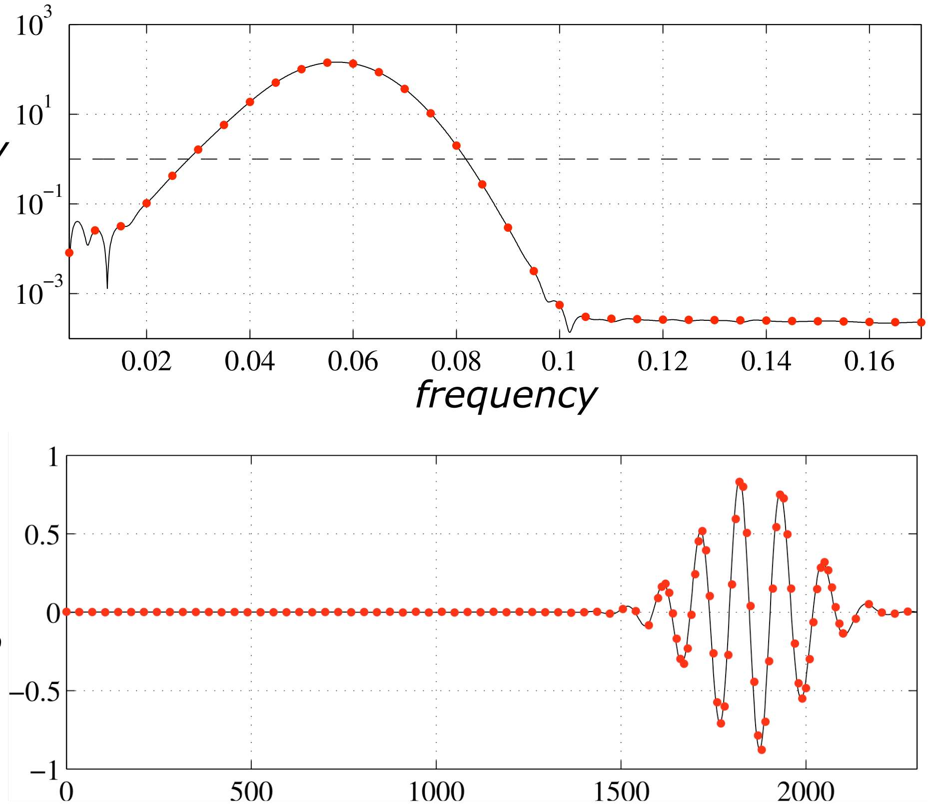
Validation of Reduced-Order Model



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- Reduced-order model & Navier-Stokes show same input-output behavior

Two Steps

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1. Develop a low-dimensional model that captures the input-output behavior of high-dimensional Navier-Stokes system
2. Use the low-dimensional model to construct a controller

Papers: 1, 2, 3 & 4

Control Design

- Linear quadratic Gaussian (LQG)
  - Based on noisy sensor measurements, find control signal that minimize effects of disturbances in a subdomain

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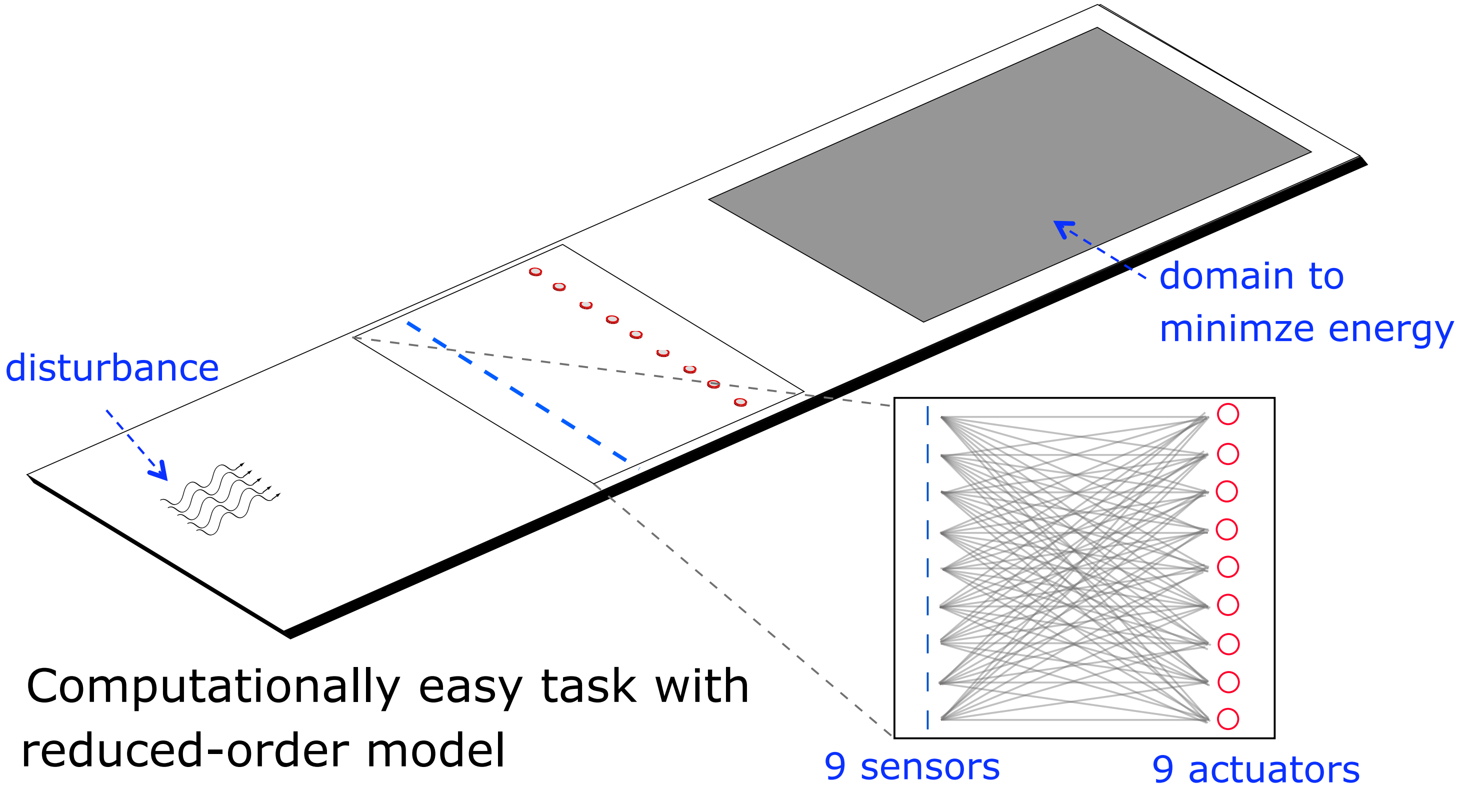

- Computationally easy task with reduced-order model

Controlled Flow

- Disturbance energy reduced orders of magnitude

  Using 9 small sensors & 9 actuators

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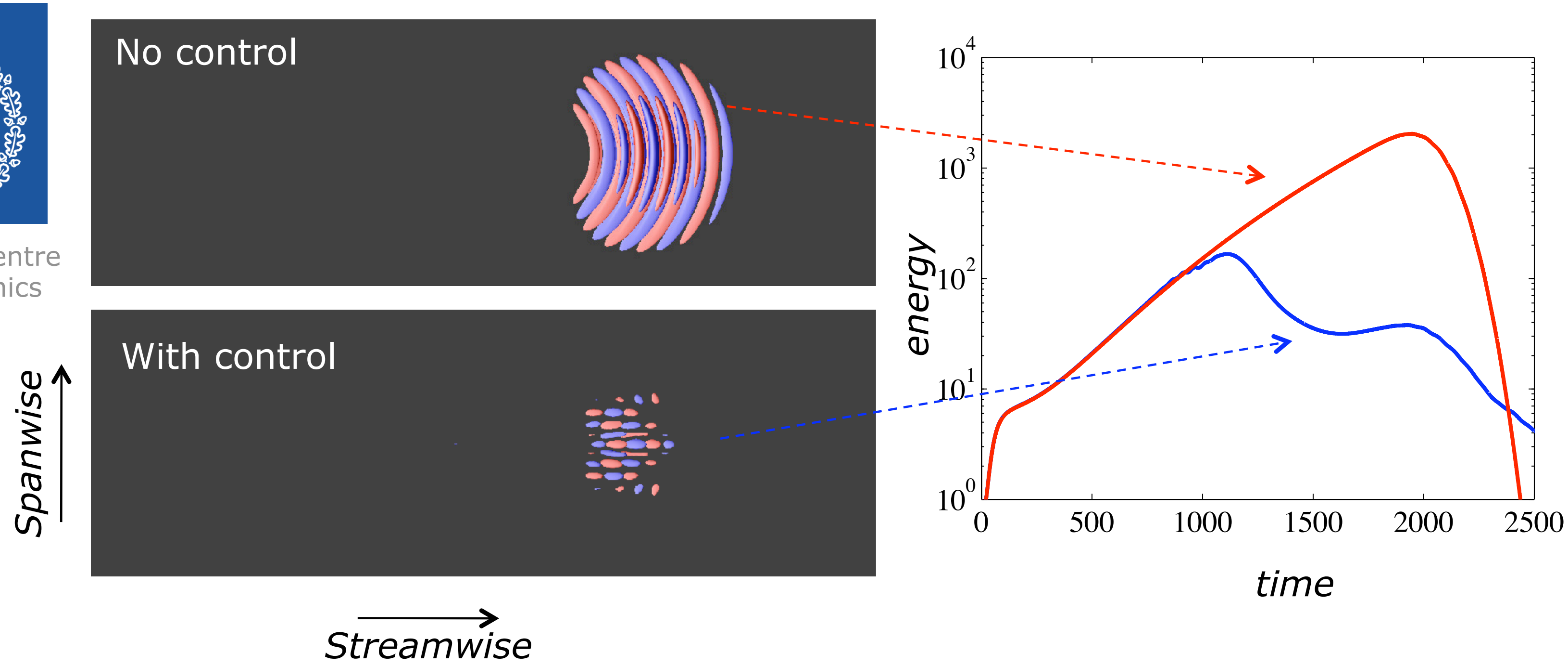

Positive u
Negative u

Summary of Part II

- Using control theory we are able to account for
  - measurement noise
  - control penalty
  - optimality and robustness

- Balanced modes
  - take into account controllability & observability
  - able to capture input-output behavior of Navier-Stokes eqs

- Disturbance energy can be reduced by orders of magnitude using localized sensing/acting

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Future Directions

- Flow analysis of the Jet in Crossflow:
  - *Sensitivity analysis*: identifty locations where steady flow is sensitive to external modifications
  - *Bifurcation analysis:* stability properties depend on the velocity ratio
- Flow control of the flat-plate flow:
  - *Delay transition:* validate numerically using low-order controller
  - *Wind-tunnel experiments:* Use the low-order controller to delay transition

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