Spontaneous Symmetry Breaking of Hinged Flapping Filament Generates Lift

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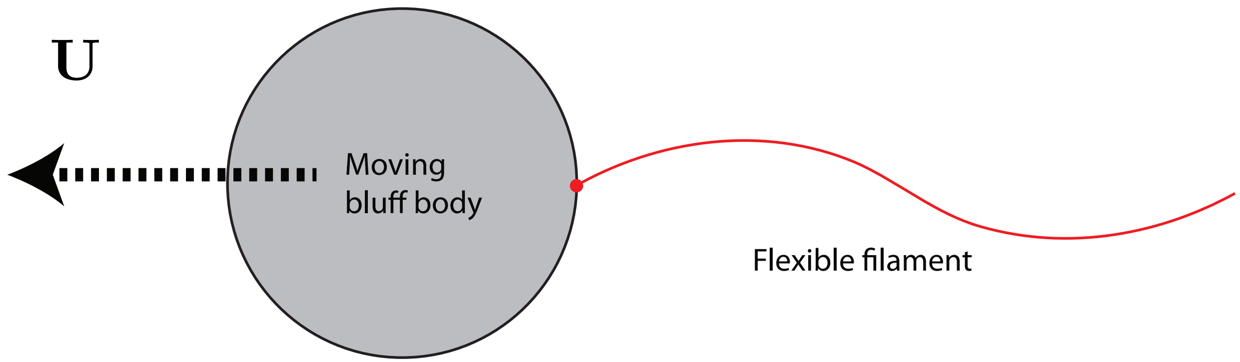
Bio-Inspired Flow Control

How does non-smooth flexible surfaces, appendages affect moving bodies?

Configuration

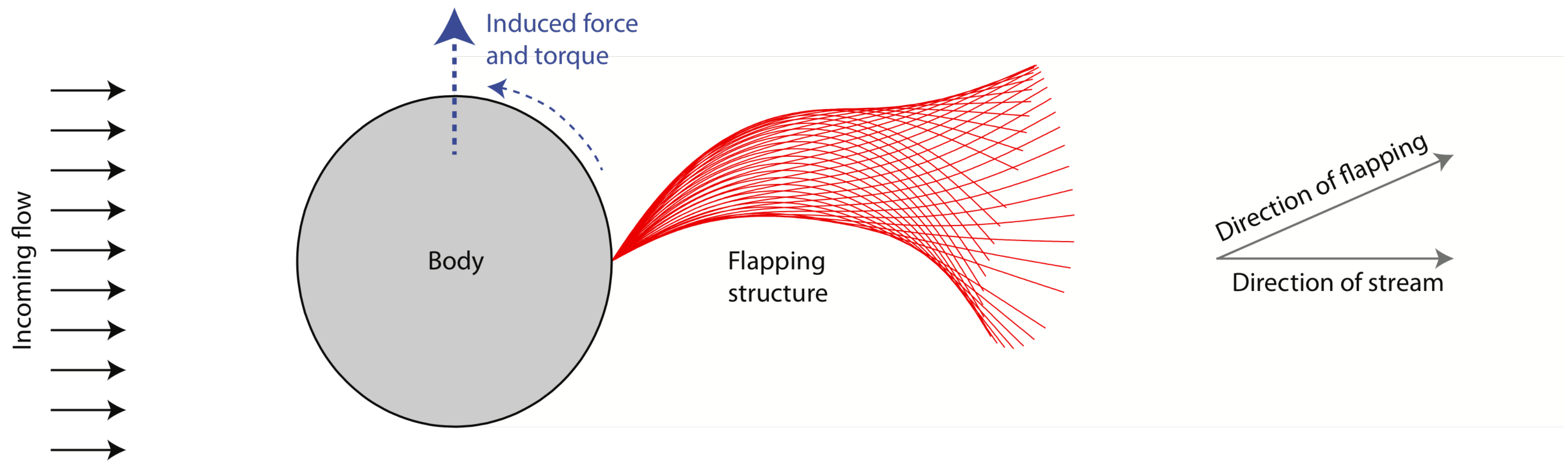
- A moving body with a hinged flexible filament



- How does the filament
  - interact with the fluid?
  - modify the motion of the body?

Symmetry Breaking

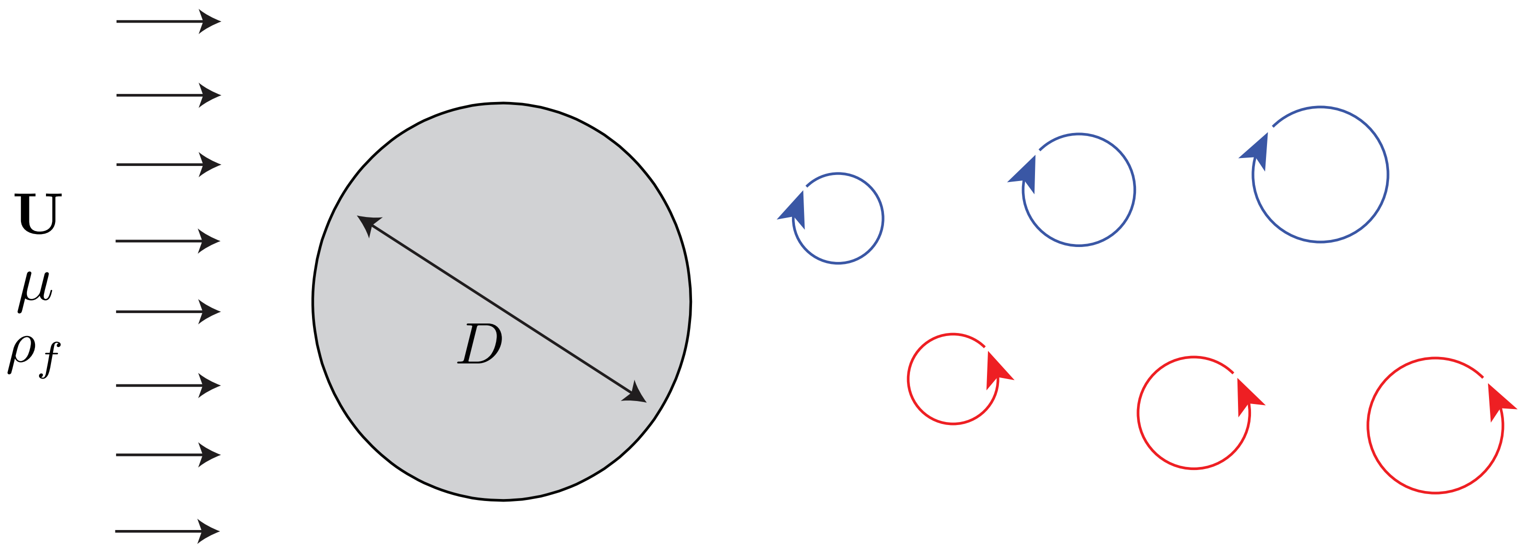
- Filament flaps asymmetrically
  - → a net force/torque on body
  - → reduced drag on body



Flow Past Body

– Reynolds number $Re = \dfrac{\mathbf{U} D \rho_f}{\mu}$

– Vortex shedding for $Re > Re_c$ with frequency f_c



(Williamson, *Ann. Rev. Fluid Mech. 1996)*

Flow Past Filament

- Reynolds number

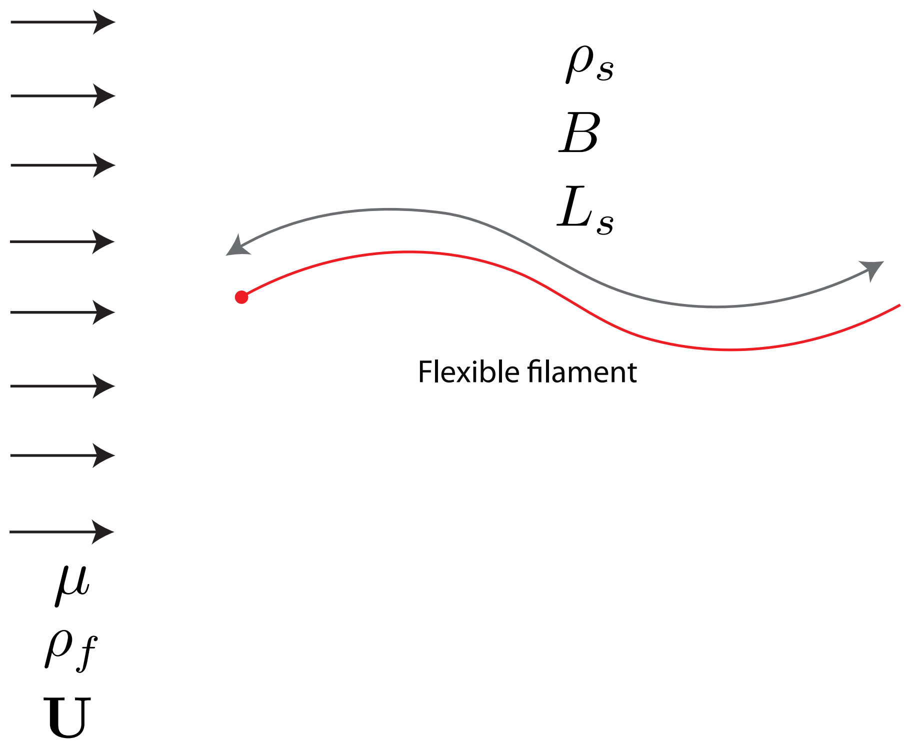
$$Re = \frac{\mathbf{U} L_s \rho_f}{\mu}$$

- mass

$$R_1 = \frac{\rho_s}{\rho_f L_s}$$

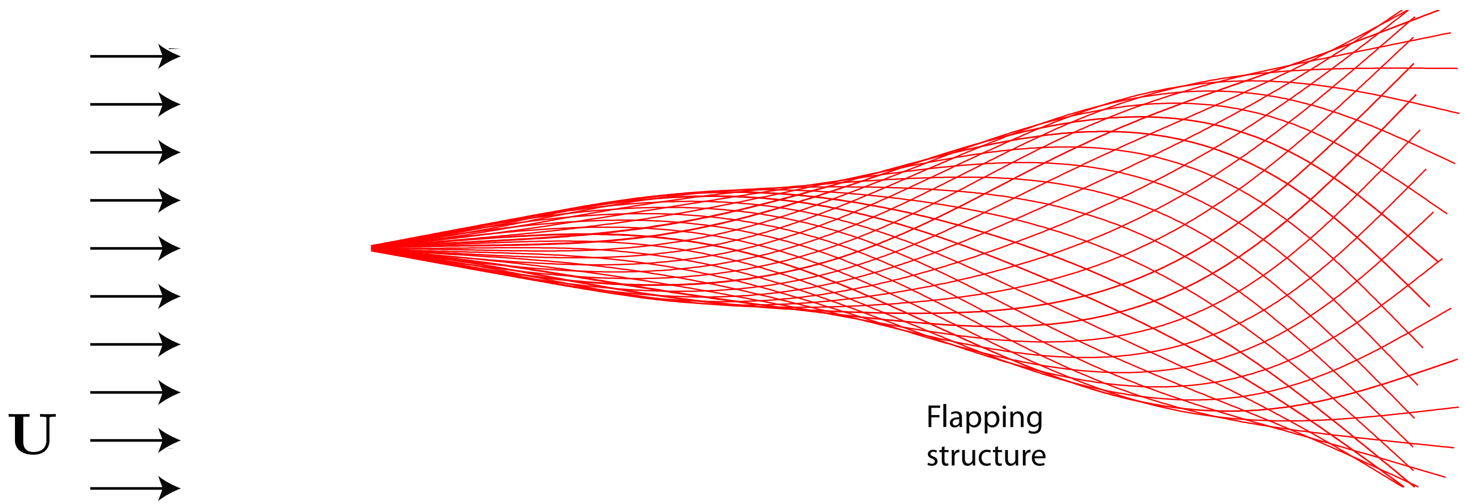
- rigidity

$$R_2 = \frac{B}{\rho_f U^2 L_s^3}$$



Flow Past Filament

- Flapping when $Re > 10^3$ $R_1 > 0$ $R_2 < R_{2,c}$

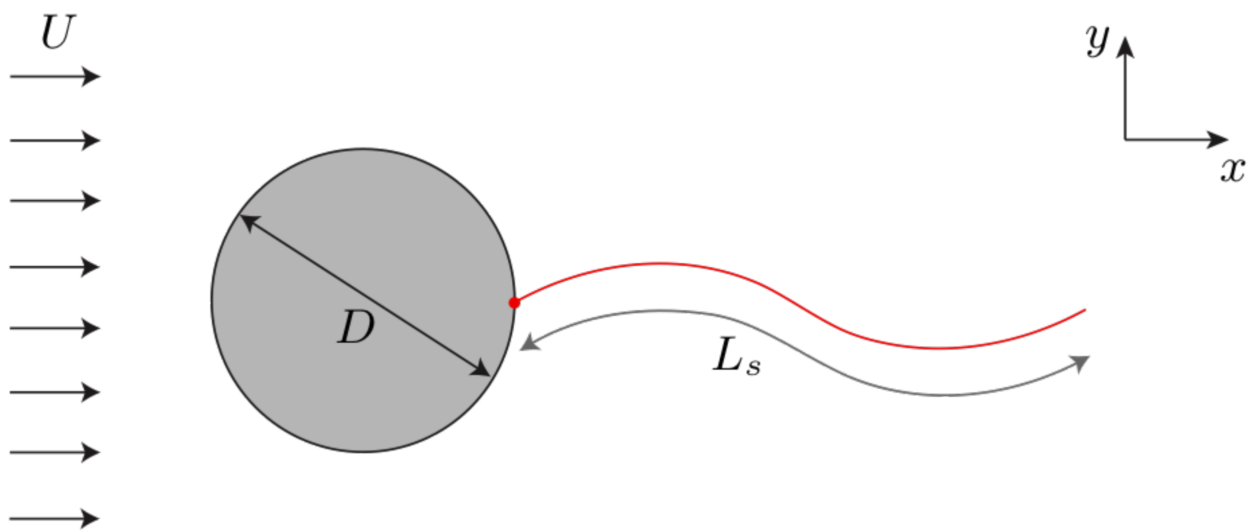


(Zhang & Shelley, *Ann. Rev. Fluid Mech. 2012)*

Numerical Treatment

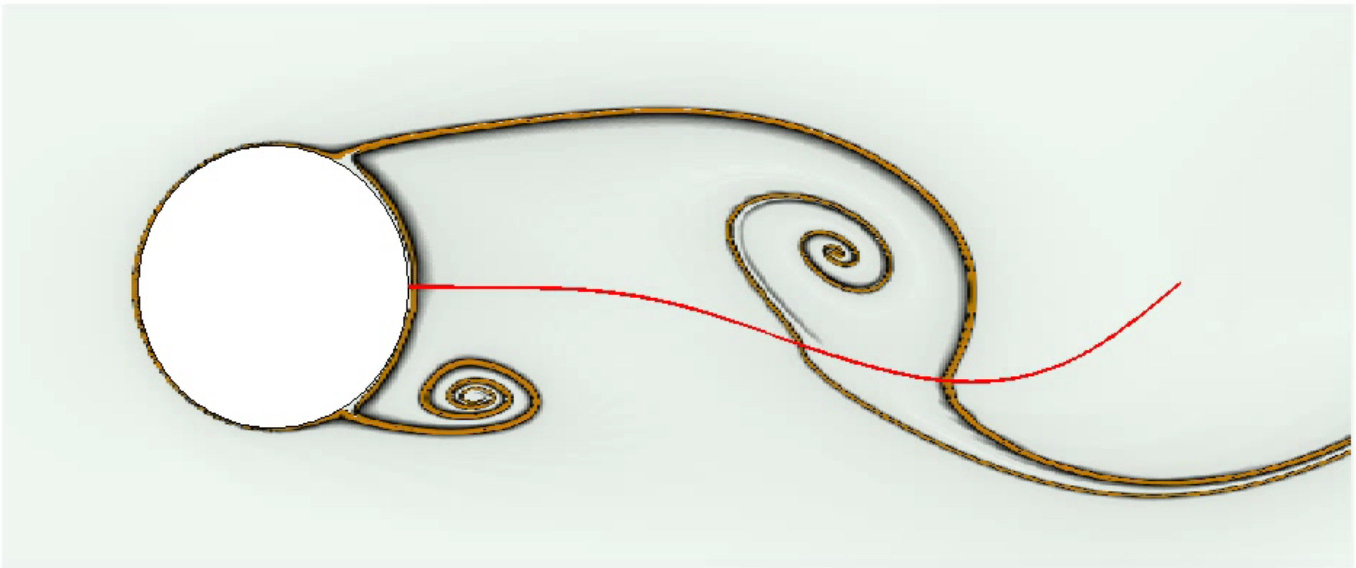
- Flow dynamics (Navier-Stokes)
- Filament dynamics (Euler-Bernoulli Beam)
- 4 parameters

$$L = \frac{L_s}{D}, \quad Re = \frac{UD}{\nu} \quad R_1 = \frac{\rho_s}{\rho_f D} \quad R_2 = \frac{B}{\rho_f U^2 D^3},$$



Long Filament

time = 264.55   L = 3.00



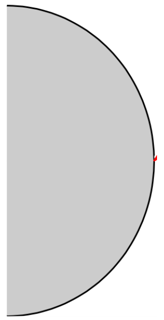
$Re = 100 \quad R_2 = 0.05 \quad R_1 = 0.1 \quad L = 3$

Short Filament

$Re = 100 \quad R_2 = 0.05 \quad R_1 = 0.1 \quad L = 1.5$

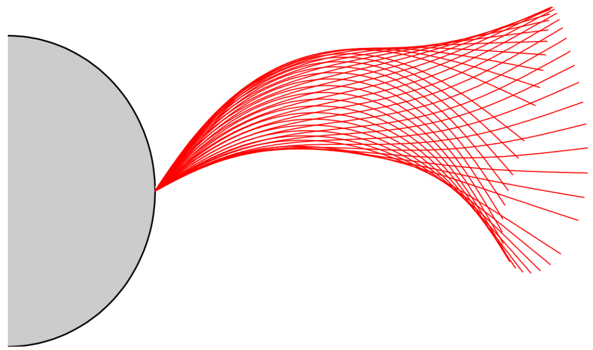
Symmetry Breaking

$L = 0$



$$\langle C_D \rangle = 1.36 \quad \text{(drag)}$$
$$\langle C_L \rangle = 0 \quad \text{(lift)}$$
$$\langle C_q \rangle = 0 \quad \text{(torque)}$$

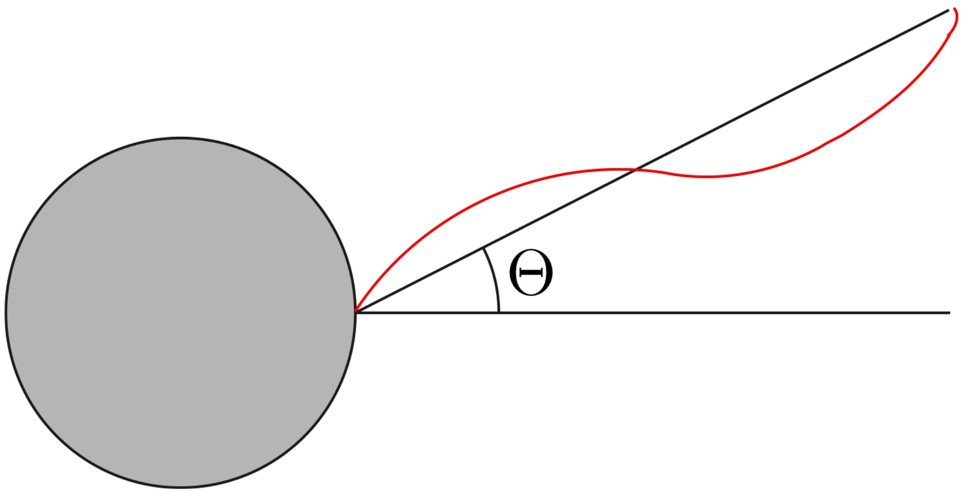
$L = 1.5$



$$\langle C_D \rangle = 1.32 \quad \text{(drag)}$$
$$\langle C_L \rangle = 0.18 \quad \text{(lift)}$$
$$\langle C_q \rangle = 0.01 \quad \text{(torque)}$$

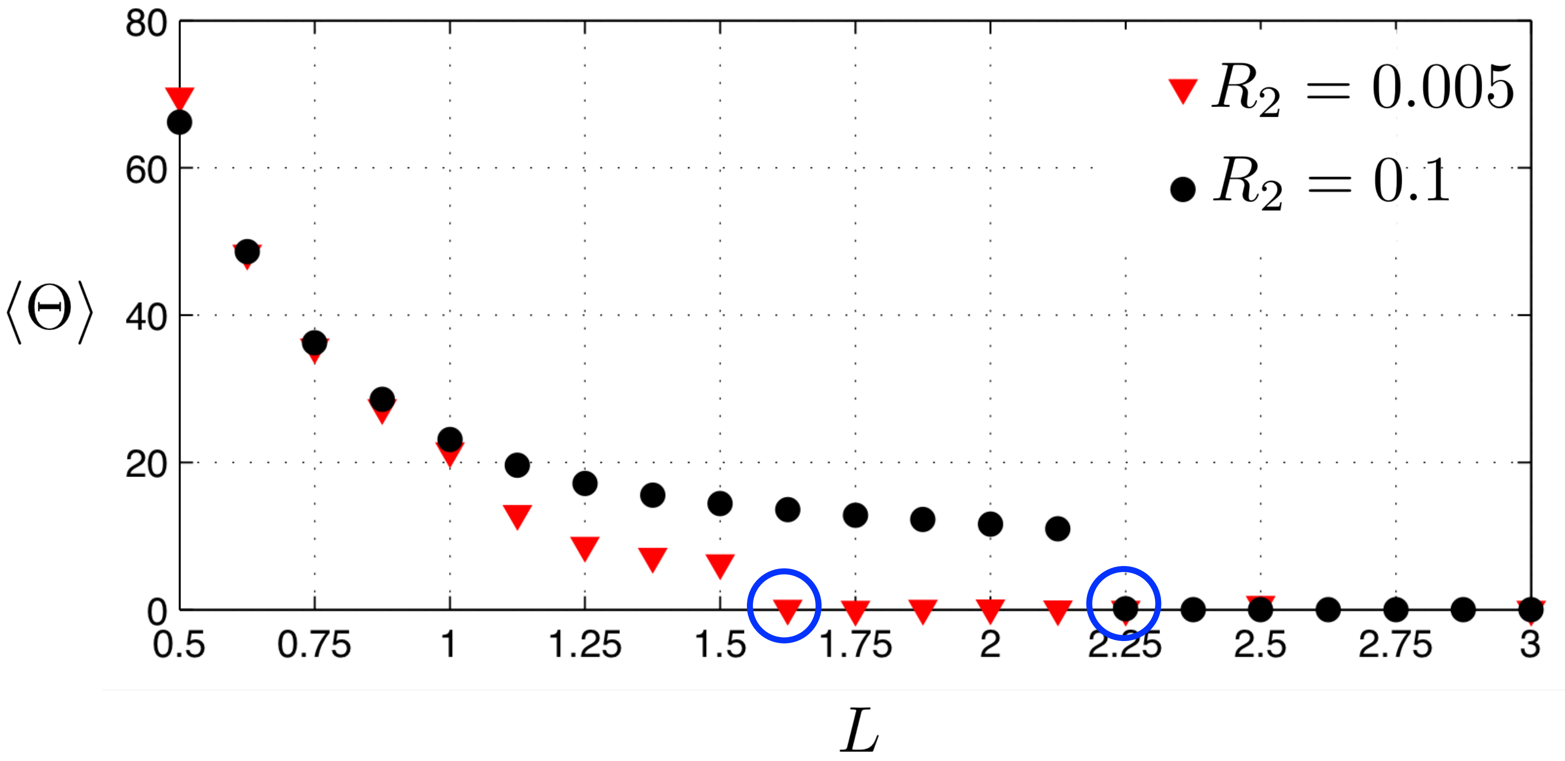
Choice of Observable

- Angle of horizontal line & line connecting filament tail



- Consider 2 cases
  - Rigid filament: $R_2 = 0.005$
  - Flexible filament: $R_2 = 0.1$

Bifurcation



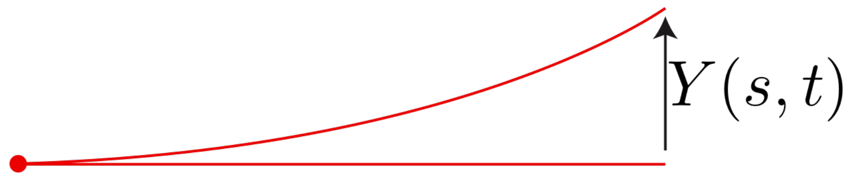
- Bifurcation: $L_c = 1.6$ (flexible filament)

  $L_c = 2.25$ (rigid filament)

Beam Equation

- Equation governing unforced beam

$$R_1 Y_{tt} + R_2 Y_{ssss} = 0$$



- Eigenfrequency

$$f_s = \sqrt{\frac{R_2}{R_1 L^4}}$$

Resonance Condition

- Free vibrations of filament f_s
- Vortex shedding frequency f_c
- If $f_s \ll f_c$ filament very slow reaction time
- If $f_s \gg f_c$ filament react instantaneously
- Thus $f_s \sim f_c$ separates two different regimes
- Gives resonance condition: $$L_r = \left(\frac{R_2}{R_1 f_c^2} \right)^{1/4}$$

Filament Energy

– Energy $$E = \frac{1}{2}\int_0^L R_1|\mathbf{X}_t|^2 + R_2|\mathbf{X}_{ss}|^2 \; ds$$

– Rescaled with filament density and length

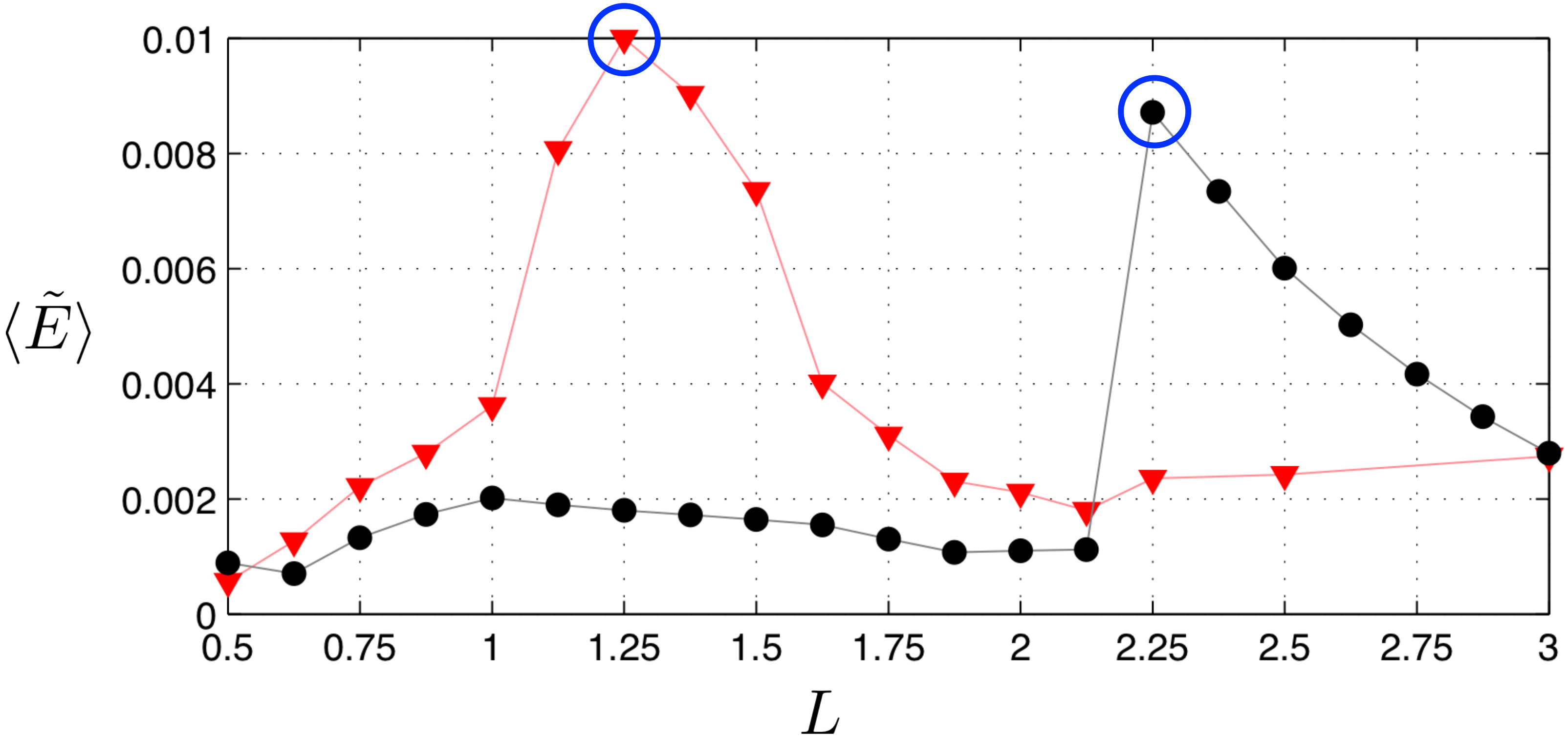
$$(\rho_f, D) \to (\rho_s, L_s)$$

– Flapping synchronized with vortex shedding, time scale

$$U/D$$

→ rescaled non-dimensional filament energy $$\tilde{E} = \frac{R_1}{L^3}E$$

Resonance



- Resonance: $L = 1.25$ (flexible)
  $L = 2.25$ (rigid)

Resonance

	Resonance (theoretical)	Resonance (computed)	Bifurcation (computed)
Flexible	1.25	1.25	1.6
Rigid	2.6	2.25	2.25

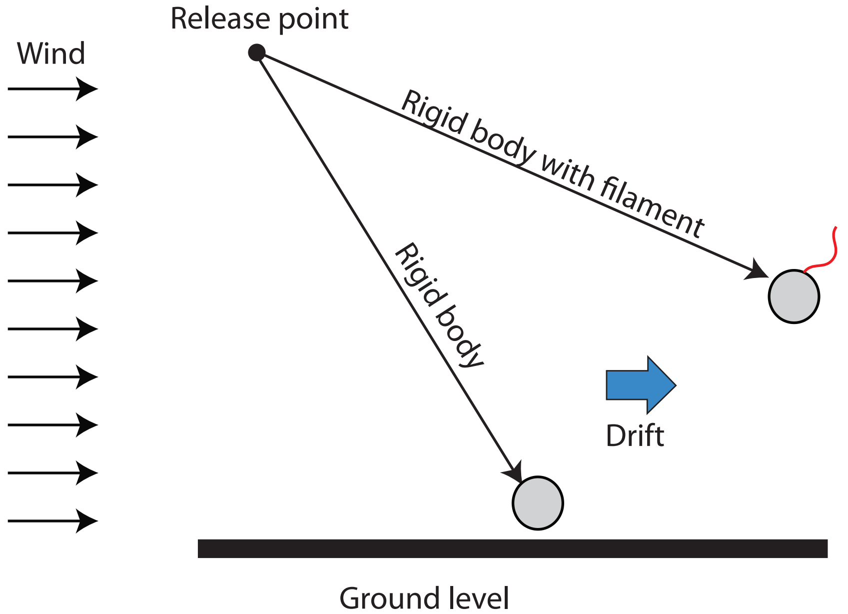
Can Filament Alter Motion?

- Swimming sea slug
  - flapping of wings (Re>10)
  - beating of cilia (Re<1)
- Inert cilia alter motion
  - interaction with fluid
  - without energy expended

(Childress, & Dudley, *JFM* 2004)

Can Filament Increase Drift?

- Efficient wind-borne seed dispersal
  - Side force due to symmetry breaking may increase drift



(Burrows, *New Phytol*. 1975)

Thank you!

Reference:

Bagheri, Mazzino & Bottaro, *PRL, 109, 2012*

See also:

Lisa Zyga, *PhysORG*, 22nd Oct

(http://phys.org/news/2012-10-symmetry.html#ajTabs)

Outline

- General physics of
  - flow past a cylinder
  - flow past a filament
- Symmetry breaking of cylinder + filament
  - resonance between fluid & structure
  - generation of net lift, torque
- Immersed boundary method
- Conlusion & outlook

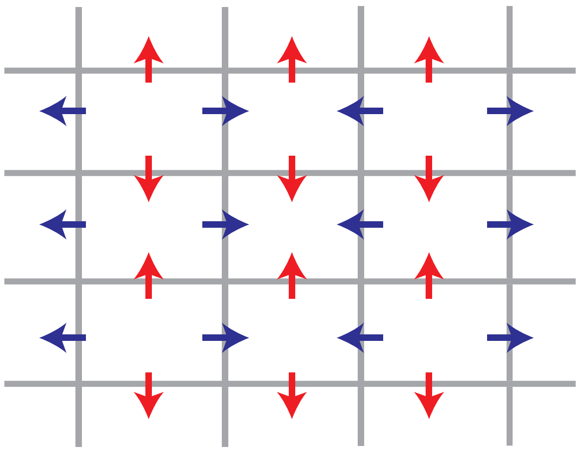
Fluid Equations

- Viscous incompressible fluid

Momentum $$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}$$

Continuity $$\nabla \cdot \mathbf{u} = 0$$

- Flow solver
  - Discretize on Cartesian grid
  - No dynamic equation for pressure
  - Projection method



(Chorin, 1968 & Temam, 1969)

Flow Past Rigid Body

- Viscous incompressible fluid

Momentum

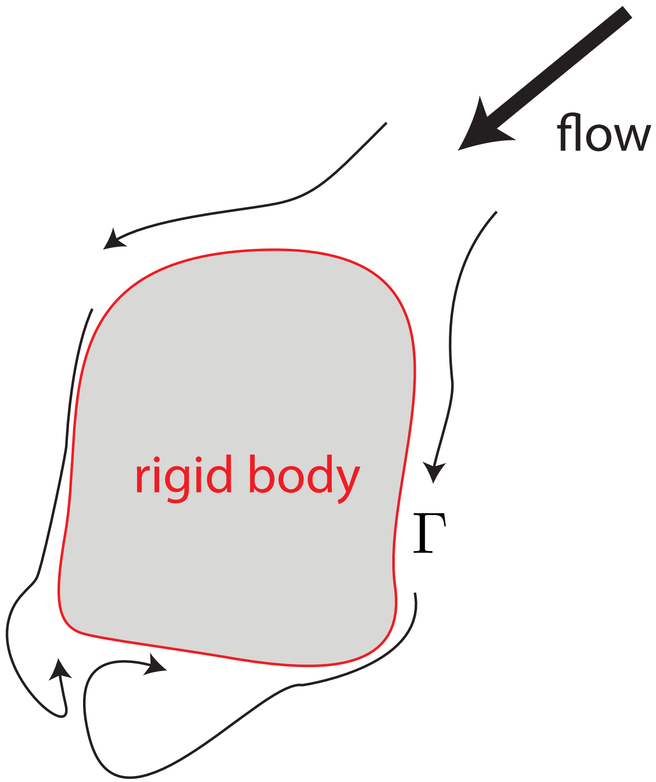
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}$$

Continuity

$$\nabla \cdot \mathbf{u} = 0$$

No-slip

$$\mathbf{u} = 0 \qquad \text{on} \quad \Gamma$$



Immersed Boundary Method

- Viscous incompressible fluid

Momentum

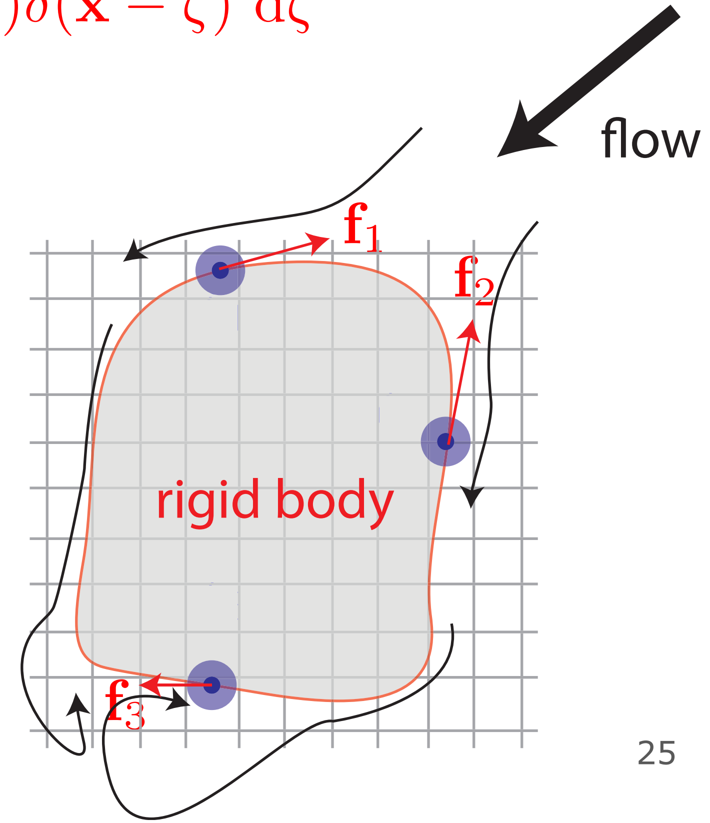
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} + \int_\Gamma \mathbf{f}(\zeta)\delta(\mathbf{x} - \zeta)\ \mathrm{d}\zeta$$

Continuity

$$\nabla \cdot \mathbf{u} = 0$$

No-slip

$$\mathbf{u} = 0 \qquad \text{on} \quad \Gamma$$



Immersed Boundary Method

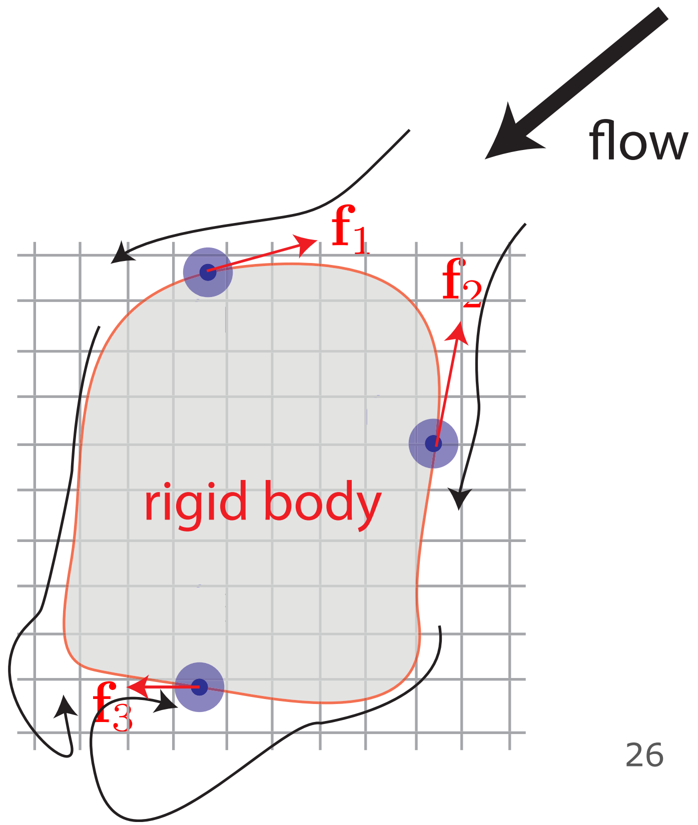
- Immersed boundary method

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} + \int_\Gamma \mathbf{f}(\zeta)\delta(\mathbf{x} - \zeta)\, \mathrm{d}\zeta$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u} = 0 \qquad \text{on} \quad \Gamma$$

  - Flow field: Eulerian (Cartesian grid)
  - Boundary: Lagrangian points
  - Boundary force to enforce no-slip
  - Projection method



(Taira & Colonius, *JCP*, 2005)

Flow Past Flexible Filament

- Viscous incompressible fluid

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} + \int_\Gamma \mathbf{f}(\zeta)\delta(\mathbf{x} - \zeta)\, \mathrm{d}\zeta$$

$$\nabla \cdot \mathbf{u} = 0$$

No-slip

$$\mathbf{u}(\Gamma) = \dot{\zeta}$$

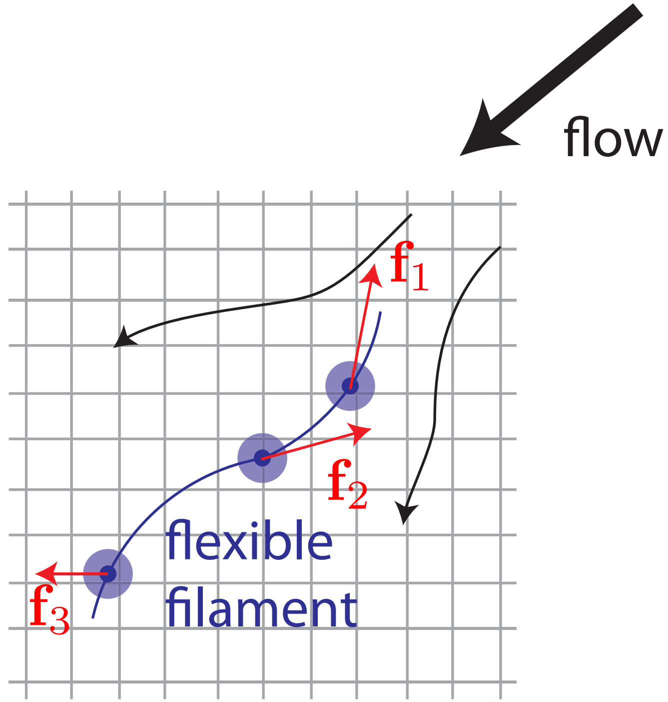
Filament dynamics

$$\rho_s \ddot{\zeta} = \partial(T\hat{\tau}) - B\partial^2(C\hat{n}) + \mathbf{f}$$

Inertia

Tensile force

Bending force



(Peskin, 1997, 2002, Kim & Peskin 2007)

Current Work

Problems to be tackled:

1. Free falling bluff body with filament
2. Interaction among particles with filament
3. Bodies with distributed, anisotropic coatings

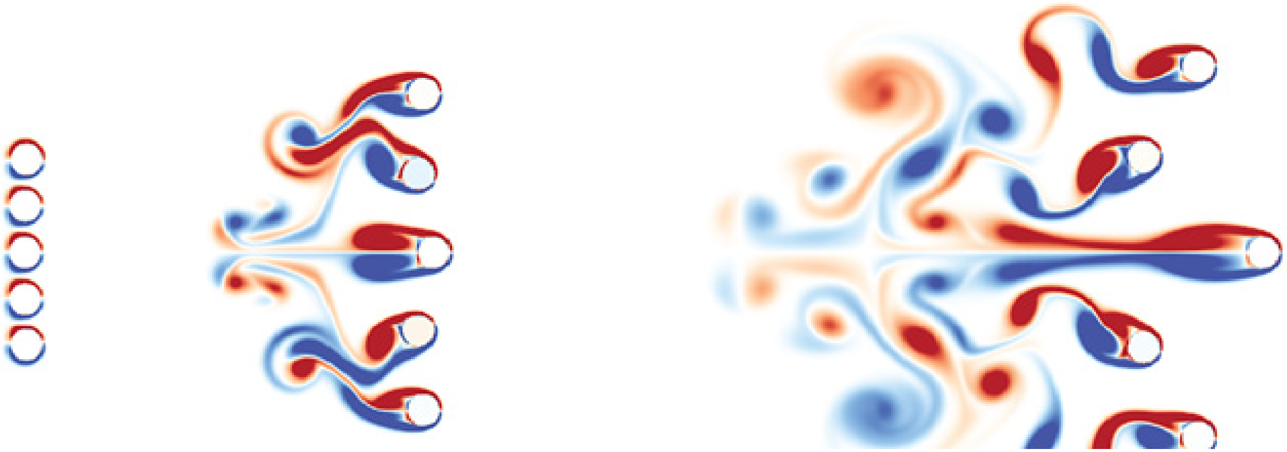
Approach:

1. Numerical (Lagrangian methods)
2. Experimental (soap film experiments)
3. Theoretical (stability/bifurcation/resonance analyses)

FSI for Multiple Moving/Flexible Bodies

Developing direct numerical simulation of fluid/structure

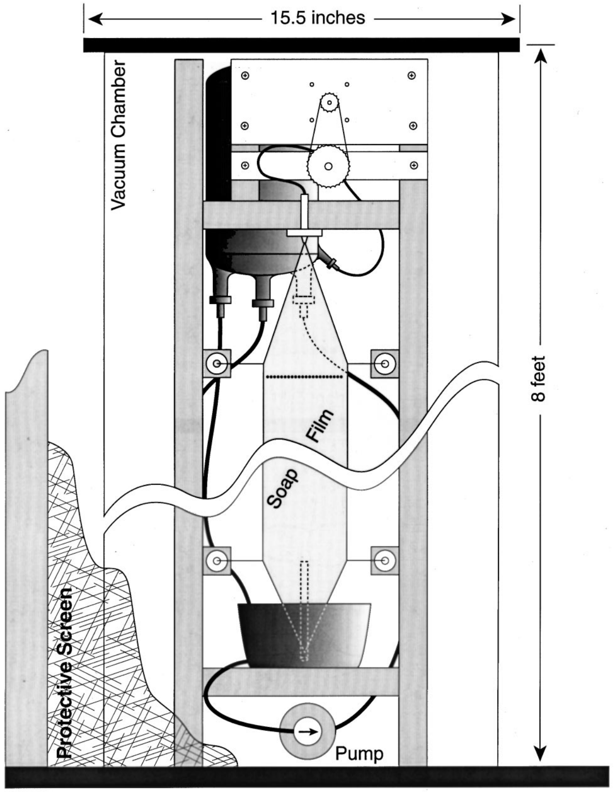
→ combination of vortex methods and immersed boundary methods



(Gazzola et al, *JCP, 2011)*

Soap Film Experiments

- Developing experimental facilities for fluid/structure
  - → soap film, water tank et



(Zhang etal, *Nature*, 2000*)* (Rutgers etal, *Rev. Sci. Inst*. 2001)

Discretization of Fluid Equations

- Viscous incompressible fluid

Momentum $$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}$$

Continuity $$\nabla \cdot \mathbf{u} = 0$$

- Discretize (Adams-Bashforth+Crank-Nicolson)

Momentum $$\frac{u^{n+1} - u^n}{\Delta t} + \frac{3}{2} N(u^n) - \frac{1}{2} N(u^{n-1}) = -G p^{n+1} + \frac{1}{2Re} L(u^{n+1} + u^n)$$

Continuity $$D u^{n+1} = 0$$

Algebraic system

- Algebraic system
  - Linear system

$$\begin{bmatrix} A & G \\ D & 0 \end{bmatrix} \begin{bmatrix} u^{n+1} \\ p \end{bmatrix} = \begin{bmatrix} r^n \\ 0 \end{bmatrix}$$

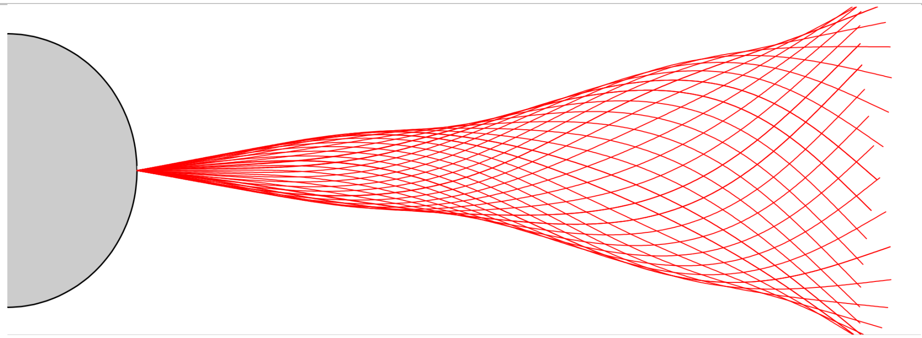
  - LU Factorization

$$\begin{bmatrix} A & G \\ D & 0 \end{bmatrix} = \begin{bmatrix} A & 0 \\ D & -DA^{-1}G \end{bmatrix} \begin{bmatrix} I & A^{-1}G \\ 0 & I \end{bmatrix}$$

  - Projection/Fractional step method
    - Momentum $Au^* = r^n$
    - Pressure Poisson $DA^{-1}Gp^{n+1} = Du^*$
    - Projection $u^{n+1} = u^* - A^{-1}Gp^{n+1}$

Symmetry Breaking

$L = 3$

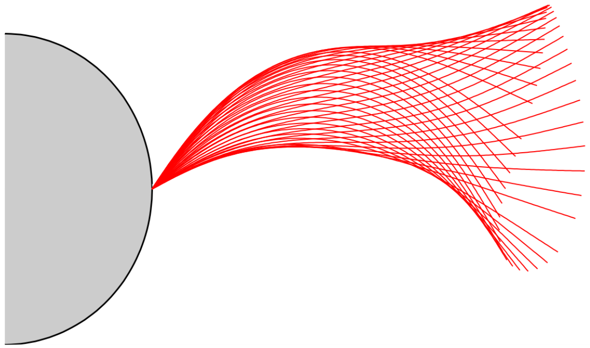


$\langle C_D \rangle = 1.28$ (drag)

$\langle C_L \rangle = 0$ (lift)

$\langle C_q \rangle = 0$ (torque)

$L = 1.5$

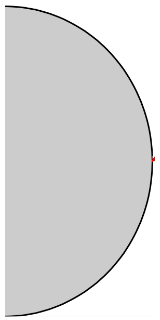


$\langle C_D \rangle = 1.32$

$\langle C_L \rangle = 0.18$

$\langle C_q \rangle = 0.01$

$L = 0$



$\langle C_D \rangle = 1.36$

$\langle C_L \rangle = 0$

$\langle C_q \rangle = 0$